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Control Charts for Some Discrete and Continuous Distributions

By

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Ελισάβετ Δεμερτζή

ΔΙΑΤΡΙΒΗ
Που υποβλήθηκε στο Τμήμα Στατιστικής
του Οικονομικού Πανεπιστημίου Αθηνών
ως μέρος των απαιτήσεων για την απόκτηση
Διδακτορικού Διπλώματος στη Στατιστική

Αθήνα
Δεκέμβριος 2024

DEDICATION

To my mother and my aunt for their endless support
To those inspiring everyday “heroes” who struggle through difficulties to achieve their goals and make their dreams come true
To my grandfather who died three years ago. He was like a father to me and I miss him so much
To God who helped me and my family so many times

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ABSTRACT

Control charts are the most important tool of Statistical Process Control (SPC) which helps maintain good quality of products and services or even better improve it. As good quality becomes more important in our everyday lives, control charts related research efforts increase. The first control charts ever constructed were based on the assumption of a Normal distribution for the quality characteristic of interest. This assumption, however, has been proved to be rather invalid in practice. Therefore, control charts have been constructed for monitoring a quality characteristic under the assumption of non-Normal distribution.

A lot of distributions have been considered in the relevant literature. There are, though, some distributions with lots of applications in various fields of our everyday lives, which have not been considered yet or have not still been addressed well enough in the field of SPC. Examples of the former case are the Logarithmic and Lindley-related distributions, while a case belonging to the latter category is the Pareto distribution. This PhD thesis is an attempt to fill in this gap in literature.

There are a lot of cases, nowadays, of monitoring single observations instead of samples of more than one unit either due to automatic inspection which allows inspection of all units or due to natural limitations. Therefore, the individual measurements case is going to be addressed here for the construction of control charts for the aforementioned distributions.

More specifically, this study proposes individual control charts for the original one-parameter Lindley distribution and a two-parameter extension of it, as well as the Logarithmic and Pareto I distributions. Individual control charts for these distributions are first constructed with probability-type control limits. Then individual Shewhart-type and EWMA control charts are considered along with some skewness correction method in order to enhance

their performance, since all the distributions of interest are skewed. Two different skewness correction methods are used in this essay and their performances are compared. The performances of all charts are investigated and compared to each other in terms of the charts' average run length (ARL) and illustrated with both simulated and real data. Conclusions and suggestions for further research are also provided in the last chapter of this thesis.

ΠΕΡΙΛΗΨΗ

Τα διαγράμματα ελέγχου είναι το πιο σημαντικό εργαλείο του Στατιστικού Ελέγχου Ποιότητας (ΣΕΠ) που βοηθά στη διατήρηση της καλής ποιότητας προϊόντων και υπηρεσιών ή ακόμα και τη βελτίωσή της. Καθώς η καλή ποιότητα γίνεται όλο και πιο σημαντική στην καθημερινή μας ζωή, οι σχετικές με τα διαγράμματα ελέγχου ερευνητικές προσπάθειες αυξάνονται. Τα πρώτα διαγράμματα ελέγχου που κατασκευάστηκαν βασίζονταν στην υπόθεση της Κανονικής κατανομής για το ποιοτικό χαρακτηριστικό που μας ενδιαφέρει. Αυτή, όμως, η υπόθεση έχει αποδειχτεί ότι μάλλον δεν ισχύει στην πράξη. Για το λόγο αυτό, έχουν κατασκευαστεί διαγράμματα ελέγχου για ποιοτικά χαρακτηριστικά που υποθέτουμε πλέον ότι δεν ακολουθούν την Κανονική κατανομή.

Στη σχετική βιβλιογραφία έχουν κατασκευαστεί διαγράμματα ελέγχου για πολλές κατανομές. Υπάρχουν, όμως κάποιες κατανομές με πολλές εφαρμογές σε διάφορα πεδία στην καθημερινή μας ζωή, οι οποιές δεν έχουν ληφθεί ακόμη υπόψη ή δεν έχουν ερευνηθεί αρκετά όσον αφορά τον ΣΕΠ. Παραδείγματα της πρώτης περίπτωσης είναι η Λογαριθμική κατανομή, η κατανομή Lindley και οι σχετικές με αυτήν κατανομές, ενώ μια περίπτωση που ανήκει στη δεύερη κατηγορία είναι η κατανομή Pareto. Αυτή η διδακτορική διατριβή είναι μια προσπάθεια να συμπληρωθεί αυτό το κενό στη βιβλιογραφία.

Υπάρχουν πολλές περιπτώσεις, σήμερα, όπου ελέχουμε μεμονωμένες παρατηρήσεις αντί δείγματα που να αποτελούνται από περισσότερες από μία μονάδες είτε λόγω αυτοματοποιημένου ελέγχου που επιτρέπει τον έλεγχο όλων των παραγόμενων μονάδων είτε λόγω φυσικών περιορισμών. Επομένως, εδώ θα καλυφθεί η περίπτωση χρήσης μεμονωμένων παρατηρήσεων για την κατασκευή των διαγραμμάτων ελέγχου για τις προαναφερθείσες κατανομές.

Πιο συγκεκριμένα, αυτή η μελέτη προτείνει διαγράμματα ελέγχου για μενονωμένες παρατηρήσεις από την αρχική μονοπαραμετρική κατανομή Lindley και μια διπαραμετρική μορφή της, καθώς και για τη Λογαριθμική κατανομή και την κατανομή Pareto. Τα διαγράμματα ελέγχου για μεμονωμένες παρατηρήσεις από αυτές τις κατανομές κατασκευάζονται πρώτα με όρια ελέγχου που βασίζονται στην πιθανότητα σφάλματος τύπου Ι. Στη συνέχεια, κατασκευάζονται διαγράμματα ελέγχου τύπου Shewhart καθώς και EWMA διαγράμματα για μεμονωμένες παρατηρήσεις χρησιμοποιώντας κάποια μέθοδο διόρθωσης ασυμμετρίας για τη βελτίωση της συμπεριφοράς των διαγραμμάτων, μιας και όλες οι κατανομές που μας απασχολούν είναι μη συμμετρικές. Δύο διαφορετικές μέθοδοι διόρθωσης ασυμμετρίας χρησιμοποιούνται σε αυτήν την εργασία και οι συγκρίνονται οι συμπεριφορές τους. Οι συμπεριφορές όλων των διαγραμμάτων ερευνούνται και συγκρίνονται μεταξύ τους σε σχέση με το μέσο μήκος ροής (ARL) και γίνεται επίδειξη αυτής της συμπεριφοράς μέσω προσομειωμένων αλλά και πραγματικών δεδομένων. Συμπεράσματα και προτάσεις για περαιτέρω έρευνα παρέχονται επίσης στο τελευταίο κεφάλαιο αυτής της διατριβής.

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CHAPTER 1

INTRODUCTION

Quality is part of every aspect of our everyday lives. Statistical Process Control (SPC) aims to help businesses and organizations to control their quality of products and services and keep it steady or, even better, improve it. An overview of the research on SPC and control charting methods was provided by Woodall and Montgomery (1999). A brief history of quality control methods can be found in Montgomery (2009). One of the most important tools of SPC is the control chart. The concept of the control chart was first introduced by Walter A. Shewhart in 1924 in a technical memorandum of Bell Telephone Laboratories and has been studied and extended a lot ever since. Control charts (which are basically plots of data over time) help professionals visualize a process and see if any patterns or anomalies occur within it and, therefore, determine if the process of interest functions as it was supposed to or not, evaluate its stability and improve the process if required. Every process presents variations over time. The usual variations are the common cause variability. If, however, variation presents unusual patterns, then we talk about special cause variability and this is an indication of quality deterioration or sometimes improvement (depending on the monitored quality characteristic). Therefore, the special or assignable cause of variation can be wanted (if it has a positive effect) and, therefore, made part of the process when identified, or avoidable (if it has a negative effect), but definitely not inevitable and not allowable (as is the case with the common cause of variation). This means that it should be detected as soon as possible and a helpful tool for that purpose is an appropriate control chart.

The original control charts proposed by Shewhart and mostly used in practice in businesses until today have their control limits placed at ±3 times the standard deviation of the quantity plotted in the chart. This construction of control charts is based on the assumption that the distribution of the quality characteristic under study follows the Normal distribution, which, however, is usually not true when it comes to real-world data. This issue has been addressed in literature by the construction of control charts for various distributions, as presented later. Nevertheless, there are still some useful distributions for which control charts have not been constructed yet. Filling this gap is the aim of the current essay, which comes to propose control charts for some distributions with lots of applications in our everyday lives, for which control charts have not yet been developed such as the Logarithmic distribution and the one-parameter and two-parameter Lindley distributions. Moreover, similar control charts are proposed for the Pareto distribution, to contribute more to the control charts already addressed in the relevant literature.

The structure of the thesis is as follows. In Part I, Chapter 1 presents an overview of statistical process control charts, while Chapters 2-4 give a review of the Lindley-, Pareto- and Logarithmic-related distributions. In Part II, Chapters 5-8 deal with the construction of control charts for individual observations from the one-parameter and two-parameter Lindley, Logarithmic and Pareto distributions with probability control limits, Shewhart-type control limits using both skewness correction and scaled weighted variance method and EWMA charts with both these methods for taking into consideration the skewness of each distribution. The constructed control charts with each of the three methods are compared with each other and illustrated through both simulated and real data examples and their performance is investigated in terms of the ARL. Conclusions and further research recommendations are offered in the last chapter of this dissertation. The contents of each Part are described in details in the corresponding Part's introduction.

PART 1

Introduction to Part 1

Statistical Process Control (SPC) charts are the most important tools for assuring and improving quality by reducing process variability. These notions along with some literature review on SPC charts and particular distributions are going to be covered in this part. It should be noted that only the univariate case is addressed here, although a multivariate chart is more effective in monitoring a multivariate process than several separate univariate control charts. A lot of research exists on the field of multivariate and multiattribute control charts [e.g. Lowry and Montgomery (1995), Knoth and Schmid (2004), Yeh et al. (2006), Bersimis et al. (2007, 2017), Topalidou and Pasarakis (2009), Butte and Tang (2010), Rogalewicz (2012), Haridy et al. (2014a), Perdikis and Psarakis (2019), Ajadi et al. (2021)], and including all the relevant efforts would substantially increase the volume of this thesis. Besides, the multivariate case is beyond the scope of this essay, since only univariate control charts are proposed in Part 2. Therefore, only the univariate case will be discussed at this point.

Part 1, in general, is dedicated to an overview of what has already been done in the literature. More specifically, Chapter 2 presents an overview of SPC charts with special sections on control charts for non-normal distributions and individual observations, which are the core of this thesis, while the next three chapters explore the literature on three distributions which are going to be used in Part 2 for the development of new control charts. In particular, Chapter 3 contains literature review for the Lindley distribution, Chapter 4 discusses the Logarithmic distribution and Chapter 5 addresses the Pareto distribution.

CHAPTER 2

OVERVIEW OF STATISTICAL PROCESS CONTROL CHARTS

2.1 Introduction

Quality plays a very important role and is required in every aspect of our everyday lives. Statistical Process Control (SPC) is a statistical way to monitor and improve quality, and control charts are the most important tool for this purpose. This chapter deals with the concepts of quality and SPC and presents an overview of some of the literature on SPC charts.

2.2 The Concept of Quality

Before talking about control charts we should first deal with the concept of quality and its definition. Although quality is very important in all the sectors of our everyday lives there is no single and generally accepted definition for it. It can be defined either based on the companies or the users and can include both attractiveness and utility, as well as value of design and product support [Mukherjee (2018)]. Quality is related to the desired characteristics that a product or service should exhibit at an established standard in order to meet the requirements of its users and satisfy them and this is usually the customers' primary factor for choosing among various competing products and services or discriminating between products of the same kind. Quality does not refer only to special characteristics that are useful for the comparison of products and services of rival companies, but also includes those characteristics which are helpful for grading outcomes from the same process. All of the above contribute to the degree of importance of quality for businesses, since good quality plays a crucial role in their success and coping with the competing market. Quality can be defined as either the degree to which a product conforms to the requirements of the design or the degree of excellence at an acceptable price for the customers and control of variability at an acceptable cost for the companies so that both customer satisfaction and supplier's profitability are achieved. Some characteristics that can be considered when defining quality are materials, dimensions, shape, design, chemical components, appearance, functionality, fitness for purpose and applicability. According to Garvin (1987) there are eight components or dimensions of quality: performance, reliability, durability, serviceability, aesthetics, features, perceived quality and conformance to standards.

Therefore, quality is a multilateral and dynamic concept, since it can be defined and evaluated in many ways subject to the context in which people use it and it is continuously changing over time in the sense that the standard characteristics required by customers keep becoming higher as time passes. This leads to a constant need for quality improvement in order for a business to be successful and prevail over its competitors. This can be achieved with the help of SPC.

