COURSE OUTLINE

(1) GENERAL

SCHOOL	SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY		
ACADEMIC UNIT	DEPARTMENT OF STATISTICS		
LEVEL OF STUDIES	1st Cycle (UNDERGRADUATE)		
COURSE CODE	**6116**	**SEMESTER**	**6th**
COURSE TITLE	**Probability theory**		

INDEPENDENT TEACHING ACTIVITIES	WEEKLY TEACHING HOURS	CREDITS
Lectures	4	7
Tutorials	2	
Labs		

COURSE TYPE	Elective – Scientific Field
PREREQUISITE COURSES:	Probability I, Probability II, Calculus I, Calculus II, Introduction to Mathematical Analysis
LANGUAGE OF INSTRUCTION and EXAMINATIONS:	GREEK
IS THE COURSE OFFERED TO ERASMUS STUDENTS	NO
COURSE WEBSITE (URL)	https://www.dept.aueb.gr/en/stat/content/probability-theory-8-ects

(2) LEARNING OUTCOMES

Learning outcomes

Upon successful completion of the course, students should be able to: determine the probability space of a random experiment with uncountable sample space according to the Lebesgue - Caratheodory extension theorem, to apply advanced probability calculus according to Kolmogorov's axioms, manage random variables as measurable mappings of a given probability space to the Borel line, determine the type of a random variable according to its probability distribution induced on the Borel line (discrete, continuous, mixed), calculate its expected (or mean) value as a Lebesgue integral on the Borel line, to distinguish and verify modes of stochastic convergence of a given sequence of random variables, to apply the laws of large numbers and the central limit theorem.

General Competences

- Search, analysis and synthesis of data and information, using the necessary technologies
- Adaptation to new situations
- Autonomous work
- Promotion of free, creative and inductive thinking

(3) SYLLABUS

Non-countable sets impose the need for mathematical foundation of the concept of measurability restricted to appropriate subsets called events, restricting also the definition of any probability measure solely to such measurable/event subsets, thus shaping the constrained concept of probability space (σ-algebra of events abiding by σ-additivity of the probability measure) as Kolmogorov's basic axiomatic framework for the development of contemporary probability theory on non-countable sets, in unity with classical theory on countable or finite sets where all subsets are considered events. Classes of subsets and the concept of the smallest/least class with specified structure (e.g. π-system, semi-algebra, algebra, monotone class, λ-system, σ-algebra) generated from a given arbitrary non-empty class of subsets, Borel σ-algebras generated from classes of intervals or rectangular subsets in finite dimensional Euclidean spaces. General properties of probability measures, equivalence of σ-additivity and continuity. Extension of a probability measure defined initially on an algebra (or a semi-algebra) of subsets, and uniqueness of its extension to the (complete) σ-algebra of subsets measurable in the Caratheodory-Lebesgue sense with respect to exterior measure induced (on the powerset) by the initial probability measure. Applications of the extension theorem in constructing probability measures on (Euclidean) Borel spaces from a given probability distribution function (univariate on intervals or multivariate on rectangles), on spaces of functions from a given (Kolmogorov consistent) family of finite dimensional distributions on the so-called cylinder-like subsets of functions, and on product probability spaces. Decomposition of a probability measure to a (unique) convex linear combination of potentially three component distributions (discrete, absolutely continuous, singular continuous) with respect to Lebesgue measure on the same (Euclidean) Borel space, connections with the theory of probability distribution functions, Radon-Nikodym characterization of the absolutely continuous distributions as dominated by Lebesgue measure, Cantor's distribution counterexample and other examples.
Random variables defined as measurable real-valued functions, pointwise convergence of a sequence of random variables defined in the same space, measurability of the limit and of the domain of convergence, Borel measurability and other properties. Stochastic independence among classes of events or random variables, Borel-Cantelli lemmas, terminal (or tail) σ-algebra, Kolmogorov's 0-1 law. Expectation of a random variable constructed as Lebesgue integral with respect to a probability measure in the space where the random variable is defined and equivalently as Lebesgue integral with respect to the probability distribution (or law) induced by the random variable on the Borel real-line, integrability of a random variable, properties of expected values. Modes of stochastic convergence for sequences of random variables (almost surely, in mean of p-th order, in probability, in law or weakly), their inter-relationships and equivalent characterizations of the two weaker modes, limit theorems (weak and strong laws of large numbers, central limit theorems), expectation continuity theorems (monotone convergence, Fatou's lemma, dominated or bounded convergence, uniform integrability convergence), characteristic functions, Fourier inversion theorem, method of moments.
Conditional expectation of an integrable random variable with respect to a given sub-σ-algebra of events of the probability space and proof of existence of almost surely equivalent versions of it as Radon-Nikodym derivatives of suitable finite measures dominated by the probability measure on the conditioning sub-σ-algebra, properties and examples of conditioning, conditional probabilities of events as conditional expectations of indicator functions corresponding to the same events.

