

## COURSE OUTLINE

### (1) GENERAL

<b>SCHOOL</b>	SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY		
<b>ACADEMIC UNIT</b>	DEPARTMENT OF STATISTICS		
<b>LEVEL OF STUDIES</b>	1st Cycle (UNDERGRADUATE)		
<b>COURSE CODE</b>	6256	<b>SEMESTER</b>	8 <sup>th</sup>
<b>COURSE TITLE</b>	<b>Special Topics in Statistics and Probability (STSP): Introduction to Measurement Theory with reference to Probability and Statistics</b>		
<b>INDEPENDENT TEACHING ACTIVITIES</b>		<b>WEEKLY TEACHING HOURS</b>	<b>CREDITS</b>
Lectures		4	7
<b>COURSE TYPE</b>	Elective		
<b>PREREQUISITE COURSES:</b>			
<b>LANGUAGE OF INSTRUCTION and EXAMINATIONS:</b>	GREEK		
<b>IS THE COURSE OFFERED TO ERASMUS STUDENTS</b>	No		
<b>COURSE WEBSITE (URL)</b>	<a href="https://www.dept.aueb.gr/en/stat/content/introduction-measurement-theory-regard-probability-and-statistics-8-ects">https://www.dept.aueb.gr/en/stat/content/introduction-measurement-theory-regard-probability-and-statistics-8-ects</a>		

### (2) LEARNING OUTCOMES

<b>Learning outcomes</b>
After successfully attending the course students will become familiar with the basic concepts of measure theory and integration and will be able to use some of its basic tools. Thus, they will be able to approach the techniques used in the probabilities and statistics from a point of view of measurement theory, as well as the techniques of statistical/mechanical learning.
<b>General Competences</b>

### (3) SYLLABUS

--

Sets and functions. Algebra and  $\sigma$ -algebra of sets. Open, closed and solid subsets of the real numbers. Constructing the Lebesgue measure in real numbers. Measurable sets according to Borel and Lebesgue. The Cantor set and the Cantor function. Non-measurable sets according to Lebesgue.

Measurable functions according to Lebesgue. Borel Functions. Random variables. Sequences of functions and random variables and convergence concepts (almost certain, in measure).

The Lebesgue integral, construction and properties. Basic convergence theorems, (the Fatou Lemma, monotonous convergence theorem, dominated convergence theorem). Expected price. Convergence in distribution and applications in statistics (estimation, simulation, etc).

Lebesgue spaces of integrable functions and random variables and their structure as metric spaces. Holder and Minkowski inequities, the Beppo-Levi theorem and completeness. Convergence in Lebesgue spaces and applications. The case of  $L^2$ , its structure as a Hilbert space, the projection theorem and its relation to conditional mean value, bases and expansions (eg Karhunen-Loeve transform, etc.).

Product measure, construction and properties and relation to independence. Integration and product measure, Fubini theorem.

Absolute continuity and measure singularity. Hahn-Jordan decomposition. Radon-Nikodym derivation. Measure space as an extension of the functions. Applications in statistics (the conditional average value under a new prism, likelihood, extreme event simulation, consistency) in finance.

Measure space as a metric space and applications. Total change distance, Helinger distance, Kuhlback-Leibler distance (entropy), transportation distance. Applications in model selection statistical and machine learning, etc.

No prerequisites are formally required, however basic knowledge of Linear Algebra, Calculus, Probability, Statistical Inference, and Stochastic processes will be useful.

#### (4) TEACHING and LEARNING METHODS - EVALUATION

<b>DELIVERY</b> <i>Face-to-face, Distance learning, etc.</i>	Face-to-face	
<b>USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY</b>	Eclass Email Use of Computer	
<b>TEACHING METHODS</b>	<b>Activity</b>	<b>Semester workload</b>
	Class Lectures	52 hours
	Study & Analysis of the Bibliography	38 hours
	Exercise/Project	10 hours
	Course total	<b>100 hours</b>
<b>STUDENT PERFORMANCE EVALUATION</b>	Assignments. Written exam at the end of the semester and/or Project with presentation.	

#### (5) ATTACHED BIBLIOGRAPHY

- Athreya, Krishna B., and Soumendra N. Lahiri. Measure theory and probability theory. Springer Science & Business Media, 2006.
- Billingsley, P. 2008. Probability and measure. John Wiley & Sons.
- Capinski, M., & Kopp, E., (2003). Measure, Integral and Probability. Springer-Verlag.
- Jacod, J., & Protter, P. E. (2003). Probability essentials. Springer Science & Business Media.
- Καλπαζίδου, Σ. (2002). Στοιχεία μετροθεωρίας πιθανοτήτων. Εκδόσεις ΖΗΤΗ.
- Professor's Lecture Notes