Moreover, besides the above "fitness for purpose or use" or "satisfying customers' requirements" or "conformance to designer's specifications" definitions, Montgomery (2009) gives another definition for quality (and quality improvement, in extension) based on its relationship to the variability of the process, since a decrease in unwanted or harmful variability of a process leads to better quality of its product. Therefore, a more complete definition of quality would combine conformance to specifications with minimum variance. This is exactly where SPC comes in to help the companies recognize the variation in their process outcomes, monitor it and identify its sources in order to be able to detect any possible variation changes and their causes. Identification of causes will be helpful in their efforts to reduce or eliminate them (if their outcome is negative) or adopt them (if their outcome is positive) and, therefore, improve quality of their products and services. Examples of such causes of variation can be the quality of raw material or equipment, the way of handling tools and machines, the skills and education of the employers, the negligence or carefulness of the operators and the environmental conditions such as temperature, humidity, acidity or pressure.

2.3 Statistical Process Control

Statistical Process Control (SPC) is a powerful collection of a variety of statistical tools and methods used to monitor and reduce variability and, therefore, achieve process stability and improve process capability which leads to improving the performance of a process in order to ensure high quality of the products produced and services offered to consumers [Ryan (2000), Wadsworth et al. (2002), Montgomery (2009) and many others]. Process stability is an indication of a process in a state of control and is accomplished if only inherent and inevitable causes (called common causes or random causes) of variability are present in the process (opposite to special causes or assignable causes, which we want to be detected and eliminated from the process). [Special causes are due to irregular or unnatural causes, make the process unstable and, consequently, unpredictable and affect some aspects of the process but not necessarily all of them, while common cause variations are regular or natural causes which affect all the outcomes of the process. The most usual special causes can be classified as people, equipment, procedures, materials and environment. A more analytic list is given by Oakland (2003).] Process stability shows the ability of the process to be consistent and, thus, predicted. Process capability, on the other hand, corresponds to the performance of a process under control and reveals the ability of the process to meet customers' specifications, which can also be used as a prediction for future production, since the process is stable. Process capability can also be considered as the variation of the quality characteristic under study when the process is in statistical control [Mitra (2021) and Burr (2004)]. A common measure of the process capability is the process spread (6σ), which will include almost all observations of the quality characteristic under study (Under the assumption of Normality, 99.73% of the distribution lies within $\pm 3\sigma$ limits around the mean). Process capability is usually examined with respect to pre-defined specification limits, with various capability ratios which reveal the ability of the process to meet requirements. Besides these capability indices, process capability can also be evaluated through histograms and probability plots [Montgomery (2009)]. Process capability indices can be computed only when the process in under control, because they use the mean, spread and quantiles of the process distribution, which change when the process is out of control (unstable). Therefore, there are two important steps that should be followed before investigating the process capability: First of all, process stability should be insured and then the Normality assumption should be checked [Bothe (1997)]. There have been, however, many attempts in the literature to extend or modify the traditional definitions of process capability indices in order to be used for non-normal distributions which can be found in references such as Rodriguez (1992), Luceño (1996), Somerville and Montgomery (1996) and Kotz and Lovelace (1998).

From all the above, it becomes obvious that SPC is very important in every process of our everyday lives. For instance, SPC has found many applications in food industries [e.g. Ittzés (2001), Augustin and Minvielle (2008), Dalgiç et al. (2011), Lim et al. (2014, 2017)], textile industries [e.g. Maroš et al. (2011), Yılmaz and Yanık (2020), Abdulghafour et al. (2021)], cement industries [e.g. Tegegne et al. (2022)] and many others. Although the main use of SPC is in industrial manufacturing environments, it can also be applied in many other areas as we will see below and, therefore, besides produced objects we can talk about other processes and offering of services, too. According to Keller (2011) a process "consists of repeatable tasks, carried out in a specific order". This means that the actions carried out during a process are generally the same for a particular set of inputs. Thus process can be every set of repeating actions that turn an input into an output for customers, with the output being not only manufactured products but services as well, such as "patient care, government or legal processes" as well as "late flight arrivals, mis-diagnoses, traffic accidents, injuries, system downtime events" and "time waiting in a queue, order-processing time, time to complete a project, etc." [Stapenhurst (2005)]. Stapenhurst (2005) also mentions as areas of application of SPC sectors such as "health care, travel, education and training, oil and gas, distribution, public services, government, information technology (IT), construction, finance, chemical monitoring, health and safety, planning, projects, design and most other areas of an organisation" and presents examples covering most of them. Osanaiye and Talabi (1989) were among the first researchers who dealt with non-manufacturing applications of control charts. Moreover, Berthouex (1989) and Morrison (2008), among others, discussed the application of control charts for environmental data monitoring. Melvin (1993) described the application of control charts to educational systems, while Clark and Clark (1997) addressed the use of control charts for athletic performance monitoring. Wu and Meaker (2002) presented the use of control charts for monitoring warranty data. Shore (2006) used control charts for monitoring queue length. Alemi and Sullivan (2001) presented risk adjusted $\overline{X}$-charts with applications to diabetes monitoring. Sego (2006) dealt with the use of control charts in medicine and epidemiology. Limaye et al. (2008) and Morton et al. (2009) presented the use of SPC to monitor and reduce hospital-related infections. Sachlas et al. (2019), among many other researchers, discussed Risk Adjusted control charts with health applications, for taking into account, for example, the pre-operative severity of illness or risk related with the patient. Mitra (2021) presented the construction of various types of Risk Adjusted and Variable Life-Adjusted Display charts for monitoring health-care processes while taking into account the risk of each patient. Simões et al. (2022) discussed the use of SPC charts in psychology. The use of control charts in other health-related process monitoring can be found in Rossi et al. (1999), Hart et al. (2003, 2004), Albers (2010a) and Tomak and Bek (2017), as well as in the reviews provided by Benneyan et al. (2003), Sonneson and Bock (2003), Grigg and Farewell (2004), Woodall (2006), Woodall et al. (2010) and Suman and Prajapati (2018). SPC has also been applied to DNA microarray data and individual gene expression [Chimka and Oden (2008)], while, for example, Tsung et al. (2007) and Golosnoy et al. (2010) applied SPC to financial cases. Coup (2009) discussed the use of control charts in forestry and Gupta et al. (2009) used control charts for maintenance policy selection. De Vries and Reneau (2010) provided a review of control charts for animal production systems monitoring. Megahed et al. (2011b, 2012) presented the use of SPC for image data monitoring. Yashchin (2012) dealt with monitoring warranty data streams of computer system components with dynamically changing observations. Qiu (2014) states the use of SPC for monitoring sequential processes such as "production lines, Internet traffic, medical systems, social or economic status". Saulo et al. (2015) used control charts for monitoring environmental risk. Zhao and Gilbert (2015) discussed control charts for monitoring the waiting time of customers. Spirić et al. (2016) presented control charts for determining a set of suspicious electricity customers. Aslam et al. (2021) talk about application of SPC for managing human resource and monitoring various incidents of misbehaviour in working environments. Control charts have also been applied for monitoring various parameters related to the COVID-19 pandemic, as presented by Mbaye et al. (2021). According to Demmy (1989) and Mahanti and Evans (2012), who dealt with the implementation of SPC in the software industry, any process that is "well defined, measurable, repetitive and sufficiently critical to justify monitoring effort" is suitable for SPC. As stated by Oakland (2003) any process consisting of a transformation of a set of inputs to a set of outputs should be monitored in order to meet customers' requirements. The output can be not only products but services as well (such as deliveries, etc.) or even information. The inputs, too, can be not only materials or tools and other equipment, but any action or method as well, even people along with their knowledge, training and skills. All that is needed for SPC to be applied in non-manufacturing processes is an accurate definition of the inputs and their suppliers, the outputs and their customers, the requirements for the outputs and precise instructions for the methods and procedures that will lead to the outputs. Once everything is absolutely clarified, the data collected about the process will be reliable and appropriate for the application of SPC which will help avoid failures whatever that process is.

Now that we have defined the concept of SPC and mentioned its uses in our everyday lives, we should talk about its implementation, goal and benefits. The ideal way of implementing SPC is for every process contributing to the quality of a final output (product or service). This, however, is not practical, but SPC should not be applied in the first convenient (with respect to time and cost) instance, either. In order to get the best out of it, process prioritization is required for the implementation of SPC. This prioritization, as stated in Goh and Xie (1998), should be done with respect to their technical and statistical criticality, with the term technical criticality being referring to the importance of the process to the quality of the final output, while the term statistical criticality concerns the statistical stability and capability of the process. Another important issue that should be avoided for a successful SPC implementation is overadjustment or tampering (which we will talk about later in section 2.6). Once all the aforementioned points are considered, SPC implementation can be very effective. According to Deming (1967) SPC can help achieving a balance for the economic loss from two usual mistakes: either looking for special causes too often or not looking often enough. The basic idea in SPC is continuous examination of the process through random samples drawn regularly in order to ensure that certain quality requirements are met and a high level of performance is maintained while changes in the process are quickly detected. So the goal of SPC is to achieve the decrease of defects and defective outputs which in sequence will lead to both increased quality and reduced cost. This means that SPC offers many benefits to both companies and their customers. Process stability and process improvement (which are both results of SPC implementation) mean that the complexity of the process is reduced, errors, non-conforming items and failure costs are decreased, on-time delivery is achieved, safety is increased, consistency of the process output becomes greater (due to variability reduction), effectiveness and productivity are improved (by reducing scrap and rework), machinery malfunctions are early diagnosed, rework of products or waste of time and materials is reduced (and, as a result, company's profits are increased), future performance can be predicted, assessment of the ability of the process to produce according to specific standards can be performed and process capability can be improved. All of the above lead to enhanced reliability and competitiveness of the company and greater customers' satisfaction.

As mentioned at the beginning of this section, a lot of tools are used in SPC. Seven of them are the most important and mostly applied in SPC due to their great usefulness and are, therefore, called the seven quality control tools or the seven basic tools or the seven tools of SPC or the magnificent seven. They are all very powerful yet simple in their implementation and, therefore, used in every quality improvement scheme. The seven quality control tools were first highlighted by Kaoru Ishikawa [Ishikawa (1985, 1986)]. He believed that almost all quality-related problems in industries can be solved with those fundamental tools. His original seven tools were: cause-and-effect (or Ishikawa or fishbone) diagram, check sheet (or tally chart), Shewhart's control charts, histogram, Pareto chart, scatter diagram (or scatter plot), and stratification. Ever since, many writers on quality control, following his steps, mentioned the seven tools, and even though they all list exactly seven tools, their lists are not always identical. They all include most of the original Ishikawa's seven tools but usually omit stratification and replace it with flowcharts (or process maps), run charts, bar charts, tolerance diagrams (or tier charts) or defect concentration diagrams. Below we focus only on the control charts which are the most important and more utilized of all the aforementioned tools.

Control charts are basically run charts with lines connecting consecutive plotted points and control limits drawn on the charts in order to help us understand when the range of the plotted values is too high to be caused by common causes of variability. There is also a central line drawn at the average of the plotted values helping us see how successive observations are behaving in relation to their average. This way it is easier to notice any non-random patterns in our process (regarding the critical-to-quality characteristic of interest). Control charts also give an indication of when the change in the process occurred (due to the time reference). The control limits are drawn in such a way that almost all of the observations are expected to fall between them and there is a very small probability that a point will be plotted outside them if the process does not change. Therefore, points outside the control limits are strong indication that the monitored process distribution has changed. Control charts are very useful tools for on-line process monitoring and more details on them are going to be presented in the next section.

Another important thing that should be mentioned about the implementation of SPC (before moving to further details on control charts) is the distinction between Phase I and Phase II, which are two different phases employing different SPC methods. During Phase I [which was the only focus of Shewhart (1931, 1939)], we are trying to properly set up the process in order to make it stable and, therefore, we do not know much about it at the beginning. For this reason this phase is more exploratory. Controllable input variables are set at such levels that the affected quality characteristic under study will roughly meet the designed requirements and then the collected data (usually 20 or 25 subgroups) are plotted on a control chart with trial limits calculated based on these data. Points outside the control limits are investigated for potential assignable causes. Any unnatural patterns found, also lead to further investigation. When assignable causes are identified, adjustments are made to the controllable input variables, out-of-control points are excluded and then a new set of data from the process is plotted on the control chart with the new control limits. This procedure is repeated until all special causes have been removed and the process is stable. Then we end up with data collected from a stable operating process, which are, therefore, representing the actual process performance and can be used for the estimation of the in-control distribution of the quality characteristic of interest. Consequently, in Phase II the process is considered to be known and in-control at the beginning and then the main objective is to monitor the process on-line in order to ensure that it stays in control. If an assignable cause occurs in the process, an out-of-control indication is given by the control chart and the process is stopped in order to investigate the special cause and fix the process. Otherwise, as long as the process remains in control, we have two options: to deal with the improvement of the process by reducing common cause variation too, or to proceed with the monitoring and improvement of another process and do nothing further with the process we were dealing with so far. So, during Phase I control charts are used to determine if the process is in statistical control by examining past data (retrospective data analysis), while control limits determined at the end of Phase I can be used for future data during Phase II in which we use recent data collected sequentially over time for online monitoring in order to determine if the process remains in control (prospective analysis).