(4) TEACHING and LEARNING METHODS - EVALUATION

DELIVERY *Face-to-face, Distance learning, etc.*	Face-to-face
USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY	In communicating with the students.
TEACHING METHODS	see table below
STUDENT PERFORMANCE EVALUATION	Written examination at the end of the semester (100% of the grade)

Activity	*Semester workload*
Lectures	52
Studying and Analyzing Bibliography	7
Tutorial	26
Self-Study	115
Course Total	200

(5) ATTACHED BIBLIOGRAPHY

Recommended Bibliography:

1. Rosenthal, J. S. (2006): A First Look at Rigorous Probability Theory, 2nd Edition, World Scientific.
2. Ρούσσας, Γ. Γ. (1992): Θεωρία Πιθανοτήτων, Εκδόσεις ΖΗΤΗ, Θεσσαλονίκη (11057-ΕΥΔΟΞΟΣ).
3. Καλπαζίδου, Σ. (2002): Στοιχεία Μετροθεωρίας Πιθανοτήτων, Εκδόσεις ΖΗΤΗ, (11379-ΕΥΔΟΞΟΣ).

Recommended Supplementary Bibliography:

1. Shiryaev, A.N.: Probability, 3rd Edition, (75491026 – Vol. I, 2016 & 91693435-Vol. II, 2019), Problems in Probability (73251304-ΕΥΔΟΞΟΣ, 2012), Springer-Verlag.
2. Billingsley, P. (1995): Probability and Measure, 3rd Edition, John Wiley & Sons, New York.
3. Bhattacharya, R. and E.C. Waymire (2016): A Basic Course in Probability Theory, 2nd Edition, Springer (75480799-ΕΥΔΟΞΟΣ, Αρχική έκδοση 2007: 176605-ΕΥΔΟΞΟΣ).
4. Roussas, G.G. (2005): An Introduction to Measure-Theoretic Probability, Elsevier-Academic Press.
5. Leadbetter, R, S. Cambanis and V. Pipiras (2014): A Basic Course in Measure and Probability – Theory for Applications, Cambrige University Press.
6. Chung, K.-L. (1974): A Course in Probability Theory, Academic Press, San Diego.
7. Durrett, R. (1996): Probability: Theory and Examples, Duxbury, Belmont.
8. Port, S.C. (1994): Theoretical Probability for Applications, John Wiley & Sons, New York.
9. Capinski, M. and Kopp P.E. (2004): Measure, Integral, and Probability, 2nd Edition, Springer.
10. Athreya, K.B. and S.N. Lahiri (2006): Measure Theory and Probability Theory, Springer (173065-ΕΥΔΟΞΟΣ).
11. Skorokhod, A.V. (2005): Basic Principles and Applications of Probability Theory, Springer (172343).
12. Gut, A. (2005): Probability: A Graduate Course, Springer (169327-ΕΥΔΟΞΟΣ).