Discussions of the use of control charts in Phase I and related matters can be found in Woodall (2000), Borror and Champ (2001), Champ and Chou (2003) and Human et al. (2010b). The differences between Phase I and Phase II SPC analysis were described by Vining (2009). Overviews of Phase I control charting can be found in Chakraborti et al. (2009) and Jones-Farmer et al. (2014). A thorough review of Phase I SPC analysis with efforts to bridge the gap between theory and practice was given by Woodall (2017).

Shewhart control charts are very effective for Phase I due to their easy construction and interpretation and their effective detection of large sustained or sudden (outliers) shifts, measurement errors and data recording mistakes. Assignable causes that usually arise during Phase I produce large and transient shifts and Shewhart control charts are most effective in detecting them. On the contrary, these charts are less effective in Phase II (where large shifts are less common), because of their insensitivity to smaller shifts. For this reason, other charts that we will discuss later such as the CUSUM and EWMA charts are more effective during Phase II, because they are good for detecting small and persistent shifts which are our major concern in Phase II.

Regarding Phase I, the selection of the observations (called "baseline") that will be used for the determination of the trial control charts needs to be considered for more efficiency of the control charts. Zhang et al. (2010) proposed a method for identifying the baseline period, while earlier Kang et al. (2007) had pointed out the need for the data values used in Phase I to extend over the range of data values for which the control chart is to be used. The identification of the shifts or outliers and their locations is also important for deciding on keeping or discarding parts of the historical data when calculating the control limits that will be used for the remainder of Phase I and more importantly for Phase II. An algorithm clustering individual observations for the detection of multiple shifts and/or outliers in historical data was presented by Sullivan (2002).

The size of Phase I data is very important for the performance of control charts in both Phase I and Phase II. In fact it is critical for Phase II for the following reason. In order to move to Phase II we need estimations of the process parameters, especially process variability, from Phase I data. The run length properties of the control chart in Phase II are influenced significantly by estimation from Phase I data. An accurate and precise estimation of the parameters in Phase I, will lead to satisfactory performance of control charts in Phase II, while a not so good estimation may lead to more false alarms than expected. A good estimation is more likely with larger sample sizes in Phase I. Moreover, the larger the number of available reference data in Phase I, the more the Phase II control limits can perform with properties approximately the same as the ones in the known parameter case. These are the reasons for the importance of the sample size of Phase I data. While 100 observations (either individual or in subgroups) would be a sufficient number for Phase I, Quesenberry (1993) suggested that at least 300 observations are required for the computation of control limits that will be used during Phase II. More over, The Phase I reference data are usually made up of m subgroups each of size n, which means that there are totally m*n observations in Phase I reference data available for parameter estimation and setting up the control limits for the Phase II. Champ and Jones (2004) dealt with the design of Phase I control charts for various values of m and n, while Yao et al. (2017) extended their work for larger values of m and provided an R package for the calculations on demand. Jensen et al. (2006) and Psarakis et al. (2014) concluded that the number of phase I samples must often be quite large (in many hundreds of observations) for achieving a reasonable confidence that the control chart will perform closely to the one of the known parameter case. Moreover, Chakraborti (2006), Saleh et al. (2015) and Epprecht et al. (2016) showed that it takes a much larger Phase I sample size than usually recommended in textbooks in order for the properties of the control charts to be consistent and close to the known parameter case.

Study of Phase I monitoring has also been conducted for more specialized applications. Boyles (2000), for example, dealt with Phase I when monitoring autocorrelated processes. The case of Phase I monitoring was addressed by Mahmoud and Woodall (2004) and Mahmoud et al. (2007) for linear profiles and Ding et al. (2006) for nonlinear profiles. Riaz (2011) presented an auxiliary information-based control chart for Phase I monitoring. In the nonparametric case Phase I monitoring was discussed by Jones-Farmer et al. (2009), Graham et al. (2010), Jones-Farmer and Champ (2010), Capizzi and Masarotto (2013) and Capizzi (2015).

2.4 Statistical Process Control Charts

Control charts present the position of a statistic (average, median, range, standard deviation) of some kind of measurement of a quality characteristic relatively to three important lines, namely the upper and lower control limits and the central line. The horizontal axis displays the sample number in time order of measuring the quality characteristic of interest. The vertical axis presents the value of the observation or the statistic computed for the quality characteristic under study. The central line represents the average value of the quality characteristic corresponding to an in-control stable process [process that exhibits only common causes of variation and natural pattern (see section 2.10)]. The control limits are computed so that almost all the sample observations will lie between them in an in-control state of the process. The points corresponding to the plotted observations are connected with lines with each other in order to facilitate the visualization of the behaviour of the observations' sequence over time. The general formulas for the computation of the control limits and the central line are the following:

$$\begin{aligned} UCL &= E(\text{statistic}) + L(\text{standard deviation of statistic}) \\ CL &= E(\text{statistic}) \\ LCL &= E(\text{statistic}) - L(\text{standard deviation of statistic}) \end{aligned} \tag{2-1}$$

As mentioned earlier, Shewhart's choice was L = 3, which is founded statistically on the assumption of the Normal distribution. We will talk about the assumptions of control charts and cases of their violations (including non-normality) later.

This setting of the control limits is equivalent to setting up the critical region for a hypothesis test where the null hypothesis is that the quality characteristic's average is equal to the value at which the central line is drawn (for the specific value of the standard deviation) and the alternative hypothesis is that it is not equal. So, control charts basically test this hypothesis repeatedly over time. If the null hypothesis is rejected then the distribution of the process has changed and the process is no longer under control. This hypothesis testing framework will be useful for the performance investigation of the control charts, with which we will deal later.

Stable quality, otherwise reaching in-control state of the process and achieving the required standards or target specifications for a characteristic of interest, means that there is no unusual variability in the process and, therefore, the observations are randomly around and relatively close to the central line and definitely inside the control limits. Observations outside the control limits need further investigation for the elimination of the assignable cause of this extra variability (if it has a negative effect on our process) or its adoption (if it has a positive effect) in order to improve quality (Shewhart (1931)). Ryan (2011) presents out-of-control action plans in section 4.16 (p. 131) therein and Halim Lim and Antony (2019) in section 9.4.6 (p. 143) therein. Treatment of out-of-control signals is also dealt with in Levinson (2011). But even if all the observations are plotted inside the control limits, the existence of any systematic or non-random behaviour is a reason for further investigation, too, so the sequence of the observations is very important for a control chart.

2.5 Purposes and Benefits from Using the Control Charts

Control charts can be used for two purposes: To analyze past data and test the stability of the process or to test whether the process remains stable and in control (Phase I and Phase II SPC). In the first case we plot a preliminary set of samples to set up the control limits and then plot each sample we draw and interpret it in relation to the previous data, while in the second case we use the control chart immediately and we can plot each sample we draw as soon as we obtain it and take appropriate actions if a non-random pattern arises.

Therefore, control charts can be used to test the homogeneity of the process and reduce variability, help us monitor the performance of the process over time and keep it steady or improve it. When stability of the process has been ensured the process capability can be assessed and estimated. This way control charts can assist process improvement efforts. They also give us the opportunity to quickly detect abnormalities and out-of-control situations in the process and eliminate out-of-limits materials as soon as they are discovered in the process with immediate corrective actions. Control charts can help us reduce the defects and defective items or services produced by a process, as well as the scraps and reworks, and this way productivity is increased and costs are decreased. This can be better achieved if control charts are applied to process variables than produced units which are affected by those variables. Control charts also make it easier for the users to recognize the difference between the background noise and the abnormal variation in a process and, consequently, avoid unnecessary process adjustments which would lead to more variation and, thus, deterioration of the performance of the process instead of its improvement which is our goal. Control charts can also reveal patterns in the process which give a clue for the cause of uncommon variation and lead to appropriate corrective actions which will improve the performance of the process. Moreover, they can show the levels of process performance and, therefore, help the users know the effect of certain actions and process changes and learn their process deeper. Furthermore, they can be used for performance comparison of different groups and activities as mentioned in Stapenhurst (2005). Last but not least, control charts are easily implemented and can be used for the improvement of process performance in many businesses, thus making them competing and able to stand out against their rivals in the market. Improved process performance also makes customers more satisfied and businesses more profitable.

2.6 Common Mistakes and Things to Pay Attention to When Constructing or Interpreting a Control Chart

Many errors may risk the effectiveness of the control chart. One of the most common mistakes is the wrong choice of the type of control chart to use. Other common mistakes include the miscalculation of control limits or their substitution with the specification limits, thus causing the control limits to be wider and the ability of the control chart to detect out-of-control conditions worse.

Quality of data and measurements is also important. If measurements are missing or poor or erroneous, this affects the performance of the control chart. The same is valid when the data are not up-to-date. Out-of-control signals and non-random patterns appearing on a control chart are also very important and should never be ignored. They should always be investigated in order to improve the quality of the process.

When using control charts in Phase II, we assume that the process parameters are well estimated. This assumption is crucial because parameter estimation affects the performance of the control charts [Jensen et al. (2006)], especially the ones which are designed for monitoring small shifts such as the CUSUM charts or the Shewhart charts with sensitizing rules (Section 2.10). Therefore, in order to obtain good estimates of the process parameters, a large sample size is required for the estimation during Phase I. If the required amount of reference data in Phase I is not available, self-starting control charts can be useful. This way, successive observations in Phase II are used for simultaneously updating parameter estimates and the plotted statistic. Examples of literature on self-starting control charts include Li et al. (2010), Zhang et al. (2012), Keefe et al. (2015), Amiri et al. (2016), McClurg (2016), Amirkhani et al. (2018), Tighkhorshid et al. (2018), Khosravi and Amiri (2019), Ravichandran (2019), Subbulakshmi and Kachimohideen (2019), Cornelissen (2021) and Dogu and Noor-ul-Amin (2023). A review on self-starting CUSUM charts literature was provided by Wendler (2021). An extensive study of self-starting control charts is also presented in Laurijsse et al. (2021).

If the effects of parameter estimation are ignored then the control charts can give more false alarms than expected [Psarakis et al. (2014)] and, therefore, they become less effective and can lead to increase of cost. Substituting the parameters with their estimated values when constructing control charts will make them perform differently than they would in the known parameter case, unless a large number of data were used for the computation of those estimates (which is not usually possible). When parameters are estimated the performance of the chart is evaluated with the conditional ARL which is the average of the unconditional or marginal run length distribution for a given set of estimators over the distribution of these estimators.

When using Shewhart control charts for monitoring the unknown process variability it is recommended in the literature to not use the range chart but other charts instead, such as the S or S^2 chart [Mahmoud et al. (2010) and Epprecht et al. (2016)]. When using CUSUM control charts, one way to deal with the problem of the chart's sensitivity to parameter estimation is the self-starting CUSUM which was proposed by Hawkins (1987). This chart, however, is also based on the normality assumption and needs special attention in case of out-of-control signal. The latter is required because the self-starting CUSUM statistic will at first move upwards after a shift, but then, contrary to the ordinary CUSUM statistic, will not continue moving upwards indefinitely, but it will begin moving downwards after the shifted values are used in the calculations. Therefore, immediate investigative and corrective action and subsequent resetting of the CUSUM is required after an out-of-control signal and all the out-of-control data should be removed when resetting. Self-starting CUSUM can also be affected by outliers and solutions as presented in Hawkins and Olwell (1998).

Another very important issue with control charts is unwise operator overadjustment of equipment or other parts of a process. Tampering with the process can lead to many out-of-control or near-control-limits points on a chart. Control charts give an indication of when critical conditions for a process are present and need further investigation and when the process is performing consistently stably. Therefore, there is no need for reaction to every small appearance of variation, because this practice will not reduce variability, but increase it instead, thus leading to the appearance of more observations being plotted near or beyond the control limits, while the process would not normally produce them.

Besides all that, the choice of the control limits, the possible use of sensitizing or supplementary runs rules and warning limits and the choice of sample size and sampling frequency are also very important in order to be able to combine control chart effectiveness and prevention of unnecessary and possibly costly process investigations and adjustments. These subjects are going to be discussed next.

2.7 Performance of the Control Charts

We talked earlier about the control charts being repeated hypotheses tests. Every hypothesis test has a probability α of type I error and a probability β of type II error. Its power is equal to $1-\beta$. In the case of the control charts, we have a type I error when we decide that our process is out of control when actually it is in an in-control state, while there is a type II error when we decide that our process is in control when in fact it is in an out-of control state. An operating characteristic (OC) curve is a good means of visualizing the ability of the control chart to detect a process shift of various values of magnitude δ, with the OC curves being graphs with β presented on their vertical axis and $d=|\delta|/\sigma$ displayed on the horizontal axis. Observing the OC curves it becomes obvious that it is easier for the control charts to detect large shifts than smaller ones and that their power is increased as the sample size is increased.

A measure of the performance of the control chart is the Average Run Length (ARL). ARL is defined as the average number of points plotted until an out-of-control-limits point appears on the chart. For uncorrelated observations, for all the Shewhart charts we will discuss later, ARL is computed as the reciprocal of the probability of a point being plotted outside the control limits. There are two important ARL values, namely the in-control ARL (ARL_0) and the out-of-control ARL (ARL_1). ARL_0 is the ARL until receiving an out-of control signal while our process is in control and is, therefore, computed as $ARL_0=1/\alpha$. On the other hand, ARL_1 is the ARL until receiving an out-of-control point while the process is indeed out-of control, and is, therefore, computed as $ARL_1=1/(1-\beta)$. A control chart with good performance is associated with a large value of ARL0 and a small value of ARL1, which should become smaller as the magnitude of the shift decreases. In a Shewhart control chart with the traditional 3-sigma control limits (based on the Normal distribution assumption mentioned earlier) in-control ARL is equal to $ARL_0=1/0.0027=370$. It should be noted that when the process parameters are unknown and need to be estimated before Phase II begins, ARL cannot be computed as the reciprocal of the signaling probability because the signaling events are no longer independent thus causing the run length distribution to no loner be geometric.

Sometimes instead of ARL other ways to assess the performance of a control chart are used, such as the False Alarm Rate (FAR) which is basically the probability of type I error. Another measure of a control chart's performance is the Average Time to Signal (ATS) defined as ATS=ARL*h, where h is the length of the fixed time intervals at which samples are taken from the process [Khoo (2004c)]. In cases of inspecting all units h=n, and, therefore, ATS=ARL*n [Wu et al. (2006)]. Sometimes it may also be useful to express the performance of the chart in terms of the expected number of individual units inspected (I) which is defined as I=ARL*n, where n is the sample size. In this case, ATS=I. Adjusted Average Time to Signal (AATS) is another measure of performance proposed by Tagaras (1998). It is the expected value of the time between the occurrence of the assignable cause and the chart signal. AATS was referred to as the steady state ATS by Runger and Pigniatello (1991). Two other measures of control chart performance found in Reynolds and Stoumbos (2000a) are the Average Number of Samples to Signal (ANSS) and the Average Number of Observations to Signal (ANOS). ANSS is defined as the expected number of samples of n observations from a certain time point (usually the beginning of the process) to the time of the out-of-control signal, while ANOS is defined as the number of individual observations from a certain time point (usually the beginning of the process) to the time of the out-of-control signal. Therefore, ANOS=n*ANSS. Similar to the case of using ARL, a control chart performs better if for given values of shift magnitude and in-control ANOS the out-of-control ANOS is smaller.

All of the above performance measures are the ones used in Phase II of SPC. During Phase I, however, we confirm process stability at a given False Alarm Probability (FAP), which is the probability of at least one false alarm (an out-of-control signal while the process is in control). Similarly the signaling probability can be used, which is the probability of at least one signal from the m subgroups used (see definition of Phase I earlier).

2.8 Choice of Control Limits

The choice of the control limits is critical for a control chart because their width affects the chart's performance. If the control limits are very wide, the type I error probability decreases and the type II error probability increases. On the other hand, if the control limits are too narrow, we risk increasing the type I error probability and decreasing the type II error probability. The usual practice is choosing the width of the control limits to be a multiple of the standard deviation of the plotted statistic. This issue is addressed by Nelson (2003). Depending on the normality assumption, choosing three-sigma limits is a good option. This is also a good choice if the distribution is reasonably approximated by the Normal distribution. We will discuss the non-normality case in further details in section 2.18.

2.8.1 Probability Limits

Another choice of control limits (which is better for non-normal distributions) is the use of probability control limits. These require choosing the type I error probability first and then computing the control limits based on this choice of α, instead of computing the control limits as a multiple of the standard deviation of the plotted statistic. If the quality characteristic under study is normally distributed, there will be little difference between the three-sigma control limits and the probability limits with α chosen to be equal to 0.001.

2.8.2 Action Limits and Warning Limits

Sometimes two sets of control limits are used on the same control chart simultaneously. In this case, the outer control limits (the control limits mentioned in the previous subsection) are called action limits, because if a point exceeds them an action for identifying the assignable cause and correcting the process is immediately required. The inner control limits are called warning limits. In the case of using the three-sigma limits as the action limits, then the two-sigma control limits are used as the warning limits. On the other hand, when the 0.001 probability limits are used as action limits, the warning limits are set to be the 0.0025 probability limits. Page (1955) obtained the ARL function in the case of using both warning and action limits.

In the case of using two sets of control limits, if one or more points fall between them or very close to a warning limit, caution is required because there might be a problem with the monitored process. In such a case, more information about the process is necessary and it can be gained by increasing the sample frequency or the sample size or both. Control charts with the sample size and/or frequency changed depending on the position of the plotted values are called adaptive control charts (variable sampling interval or variable sample size control charts).

When having a variable sample size, there are three options: First, we can use the average sample size for the computation of the control limits, thus having approximate but constant control limits. This approach works best with large sample sizes and when the sample sizes do not vary more than 25% from the average sample size. This approach has the advantage that calculations of the control limits and interpretation of the chart is easier but has also the disadvantage that since the control limits are approximate, if a plotted point is close to them, we cannot be sure if it is really inside or outside the exact control limits. A second approach for variable sample sizes is using the exact (but variable) control limits which are based on the actual size of each sample. This means that the control limits are calculated for each subgroup separately, based on each subgroup's size and, therefore, they will vary as the sample size varies. This approach has the disadvantage of not looking so good as a chart with fixed control limits, but has the important advantage of the control limits being exact and, therefore, interpretation of the control chart is more direct, as usual. A third method for dealing with variable sample sizes is using standardized control limits. This entails the computation and use of standardized statistics for each subgroup and then using the approximation by a standard normal distribution which practically means that the control limits will be simply equal to -3 and 3, when using the three-sigma approach for the control limits. This approach has the advantage of constant control limits regardless of the subgroups' sample size and the disadvantages of the extra computation of the standardized statistic and the possibility of making this way the interpretation of the standardized statistics and the standardized control limits more difficult due to the fact that they are no longer in the original scale of measurement.

2.8.3 Control Limits and Specification Limits

When using control charts to monitor a process, there is one important thing that we should paint special attention to, namely the difference between control limits and specification limits. Control limits are implied by the process. They are computed using the natural variability of the process and represent the span of the values that can result from the distribution of the quality characteristic under study when the process is in control. Specification limits are completely irrelevant. They are determined externally by process designers or customers and represent the span of values that a process is desired to produce. When setting specification limits, knowledge of the process and its inherent variability is required, but there is no relationship connecting control limits with specification limits. It is definitely a mistake to use specification limits on a control chart in place of control limits. If we want to monitor the capability of the process to meet the required specifications, the process must first be insured to be in control. A process could be in control but not capable to meet the specifications, but we cannot be sure if it can meet the specification requirements if it is not in control, since it is unstable and, thus, unpredictable.

2.9 Sample Size, Sample Frequency and Rational Subgroups

As mentioned earlier, the sample size plays a very important role in the chart's ability to detect process shifts. More specifically, larger sample sizes make it easier for the control charts to detect smaller shifts. Therefore, the choice of the sample size depends on the magnitude of shift which we want to detect early. OC curves mentioned in Section 2.7 can help us choose the appropriate sample size based on ARL from a statistical point of view, depending on the power we want the control chart to have in detecting shifts of a certain magnitude. Moreover, the sample size should be decided by looking at the process variability. If the inherent process variability is large then larger sample sizes are required in order to detect the out-of-control situation, while smaller samples would be required for the case of a process with smaller inherent variation. When setting-up a control chart it is generally suggested to collect a minimum of 20-30 data points, so as to have the time required for the estimates of the process mean and standard deviation (and, consequently, the control limits which depend on them) to become accurate [Stapenhurst (2005)]. For variable measure data a sample size between 2 and

12 (usually chosen to be equal to 4 or 5) is required for setting-up a control chart, while for attributes a sample size between 25 and 250 (with most commonly chosen values being 50, 100 and 200) is required in order to obtain a reliable estimate of the process parameters [Murdoch (1979)].

Frequency of sampling is also important, mostly from the economic point of view, because the ideal choice of drawing large samples in high frequency is not always realistic. So the choice will be either to draw small samples more frequently or larger samples less frequently. The usual choice in practice is the first one. From an economic point of view, smaller and more frequent samples are preferred if the cost of producing defective items is high, because large intervals can cause many defective items to be produced before detecting a process shift. Sometimes it is useful to begin with frequent samples and to reduce the frequency later when the process becomes stable. If a point plots close to the control limits then it is reasonable to increase the sample size or decrease the sampling interval or both, because there is a high possibility of the next point plotting outside the control limits and we want to detect an assignable cause as quickly as possible. The opposite can be chosen when a point plots close to the central line.

Sampling frequency also depends on the process performance and the consequences of changes in the process. For example if process changes are disastrous or costly or if significant changes are happening frequently in the process, then frequent sampling is preferred. On the contrary, long intervals between samples are more reasonable when changes in the process happen rarely and even when they do, only a moderate loss is suffered when it takes some time to detect them. Furthermore, if there is a suspicion of cyclical behaviour of the process, samples should be drawn frequently in order to give the ability of investigating the possibility of a cyclic pattern (see definition in section 2.10).

The rate of production can also affect the choice of the sample size and sampling interval. High production rates require more frequent samples (because of the higher possibility of many nonconforming items in a short time interval) and allow larger sample sizes to be drawn economically (if testing in not costly or catastrophic). If testing is, however, expensive or disastrous then smaller sample sizes are preferred no matter how high the production rate is. If the investigation of false alarms is also expensive, then smaller sample sizes are preferred, too.

Besides the sample size and sampling frequency, the way of collecting the samples is also important for the construction of control charts. There might be cases, for example, when the level of one variable affects the behaviour of other variable(s) related to the quality characteristic under study and sometimes the combined effect of two or more variables on the quality characteristic of interest is different from the individual effect of each of those variables. One such case could be the effect of the combination of some level of pressure and some level of temperature on a quality characteristic of a chemical process. In such a case, a more effective sampling process would entail controlling one of those factors (pressure and temperature) at various specific levels and then find the effect of the other on the quality characteristic of interest for each of the first factor's values. If not such a sampling process is adopted, but samples are drawn from random combinations of pressure and temperature instead, there is a high risk of not identifying the interactive effect of those variables on the quality characteristic of interest and, therefore, not monitor the process effectively.

A method for collecting data must be rational. Rational subgrouping requires respect of the structure of the process data and the collection of samples so as to minimize the chance of variability due to assignable causes (if they are present in the monitored process) and maximize the chance of variability due to common causes. In other words the samples should be collected so as to increase the probability of variation between the samples, while keeping the within samples variation small. If the within-samples variability is large the width of the control limits increases and the sensitivity of control charts to process shifts is reduced. Nelson (1988) talked about the need for rational subgroups and emphasized the fact that data collected over a short time period will not necessarily be rational subgroups. Palm (1992) illustrated the significance of a good sampling plan for the construction of control charts and underlined the importance of rational sampling. Rational subgroups were also discussed in Reynolds and Stoumbos (2006a).

One way to minimize the within samples variation is to sample items produced consecutively by the same process (in a short enough time period in order for the process to be stable during the data collection time) with an interval between two successive samples, so that any process shifts that have occurred are presented on the chart as between-sample variation. This way, rational subgroups contain only common cause variation. Otherwise, large within-subgroup variations will be present and this will make control limits wider thus making the control chart insensitive to process shifts. This approach provides a better estimation of the standard deviation of the process in the case of variables control charts which we will discuss later. In this case the within-sample measure of variability is used to construct the control limits. When choosing the rational subgroups, it is important to insure that each item is produced by the same process. If samples contain data from different process conditions, the variation will be so large that it will make it difficult for important process changes to be noticed. Another general approach for constructing rational subgroups entails subgroups being a random sample of all units produced over the entire interval since the last subgroup was selected. Therefore, caution is required about the interval between chosen units, because if it is very wide there is a risk of an out-of-control process appearing as in control due to the wider control limits.

Subgroups should be selected in such a way that there will be no autocorrelation in the observations within the subgroups, because this makes within-subgroup variation too small (affecting the width of the control limits and, therefore, the effectiveness of the chart) and a bad predictor for the between-subgroups variation. Furthermore, the smaller the within-subgroup variation the narrower the control limits, thus giving more false alarms of out-of-control state.

Moreover, attention should be paid in the selection of the appropriate subgroup size, unless the process itself enforces the size of the rational subgroup to be equal to one, as is usually the case for example in chemical industries or processes where quality characteristics change very slowly and, therefore, consecutive samples drawn close to each other will be almost identical. Although the subgroup size is usually selected without special thought to be equal to 5, 10 or 20, it is very important for a control chart. It is crucial for the appropriateness of signals and the overall performance of the control chart as was proved, for example, by Tabim and Ferreira (2015). The subgroup size may prevent the control chart to detect significant process shifts if it is too small or may be responsible for many out-of-control signals without any significant shift. So the choice of the subgroup size should be such that the probability of detecting important shifts will be high, while probability of false alarms caused by insignificant shifts will be very small, and, therefore, only a reasonable right amount of control chart signals will be given [Razmy (2016), Manyele (2017)].

Other SPC tools discussed earlier such as the cause-and-effect diagram or the scatter diagram, may be useful when choosing rational subgroups, because they can help us identify possible causes of differences in the process or important correlations and choosing the right subgroups to detect them. For example waiting times might be affected by the department or by the number of cases which may depend on the time of the day, thus causing the waiting times to be different in various times or sectors. Use of a histogram could also be helpful for deciding if rational subgrouping is required, as is the case for example if the shape of the distribution is bimodal, because this would be an indication that two processes have contributed to the data. In case of any data pattern immerging during a process monitoring, further rational subgrouping can be useful as it could help explain the reason. We will talk more about patterns in section 2.10.

The idea behind rational subgroups is homogeneity, namely that data chosen for the subgroups come essentially from the same population and data from different populations (for example data from different shifts, different machines or different operators) are not mixed. Mixing data from them when constructing the control chart, will give a control chart for a mixture distribution (with an overestimation of the variability due to different processes) which will not be suitable for application to data coming from each population separately and will not be able to detect differences from one process to another or detect if a particular process does not perform well. Although it may not be practical to construct a control chart for each of the different populations, it might be very helpful, in order to see if each of the different processes performs well on its own. All the different processes could separately be in statistical control and this would be an indication that process improvement is required for each of them.

2.10 Patterns on Shewhart Control Charts and Shewhart Control Chart Enhancements

We have already mentioned earlier the possibility of non-random behaviour of our data in the control charts, else called patterns, which (besides observations outside the control limits) is also an indication that our process may be out of control due to an assignable cause. Any non-even distribution of the plotted points around the central line of the control chart is an indication of a non-random behaviour and needs to be further investigated.

We talk about patterns on Shewhart control charts specifically and not control charts in general, because patterns on other control charts that we will see later (such as EWMA and CUSUM charts) are not necessarily and indication of an out-of-control situation. This is due to the fact that the statistics plotted on those control charts are functions of both the current and the past observations and are, therefore, correlated. This means that patterns are expected to be present on those control charts even if the process is in control.

2.10.1 Control Chart Patterns

A sequence of observations of the same type, called a "run", is an example of a non-random behaviour but not the only possibility. Other patterns are also likely. Even if all points of a control chart are inside the control limits, if they exhibit any non-random pattern they indicate that something unusual is happening in our process and needs attention. The process must be stopped and investigated even if the plotted statistic lies between control limits. If the source causing this pattern is helpful (as it rarely might be) then it must be identified and adopted in the process, otherwise it must be detected and efforts should be made to reduce it or even better eliminate it (if possible), so as to improve the process.

So it is important not to only recognize the pattern but be able to assign the root causes to non-random patterns. Identification of causes of patterns, however, requires good knowledge and understanding of the process (equipment, operating conditions) and the impact of those causes on the quality characteristic under study. It should also be noted that the causes of a pattern in a control chart monitoring the mean of a process can be different from those for a control chart monitoring the variability of the process. Some of the most important and usual patterns that can appear on a control chart with some of their possible causes are the following [Hubbard (1990), Noskievičovα (2013)]:

(1) normal or natural: This pattern is the representation of the stable in-control process. Points on the control chart are scattered in the chart, fluctuating randomly around the central line between the control limits, not very close to them and definitely not exceeding them, with approximately half of the data below the central line and the other half above it and all the points of the control chart lying there without any non-random behaviour. This is the only pattern we would like to see on a control chart when the process performs well. Each of the following patterns is unwanted.

(2) trend (upward trend or downward trend): This pattern (Figure 2.1a,b) is displayed as a continuous gradual (increasing or decreasing) run of points in one direction caused by a factor which started to act on the process at the beginning of the change in level. This pattern can happen because of workers' fatigue or effective training (leading to gradual change in their skill level) in machine operation and/or measuring systems, inspector's or well-skilled superior's presence or any other change of supervision, change in the production rate or the number of components reaching the process, gradual change of the quality of raw materials or components over time (because of SPC implementation by the supplier), tool wear, gradual deterioration of measuring equipment or machine parts, machine warm-up and cool-down, or change of the maintenance system or inadequate maintenance. In chemical processes trends can occur due to settling or separation of components of a mixture.

(3) sudden large shifts (freaks): This pattern (Figure 2.1c) is depicted as occasional wild individual observations (often called "freaks"), namely points close to the control limits showing sudden and high changes

affecting one or more samples and can be caused by mistakes such as wrong setting, sampling mistakes, error in measurement and plotting, misplacement of inputs or raw materials, use of new tools for short test periods, incomplete or omitted manufacturing operations, breaks of power or gas supply, overcorrection, failure of a component, or equipment malfunctions.

(4) smaller sustained shift (upward shift or downward shift): This pattern (Figure 2.1d,e) is demonstrated on the chart as a sudden jump to a new level (above or below the previous one) with the process remaining at the new level, which means that there is a series of points on the same side of the central line. This can be caused by damaged equipment, new personnel, new suppliers, new production methods, new or repaired machines or equipment, change in work practices or measurement system, changes in maintenance, change in operators' skills or motivation, change in quality of raw materials, intentional data improvement when recording them, or failure to recalculate the control limits after a change in the process.

(5) systematic: This pattern (Figure 2.1f) is sometimes called "saw-tooth effect" and is exhibited as a series of consecutive alternating high and low points in a control chart. This can occur when there are two different alternating processes together, such as two machines working together or two operators with different skill levels using the same machine (for example a shift change), two alternating suppliers, or tampering. In fact, one of the most typical causes for this pattern is process overcontrol or unwise operator overadjustment of equipment, for example after just a few measurements above the average or close to the control limits. Overadjusting of a process is often called "tampering" with the process. Adjusting a process which is statistically in control increases the variation in the process. If operators try to achieve certain values of a quality characteristic of interest but the result is a little lower or higher (but still in control), the "saw-tooth" pattern appears on the chart, which leads to more adjustments and finally to a process that is definitely out of control.

(6) cyclic: This pattern (Figure 2.1g) is displayed on the chart with upward and downward movements of points recurring periodically. It is an indication of an assignable cause with periodic effects on the process. Identification of the period of variation can give a strong indication of where one should start looking for the causes, which might be for example seasonal factors (as is the case for winter or summer or holiday activities or procedures taking place at repeated times of the day, week, month or year). Other reasons might be rotation in equipment, or shift changes in machine handlers or personnel making measurements, or operator fatigue and subsequent boosting after breaks. This pattern may also result from environmental changes (such as changes in temperature, humidity or lighting), fluctuation in voltage or pressure, or some other variable in the machinery causing it to malfunction sporadically, periodic machine maintenance or lubrication, periodically alternating raw material and supplies, seasonal variation of incoming components, or due to periodicity of mechanical or chemical properties of raw material. Failure to apply a correct sampling plan for the implementation of the control chart can also cause cycles or prevent them from being revealed. Infrequent sampling may cause only the high and low points to be represented on the control chart (as mentioned earlier).

(7) stratification: This pattern (Figure 2.1h) is presented on the chart with points clustering closely around the central line lacking variability and never approaching the control limits. This may happen because of incorrect calculation of the control limits or incorrect rational subgrouping, or when not recalculating the control limits after a process improvement. It can also result from a process with very different components, each with small associated variation, such as different raw material streams or the output of different machines working simultaneously or different shifts performing differently (which could be avoided if rational subgrouping, as mentioned earlier, was used). If all the different components are mixed and samples are drawn from the mixed output, small variation will result. This happens because the range between the smallest and the largest observation in

the sample is very large (since those values come from different distributions and we basically measure the variability between the different underlying distributions) causing the width of the control limits to increase. The result is basically what one might call “too good to be true” or “too much consistency” and it is not at all a good situation, but rather worrying, because if not detected and corrected, it can prevent identification and elimination of the cause of difference between the process components and, therefore, lead to process deterioration instead of improvement. This pattern may also emerge when testing or measuring with a malfunctioning instrument or conducting chemical or biological tests with outdated reagents. It can also appear when recording data incorrectly or intentionally manipulating them (for example not recording extreme values).

(8) mixture: This pattern (Figure 2.1i) is portrayed by unexpectedly large variation, wild values (close to the control limits) and absence of points near the central line and can be the result of two different simultaneous processes one producing a set of small values and the other a set of high values. It could be for example the result of different lots of raw material with different characteristics mixed as process input. It could be caused by incorrect subgrouping (different subgroups taken from different sources). It could result for example from different work methods used by different operators, different testing or measuring tools used or various machines working simultaneously. The severity of the mixture pattern depends on the extend to which the different distributions overlap. Other causes could be change in the calibration of a measurement tool, process overcontrol or unwise operator overadjustment of equipment after just a few measurements above the average or close to the control limits, sporadic use of raw material of variable quality or from different suppliers, malfunctioning inspection device, or instability of automatic control.

The detection of unnatural patterns is crucial for increasing the sensitivity of the Shewhart control charts, since these charts study samples individually and do not consider the joint information obtained from sequential points, opposite to CUSUM and EWMA control charts for example, which combine the values of present and past samples. To increase the sensitivity of the Shewhart control charts and enhance their performance, supplementary sensitizing rules and runs rules are used in conjunction with the charts. These rules help us determine the presence of special causes from the charts and detect small shifts and patterns. They should not, however, be used when monitoring individual data, because the false alarm rate is high in this case. Attention is also needed with correlated data and this is the reason that these rules should not be used with CUSUM and EWMA control charts, either. They are also meaningless (and, therefore, not used) in case of applying control charts for comparison of different groups or actions (as mentioned in Section 2.5) where there is no logical data order.

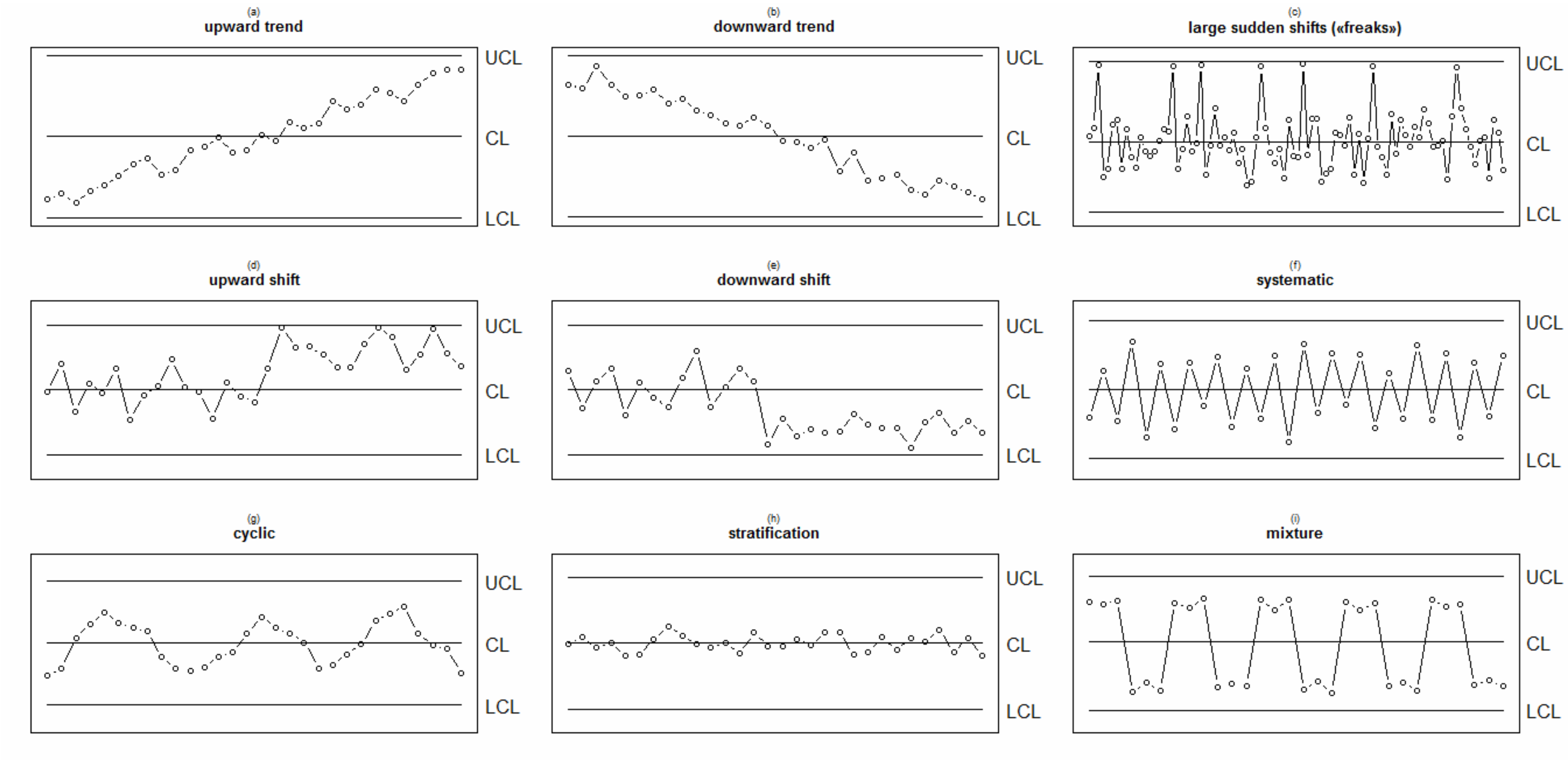

Figure 2 - 1: Patterns on Shewhart control charts

2.10.2 Sensitivity Rules

While Shewhart control charts are effective in detecting large shifts, they lack sensitivity in detecting small shifts. Their effectiveness for detecting small shifts as well as non-random patterns is enhanced with the use of supplementary sensitizing runs rules. The advantages and disadvantages of Shewhart control charts with supplementary runs rules were presented in Nelson (1985). Koutras et al. (2007) and Park and Seo (2012) reviewed the literature on the use of sensitizing runs rules in Shewhart control charts. The implementation of these rules is based on dividing the control chart area into various zones above and below the central line defined in terms of multiples of the standard deviation of the plotted statistic, as shown in Figure 2.2. For this reason these rules are also called "zone rules". The properties of control chart zone tests were studied by Roberts (1958). The performance of the zone control chart was studied by Davis et al. (1990), who also compared it with the performance of various Shewhart charts with and without runs rules. The zone chart's performance was improved with the addition of a fast initial response feature by Davis et al. (1994).

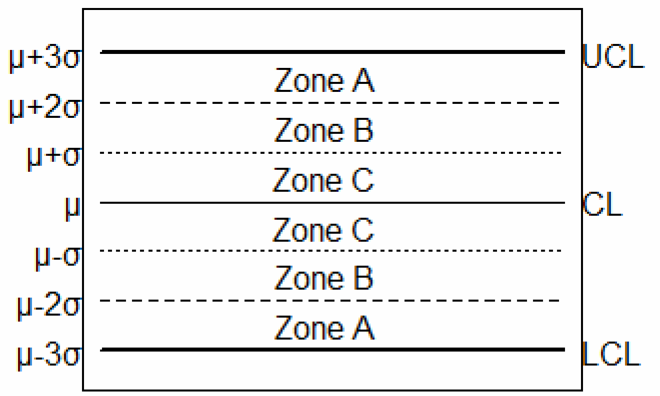

Figure 2 - 2: Shewhart control chart with "1-sigma", "2-sigma" and "3-sigma" zones ("Zone C", "Zone B" and "Zone A", respectively).

Different sets of zone rules have been proposed with the most famous set being the one by Western Electric Company (1956). Two other sets are the one in Nelson (1984) and the one from Duncan (1986). Duncan's rules are similar to the Western Electric rules but in a different order, while Nelson's rules are more and some of them are a little different as to the number of points they require. Nelson's rules cover more cases of patterns as we will see next. The only rule common in all sets is rule 1 which basically is also the only Shewhart's rule. More sets of rules can be found in Noskievičovα (2013) and Halim Lim and Antony (2019), but most of them are similar to these three. For the application of some of these rules (where 2-sigma limits come in), warning limits mentioned earlier can be proved very useful.

According to the Western Electric alarm rules the chart will signal if any of the following situations is true.

1. One point outside the 3-sigma control limits (beyond Zone A)
2. Two out of three consecutive points outside the 2-sigma limits (in Zone A or beyond) on one side of the central line
3. Four out of five consecutive points outside the 1-sigma limits (in Zone B or beyond) on one side of the central line
4. Eight consecutive points on one side of the central line (in Zone C or beyond)

Duncan's set includes the following alarm rules:

1. One point outside the 3-sigma control limits (beyond Zone A)
2. Seven consecutive points up and down or on one side of the central line (in Zone C or beyond)
3. Two consecutive points outside the 2-sigma limits (in Zone A or beyond)
4. Four consecutive points outside the 1-sigma limits (in Zone B or beyond)
5. "Obvious" cycles up and down

Nelson's alarm rules are the following:

1. One point outside 3-sigma control limits (beyond Zone A)
2. Nine consecutive points on one side of the central line
3. Six consecutive points increasing or decreasing
4. Fourteen consecutive points alternating up and down
5. Two out of three consecutive points outside the 2-sigma limits (in Zone A and beyond) on one side of the central line

6. Four out of five consecutive points outside the 1-sigma limits (in Zone B and beyond) on one side of the central line
7. Fifteen consecutive points inside the 1-sigma limits (in Zone C)
8. Eight consecutive points with none inside the 1-sigma limits (none in Zone C)

It should be noted that these rules are applied on one side of the center line at a time. For a two-sided control charts they are applied to each side separately. When using several of those rules simultaneously, usually graduated response to out-of-control signals is applied. For example, when a control chart presents an out-of-control point we stop the process immediately and we look for an assignable cause, but if one or two consecutive points get out of a 2-sigma warning limit, then we can increase the sampling frequency (adaptive sampling response) instead of looking for an assignable cause, in order to get a high probability of detecting the problem quicker than we would with the longer sampling interval. Caution is definitely required when combining some of the above rules. Each supplementary run rule increases the overall false alarm rate (FAR), although the FAR associated with it can be small on its own. The more rules used simultaneously, the higher the frequency of false alarms becomes. ARL computations for various combinations of four Western Electric alarm rules, as well as for the simultaneous use of all four of them, were conducted in Champ and Woodall (1990) revealing the decrease in ARL when the process is in control. The most recommended combination of rules in the literature is Western Electric rule 1 with Western Electric rule 4 or, respectively, Nelson's rule 1 with Nelson's rule 2. Nelson's rule 7 is recommended to be used at "a start-up of SPC rather than in an on-going control". According to Montgomery (2009), although in Phase II Shewhart control charts are not very effective when it comes to small to moderate shifts, using sensitizing rules to improve their ARL performance is likely to be an "unsatisfactory attempt" because of the increase in FAR and, therefore, "routine use of sensitizing rules to detect small shifts or to react more quickly to assignable causes in Phase II should be discouraged".

2.10.3 Relations between Unnatural Patterns and Rules

The first two Western Electric rules, or equivalently Nelson's rules 1 and 5, can recognize quickly sudden large shifts, while smaller sustained shifts can be quickly detected by Western Electric rules 3 and 4 (or Nelson's rules 6 and 2, respectively). Trends can be detected by Nelson's rule 3. Attention is required, though, for false alarms when data are correlated. Davis and Woodall (1988) showed that this rule increases the false alarm rate very much. Nelson's rule 4 can detect the systematic variation pattern. Nelson's rules 7 and 8 are connected with patterns caused by incorrect sampling strategy. The first of those two can detect stratification, while the second one can detect mixture patterns.

Wheeler (2004) states that, even if none of the above tests gives an out-of-control indication, there is still a possibility of an assignable cause being present in our process. For example there could be a process with two high points being followed by one low point on the chart repeatedly. This is definitely a repeated pattern but the previously mentioned rules do not detect it. Yet the process is not random and, therefore, not in control. For this reason, Wheeler (2004) proposes one more rule for pattern detection, which is: "An explanation should be sought anytime a pattern repeats itself eight times in succession."

2.10.4 Runs-type Signaling Rules or Supplementary Rules

These rules are used to enhance Shewhart control charts by making them more sensitive to detecting smaller shifts in the process. They are a generalization of the original Shewhart control chart's 1-of-1 rule. 1-of-1 rule means that the statistic plotted on the chart computed only from the current sample is used for testing if there is an out-of-control signal or not. It would be, however, more useful to test a few of the previous samples as well, because this might reveal a pattern in the signals. For example there might be a run of two consecutive samples giving a signal which would make the indication of an out-of-control condition stronger. This is where these supplementary runs rules come in handy. Two of the most popular rules of this kind are the 2-of-2 and 2-of-3 rules, which belong to the more general category of the k-of-k runs-type signaling rules, where k can be equal to or greater than 2. The k-of-k rule signals when k consecutive points on the chart exceed the control limit(s). There is a generalization of this rule, too, which is the k-of-w rule with $1 \leq k \leq w$. This rule signals when k out of the last w points on the chart are on or beyond the control limit(s). Control charts using these rules are more sensitive to smaller shifts than the simple Shewhart charts, but they have certain disadvantages. Their false alarm rate increases and there is also the risk of not immediately detecting large shifts in the process, because the chart's ability to detect a large shift is delayed until at least w samples have been collected. This means that caution is needed when deciding to use these rules and careful balance of cost and benefit should be considered. Many control charts with this type or rules have been proposed in the literature, such as by Champ (1992), Klein (2000), Shmueli and Cohen (2003), Khoo and Ariffin (2006), Acosta Mejia (2007), Antzoulakos and Rakitzis (2007), Lim and Cho (2009), Antzoulakos and Rakitzis (2010), Cheng and Chen (2011), and Santiago and Smith (2013a). Moreover, Champ and Woodall (1987) dealt with various out-of-control situations when k of w consecutive points fall outside the 1-, 2-, or 3-sigma limits with $2 \leq k \leq w$. Maragah and Woodall (1992), Alwan et al. (1994), and Balkin and Lin (2001) analyzed the effect of serial dependence on the runs rules charts by simulations (actually, for a retrospective application of the chart). Derman and Ross (1997) proposed two additional rules. The first of them signals when two consecutive points exceed either one of the two 3-sigma control limits and the second rule signals when 2-of-3 consecutive points exceed different 3-sigma control limits. These two rules were modified by Klein (2000) by requiring the related points to exceed a same control limit. Generalized *k*th-order runs were used by Weiß (2012, 2013) for monitoring categorical data. Khilare and Shirke (2014) studied the steady-state performance of cumulative count of conforming control charts with runs rules. Mehmood et al. (2018) investigated the performance of $\bar{X}$ chart with various runs rules for known and unknown parameters of various distributions, using the false alarm rate and power curve performance measures. Mehmood et al. (2019) discussed control charts based on various runs rules for various probability distributions and for both known and unknown parameters.

2.10.5 Recommendations for the Application of Rules for Unnatural Patterns Recognition

The most important recommendation in the literature regarding the use of the sensitizing and supplementary runs rules mentioned so far is to never routinely apply all the available tests, because they increase the false alarm rate very much when applied all simultaneously. Caution is also suggested when using these rules for correlated data as mentioned earlier. Same is valid for Moving Range charts as well as EWMA and CUSUM charts where the same observation is used multiple times for the computations and, therefore, successive values are not independent. This makes the application of the zone rules unsafe with these charts. Therefore, only the point beyond control limits rule should be applied with these charts. Lesany and Fatemi Ghomi (2021) also noted that the concept of CCPR is basically a test of whether the control limits are the same for all samples and, therefore, it is meaningless to look for behavioural patterns when control limits are computed separately for each sample as is the case with variable sample sizes. These rules also make sense only for symmetrical or almost symmetrical control limits. This is the reason that extra caution is required with individual data especially when they are highly skewed, because they will produce false signals. Therefore, only the point beyond limits rule can be applied for this kind of data. If the individual data come from a symmetric distribution, however, all the sensitizing rules can be applied. Same is true for control charts for averages due to the central limit theorem and standardized charts. For all the other Shewhart control chart types, that we will see later, namely control charts for range or standard deviation and control charts for attributes, the rules that can be applied are Nelson's rules 1, 2, 3 and 8. If, however, data come from a distribution for which the normal approximation is valid then the control limits are almost symmetrical and then the sensitizing rules can be applied without a problem for the attributes control charts with constant control limits. Noskievičovα (2013) also suggests that Nelson's rules 7 and 8 should be used at the beginning of the SPC implementation for the verification of rational subgrouping. When the distribution is stable (Phase II) it is recommended to start with Western Electric rules 1 and 4 and if it is necessary to additionally sensitize the control chart one or both of the other two Western Electric rules could be used. Stapenhurst (2015) also notes that "as the group size, n, decreases the likelihood of runs below the average increases slightly, so some analysts suggest that for n<6, a run of eight points below the average is required to signal a decrease in process variability".

2.10.6 Control Chart Patterns Recognition and Performance of Control Charts Under Drifts

The control chart patterns we presented earlier can appear on control charts either as single or concurrent patterns. Many attempts have been done in literature for pattern recognition on control charts, using various methods. For example, neural networks and other machine learning models were used by Hwarng (1991), Guh and Hsieh (1999), Guh and Tannock (1999), Guh et al. (1999), Gauri and Chakraborty (2007), Shaban et al. (2010) and Xanthopoulos and Razzaghi (2014) among others. Robustness of neural network-based control chart pattern recognition to non-normality was studied by Guh (2002). Statistical correlation coefficient method was employed by Yang and Yang (2005). Principal Component Analysis was utilized by Colosimo et al. (2007). Hassan et al. (2003) dealt with control chart pattern recognition (CCPR) using statistical features. CCPR based on Gaussian mixture models was proposed by Yu (2012). Classification methods were used by Othman and Eshames (2012). A more sophisticated technique can be found in Ebrahimzadeh et al. (2013) where a hybrid method is used combining a feature extraction module, a classification module (based on neural networks and support vector machines) and an optimization module with an algorithm which was proved to have very high recognition accuracy. Other hybrid methods were developed for concurrent CCPR by Chen et al. (2007) and Wang et al. (2009). Akaaboune et al. (2022) combined neural networks and Principal Components Analysis for concurrent CCPR. Recognition of mixture control chart patterns was addressed by Lu et al. (2011) and Zhang and Cheng (2015) by means of support vector machines. An adaptive neuro-fuzzy inference system was used for CCPR by Nikpey et al. (2014). John (2022) proposed a CCPR method for monitoring weekly customer complaints. A review of CCPR literature was presented by Hachicha and Ghorbel (2012), while a review of literature on concurrent CCPR was presented by García et al. (2022).

Davis and Woodall (1988) studied the performance of control chart trend rule under linear drift, showing that these charts are not effective for drifts detection. Gan (1991b) studied the performance of EWMA control charts under drift, while Gan (1992a) and Gan (1996) studied the performance of CUSUM charts under drifts and trends. Divocky and Taylor (1995), Chang and Fricker (1999) and Fahmy and Elsayed (2006) dealt with control chart detection of drifts. Shu et al. (2008) used a weighted CUSUM chart for the detection of patterns. EWMA charts were used to detect drifts in Ross et al. (2012). Reynolds and Stoumbos (2001a) dealt with monitoring both mean and variance of processes subject to drifts and so did Stoumbos et al. (2003) for processes subject to both drifts and sustained shifts. Zou et al. (2009) presented comparisons of control charts for monitoring process mean with drifts. Kabiri Naeini et al. (2011) developed a method for CCPR based on Bayesian inference and MLE and proved through simulation both the accuracy of the proposed method for the detection of unusual patterns and satisfactory reasults in the estimation of pattern parameters. Knoth (2012) dealt with drifts on control charts and their detection and extended pre-existing literature results. Detection of patterns was achieved through adaptive generalized likelihood ratio control charts in Capizzi and Masarotto (2012). Lesany et al. (2014) dealt with recognition of both single and concurrent unnatural patterns. Lesany and Fatemi Ghomi (2021) addressed the extraction and organization of statistical distribution functions for simulation of variations and patterns in control charts for variability.

2.11 Selection of the Type of the Control Chart

When having to construct a control chart in order to monitor a process, one important choice to be made is the selection of the appropriate type of the control chart to be used. The type and the distribution of the data are defining for the type of the control chart to be used. There are two types of data:

numerical (or variables data) and attribute data. Numerical data consist of measurements taken on a continuous scale, whose accuracy can be chosen by choosing the number of decimals to be recorded. Examples of such data are measurements of height, weight, length, width, depth, diameter, distance, volume, time, temperature, pressure, amount of money etc. Attributes data consist of countable non-measurable data with no decimals (discrete data) indicating the presence or absence of a defect or a characteristic of interest. Examples of attributes data include the number of defects or defective items (a defective item may have more than one defects, but an item with more than one defects is not necessarily defective), number of mistakes, injuries, accidents, complaints, orders, rejects, people, events, etc. Data with categories are easily turned into attributes data by considering the number or percentage of units belonging in each category. If there is an option of the data type when collecting the data, we should always bear in mind two important things. First is the fact that variables data contain much more information than the attributes data (for example exact concentration of an ingredient in each item versus the number of items containing that ingredient in a level above or less than a certain amount) and will, therefore, reveal non-random variation more easily when plotted on control charts. The other thing to consider is the cost (in time and money) for the collection and analysis of the data which is more for the variables data and less for the attribute ones. Similarly, whenever there is a choice between defects data or defective items data, we should always remember that defects data contain more information than defective (for example the number of forms containing mistakes without knowing what mistakes and how many of them versus the particular number of mistakes contained in the forms).

The above discrimination of data means that there are generally two types of control charts: control charts for variables ($\overline{X}$-chart for the process mean, R-chart for the process range, S-chart for the process standard deviation, S^2-chart for the process variability) and control charts for attributes (np-chart for the number of nonconforming observations for constant sample size, p-chart for the percentage or fraction of nonconforming observations for variable sample size, c-chart for data in subgroups with the same sample size, u-chart for data in subgroups of different sample sizes).

When choosing the type of control chart to use, it would be better if this could be done before collecting the data, because then there is an option of the way to collect the dataset and the chart to use accordingly. More specifically, we take into consideration how soon the chart can identify the out-of-control situations and select the one which does that sooner. So, we first choose the $\overline{X}$/s charts and then $\overline{X}$/R charts (the choice between them depends also on the sample size), secondly we choose the individual X/MR charts and thirdly the c- or u- charts. The p- and np- charts are the last option, since they are the slowest in detecting process shifts. If there is no option as to how to collect the data, then the choice of the chart-type depends strictly on the data at hand and the quality characteristic we want to monitor. Figure 2.3 presents the algorithm for the choice by practitioners of the appropriate type of Shewhart control chart to be used depending on the data for the quality characteristic under study. Further details on the Shewhart control charts presented on this graph are going to be presented in the following section.

This graph contains only the types of Shewhart control charts. There are, however, other control charts besides them, such as CUSUM and EWMA control charts, which were constructed in order to solve some problems (disadvantages) of the Shewhart charts and will be addressed here later. For various types of univariate control charts, a selection guide is presented in Chapter 10 of Montgomery (2009). A general recommendation summarizing that selection guide regarding all types of control charts would be to prefer CUSUM and EWMA control charts rather than Shewhart charts when interested in detecting small shifts or monitoring data that are skewed or autoccorrelated. More details on these and other weaknesses of Shewhart charts will be discussed later.

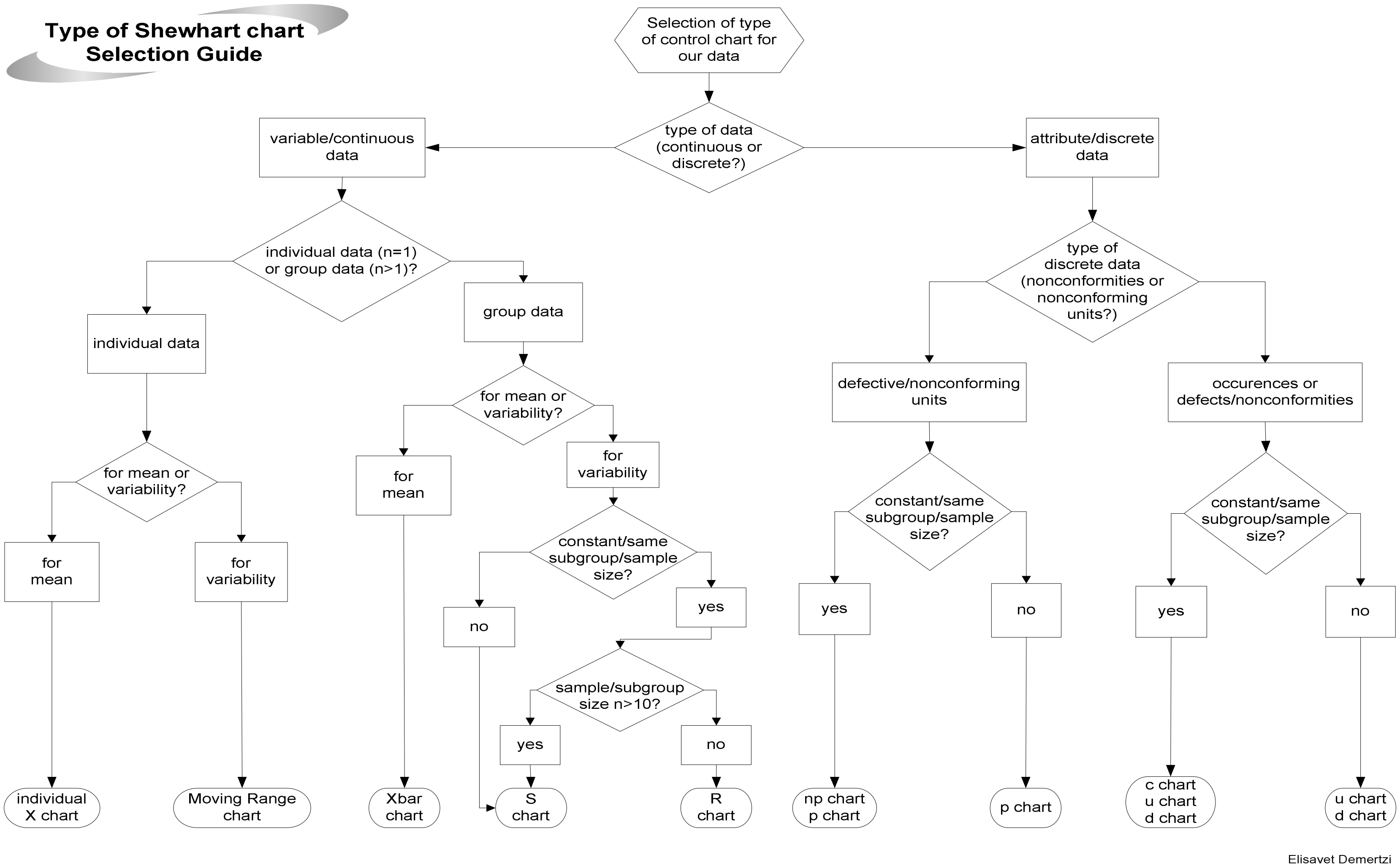

Figure 2 - 3: Flowchart guide for the selection of the appropriate Shewhart control chart type

2.12 Shewhart Control Charts

As previously mentioned, Shewhart control chats can be used to monitor data that are either variables or attributes. Each category of data is monitored by different types of Shewhart control charts. This section is dedicated to the presentation of these charts and their constructions and characteristics. One of the major differences between the variables control charts and the attributes control charts is that when dealing with variables data (individual observations or not) we need two different charts for monitoring the process mean and process variability while control charts for attributes respond to both mean shifts and variance shifts. Before starting with the control charts for variables, it should be noted that when monitoring a quality characteristic that is a variable, we should monitor both its mean and its variability, because even if the process mean stays in control the process variability might be out of control. Attention is required when interpreting those two charts in case of non-normal data. If the underlying distribution is normal then the control chart for the mean should behave independently from the control chart for the variability. If this is not the case and the underlying distribution is skewed then the two charts will "follow" each other, thus leading to wrong analysis. When interpreting patterns on these two charts, we should bear in mind that the interpretation of the chart for the mean relies on a constant variability. Therefore, the chart for variability should be analyzed first in order to determine that the variability is in control. If the chart for variability indicates an out-of-control condition we should not proceed to interpretation of the chart for the mean unless the variability has been brought in control first. If both charts indicate the presence of an assignable cause, we should first deal with the elimination of the assignable cause which will first bring in control the chart for variability. Now, before moving on to the particular types of the Shewhart control charts we need to distinguish between setting up a control chart and regularly using a control chart (Phase I and Phase II).

2.12.1 Setting-up the Control Charts

When setting up a control chart an initial considerable data set is analyzed in order to find the standard deviation for the computation of the control limits of the chart. The so called "standard procedure" for setting up a control chart is to compute the standard deviation and then examine if the dataset comes from an in-control process or not. The steps as presented by Porter and Caulcutt (1992) and Caulcutt (1995) are the following:

(a) Obtain the dataset.

(b) Put the data into subgroups.

(c) Calculate the mean and range of each subgroup.

(d) Calculate the overall mean $\left(\overline{X}\right)$ and the mean range $\left(\overline{R}\right)$.

(e) Estimate the process standard deviation by using $\overline{R}/d_n$, where d_n is Hartley's constant which is tabulated for various values of n in the appendices of many SPC textbooks.

(f) Construct the control chart for monitoring the mean or variability by using $\overline{X}$, $\overline{R}$, or $\overline{R}/d_n$, and appropriate constants, for the computation of the control limits.

(g) Plot the group means on the mean chart and the group ranges on the range chart.

(h) If the control charts show that the process is in control then these charts or other charts of a more appropriate type based on the same estimates can be used for monitoring the process in the future.

(i) If the control charts show that the process is out of control, investigate the assignable causes and take corrective actions. Then repeat steps (a)-(i).

Subgroups in step (b) should be selected in a way that makes each subgroup as homogenous as possible with maximum variation between subgroups and not within them. It should be noted that the control limits are computed so as the variation of the plotted statistic is only due to common causes and the measure of dispersion used for the calculation of the control limits is based on the within subgroup variability. This note is stressed by many authors on SPC who warn that when setting up a control chart it is not correct to use the estimate of the process standard deviations from all the data, because this estimate can be affected by the between-samples variation.

The purpose of using a control chart is to compare current observations from the process to the process expectation based on past values in order to keep the process under control. This is the reason that when setting up a control chart if the chart indicates an out-of-control situation the whole procedure is repeated until the control charts show only in-control process points and then the final estimations are established for the parameters to be used for future control charts.

2.12.2 Choosing between Variables and Attributes Control Charts

Shewhart control charts for attributes are easier to implement than the Shewhart control charts for variables since they do not require two control charts (one for the mean and one for the variability, since one chart is responding to changes to both of them) and they do not require actual measurements but just the number of nonconforming items or the number of nonconformities in a sample, which sometimes is easier than exact measurements (for example it is easier to monitor if patients survived for a specific time interval after a surgery than monitoring exactly how long they lived) or quicker or less expensive. Other advantages of the attributes control charts are that they can be used for visual inspections of items and can be applied to several different nonconformities at the same time, while for variables a separate control chart is required for each monitored quality characteristic. On the other hand, the attributes control charts need good definition of the specified requirements for an item to be categorized as conforming or nonconforming (otherwise the classification is completely subjective thus leading to inconsistencies) and require larger sample size (equal to or more than 50) than the control charts for variables (a sample size of four or five can be adequate). The sample size must be large enough to allow defects or defective items to be observed in the sample, otherwise the attributes control charts will present the wrong indication of process improvement due to many samples with zero defects or defective items. On the contrary, the items required for variables control charts are much less than those required for the attributes charts. The latter is particularly important for cases when the testing is destructive or very expensive. Variables control charts are preferable in this case even if variables inspection is more expensive and more time-consuming than attributes inspection. Moreover, if attributes control charts indicate an out-of-control situation then the number of nonconforming items that should be rejected or the number of nonconformities on an item is unacceptable, while variables control charts give a signal before the detection of something unacceptable or before the number of rejected items increases in the process and when they signal, they usually help more in identifying the special cause. It should also be noted that, as we will see later in the relevant section (2.12.5), the attributes charts are not appropriate for rare event data. Furthermore, attributes data contain less information than the variables data (for example when plotting only the concentration of an ingredient which is higher than a specific value instead of plotting all concentrations) and, therefore, attributes control charts cannot detect out-of-control shifts or give warnings as easy and quickly as the variables control charts. So, whenever it is possible to choose between the two types, variables control charts are generally preferable to attributes control charts.

There are, however, some cases when it is particularly suggested to use mostly variables or attributes control charts. More specifically, Montgomery (2009) suggests choosing variables control charts in new processes or processes which have problems continuously, when testing is destructive or expensive, when trying to diagnose problems in a process or change process specifications, or when continuous demonstration of process stability and capability is required. Moreover, when attributes charts have already been used for monitoring a process, but the process still remains out of control or is in control but the process outcome is still unacceptable, then variables control charts are definitely required. On the other hand, attributes control charts are chosen when we need to monitor processes for which measurements cannot be obtained or processes which are complex groups of operations (such as the production of computers or cars or parts of them) where the output quality is measured in the existence of defects or not and successful or unsuccessful output performance. Attributes control charts are also useful for a historical summary of the performance of the monitored process.

2.12.3 Shewhart Control Charts for Variables

When dealing with variables, it is very important to first determine the subgroup size to decide which control chart to use, because when we monitor the variability we must be careful in the choice of estimator for the standard deviation. Although the range R is usually preferable due to its simplicity, the sample standard deviation s should be preferred in cases of moderately large sample size (n>10), because then R is not statistically efficient for estimating the standard deviation, or in cases of variable sample size.

The control chart for monitoring the process mean is the X-bar chart, which is constructed as follows (when using the sample ranges for estimating the process variability):

$$UCL = \bar{\bar{x}} + A_2 \bar{R}$$
$$CL = \bar{\bar{x}}$$
$$LCL = \bar{\bar{x}} - A_2 \bar{R}$$

where $\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$ is the average of averages and $\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m}$ is the average of sample ranges of the m subgroups in the sample and $A_2 = 3/\left(d_2\sqrt{n}\right)$, with the usual 3-sigma limits convention. For the computation of A_2, $d_2 = EW = ER_n/\sigma$ (which is the expectation of the relative range W) is used, with R_n being the range of a sample size n from a normal distribution of variance σ^2. The constant A_2 is tabulated for various sample sizes in the appendices of many SPC textbooks.

If we are interested in detecting moderate to large process shifts (on the order of two standard deviation units or larger) when using this control chart, then relatively small samples (n=4, 5, or 6) are quite effective. When, however, we want to detect small shifts then larger sample sizes are definitely required (possibly n=15 to n=25). An alternative would be the use of CUSUM or EWMA control charts.

When using the standard deviation instead of the sample range, the control limits of the X-bar chart are constructed as follows:

$$UCL = \overline{\overline{x}} + A_3 \overline{s}$$
$$CL = \overline{\overline{x}}$$
$$LCL = \overline{\overline{x}} - A_3 \overline{s}$$

where $\overline{\overline{x}} = \frac{1}{m}\sum_{i=1}^{m} \overline{x}_i$ is the average of averages and $\overline{s} = \frac{\sum_{i=1}^{m} s_i}{m}$ is the average of standard deviations of the m subgroups in the sample and $A_3 = 3/\left(d_3\sqrt{n}\right)$, with the usual 3-sigma limits convention. For the computation of A_3, $d_3 = ES_n/\sigma$ is used, with S_n being the standard deviation of a sample size n from a normal distribution of variance σ^2.

The variability of a process can be monitored using either the sample range R (R chart) or the sample standard deviation (s chart). The R chart is constructed using the following equations:

$$UCL = D_4 \overline{R}$$
$$CL = \overline{R}$$
$$LCL = D_3 \overline{R}$$

where $D_3 = 1 - 3(d_3/d_2)$ and $D_4 = 1 + 3(d_3/d_2)$. Here d_2 is defined as previously and d_3 is the standard deviation of W which means $d_3 = \sigma_R/\sigma$, where σ_R is the standard deviation of R. The constants D_3 and D_4 are also tabulated for various values of n in the appendices of many SPC textbooks.

It should be noted that for small sample sizes the R chart is relatively insensitive to shifts in the process standard deviation. Therefore, sample sizes larger than n=5 are preferable, since they are more effective, but we should always remember that the efficiency decreases as the sample size increases. For n>10 a control chart for s or s^2 should be preferred. Moreover, the s chart is less sensitive than the R chart to shifts caused by just one of the observations in the sample.

The control limits and central line of the s control chart are computed as follows:

$$UCL = B_6\sigma$$
$$CL = c_4\sigma$$
$$LCL = B_5\sigma$$

where $B_5 = c_4 - 3\sqrt{1-c_4^2}$ and $B_6 = c_4 + 3\sqrt{1-c_4^2}$. Here $c_4 = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma(n/2)}{\Gamma[(n-1)/2]}$ is a constant depending on the sample size n and $E(s) = c_4\sigma$. When using the unbiased estimator $\bar{s}/c_4$ of σ, where $\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i$ is the average of the standard deviations of the m samples, then the control limits and central line of the s chart become:

$$UCL = B_4\bar{s}$$
$$CL = \bar{s}$$
$$LCL = B_3\bar{s}$$

where $B_4 = B_6/c_4$ and $B_3 = B_5/c_4$, and the corresponding $\bar{x}$ chart is constructed as follows:

$$UCL = \bar{\bar{x}} + A_3\bar{s}$$
$$CL = \bar{\bar{x}}$$
$$LCL = \bar{\bar{x}} - A_3\bar{s}$$

where $A_3 = 3/\left(c_4\sqrt{n}\right)$. Values of the constants c_4, A_3, B_3, B_4, B_5 and B_6 are also tabulated for various values of n in the appendices of many SPC textbooks.

Attention is required to the estimator of the standard deviation used when constructing the control charts so far. All the above formulas are based on the use of the unbiased estimator s^2 of σ^2 which uses n-1 in the denominator, which means that in the above formulas $s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}}$. If s is defined with n in the denominator instead of n-1, then the constants c_4, A_3, B_3, B_4 are replaced with the constants c_2, A_1, B_1, B_2, respectively, as defined in Bowker and Lieberman (1972).

A subject of discussion regarding the $\bar{x}$ and s charts concerns whether or not these two charts should be combined into a single chart, because a shift in variability can affect the performance of the $\bar{x}$ chart, although a shift in the process mean will not have an effect on the performance of the s chart other than possibly affecting a single subgroup. Combination charts have been proposed by Chao and Cheng (1996), Chen and Cheng (1998), and Hawkins and Deng (2009) among others, but when the combined chart raises a signal, there is no indication whether the shift occurred in the mean or the variance or both and this can be figured out only by looking at individual $\bar{x}$ and s charts. Moreover, when simultaneously monitoring the process mean and variability, there is always the possibility of misleading signals, namely a signal from the chart for the mean (variability) being misinterpreted as a signal from the chart for the variability (mean) as was noted by John and Bragg (1991).

So far we used either the range R or the standard deviation s to construct control charts for monitoring the process variability. If we want, however, to monitor it directly with the sample variance instead of using the sample standard deviation, then the s^2 control chart can be used, which is defined with probability limits constructed as follows.

$$UCL = \frac{\bar{s}^2}{n-1}\chi^2_{\alpha/2,n-1}$$

$$CL = \bar{s}^2$$

$$LCL = \frac{\bar{s}^2}{n-1}\chi^2_{1-(\alpha/2),n-1}$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-(\alpha/2),n-1}$ are the upper and lower $\alpha/2$ percentage points of the chi-square distribution with n-1 degrees of freedom and $\bar{s}^2$ is the average sample variance obtained from preliminary data if σ^2 is not known. If it is known, however, it can be used in the above equation instead of $\bar{s}^2$. The effect of measurement errors on the Shewhart S^2 control chart was studied by Linna and Woodall (2001).

2.12.4 Shewhart Control Charts for Individual Measurements

The last two control charts of the category of Shewhart control charts for variables are the ones for monitoring the mean and variability of a process with individual measurements. These charts are useful in many cases in which the available data consist of samples of only one observation. This is very common nowadays with modern and automated inspection and measurement technology, since this way every process unit is examined and there is no basis for rational subgrouping. There are also cases of short production runs, where the output consists of only a very small amount of items in the entire run. Moreover, there are situations in which all units are monitored such as service applications or situations in which testing can be destructive or sampling and measuring can be very expensive and/or time consuming, thus making individual sampling a one-way choice. There are also cases when the production rate is low or the data are slowly available such as in the clerical or accounting sector and other manufacturing or non-manufacturing processes. Examples of these situations include time to complete a task, equipment or machine downtime, examination marks, delays in time or costs due to, for example, late delivery or breakdowns and sales or number of complaints during a month. If the interval between two consecutive produced items is large, there is significant opportunity for process shifts between them and, therefore, by the time of the next sample the process might have already come out of control, so the control charting will become too slow to react to problems. In such cases, it is not safe to assume that even two successive units are produced under the same conditions and, therefore, the only natural choice is to draw samples consisting of only one unit. Furthermore, sometimes successive observations differ only due to measurement errors or errors during the analysis as happens very usually in chemical processes or differ very little as is the case when monitoring variables such as temperature, thickness and pressure. Other situations where samples consist of individual observations are the cases of taking multiple measurements at several different locations on the same unit or the case of using control charts to determine the capability of a process to meet specification limits. The latter is justified by Burr (2004) by the fact that consumers purchase individual items and not groups of them. Therefore, they are interested in each item's quality and that is the way it should be monitored: individually.

The data from all such processes which consist of only one observation can be monitored using a Shewhart control chart for individual observations (when we are interested in a large magnitude of shift). For a smaller shift magnitude, on the other hand, alternative control charts such as the CUSUM and EWMA, that will be presented later (sections 2.14.1 and 2.14.2), would be a better choice.

The Shewhart control chart for individual observations (X chart) is constructed as follows:

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2}$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2}$$

where $\overline{MR}$ is the average of the moving ranges (MR) of two observations, with $MR_i=|x_i-x_{i-1}|$. The term d_2 is the Hartley's constant, as earlier. If a moving range of two observations is used, then $d_2=1.128$. It should be noted that the standard deviation is estimated using moving ranges instead of the usual estimation which can be affected by outliers and non-normality.

The X control chart, despite its simplicity in construction and use, is not as good in detecting small shifts in the process mean as is the Shewhart chart for the mean and needs more samples to detect changes of the same magnitude (the power of the X chart is less than the power of the Shewhart mean chart, whose power increases as the sample size increases). Moreover, the X chart cannot distinguish between changes in the process mean or variability. Therefore, it should not be preferred when there is a choice of using another control chart. All the above are also the reason that a moving range control chart (MR chart) is usually used together with an X control chart, for monitoring the variability of data with individual observations. The control limits of the MR chart are constructed as follows:

$$UCL = D_4 \overline{MR}$$
$$CL = \overline{MR}$$
$$LCL = D_3 \overline{MR}$$

If a moving range of two observations is used D_3=0 and D_4=3.267.

The interpretation of the control charts for individual observations is similar to the interpretation of the control charts for the means. We first start with the interpretation of the MR chart, to insure that variability is in-control and then proceed with the interpretation of the X chart. If a point plots outside the control limits this is an indication of an out-of-control process. Attention should be paid, however, to the fact that the moving range points are correlated. This correlation can cause patterns of runs or cycles and, therefore, only points beyond the control limits indicate out-of-control signals on a MR control chart. On the contrary, any pattern on the X control chart should be investigated since the individual observations are assumed to be uncorrelated. If both X and MR charts present points beyond the control limits, the spike on the MR chart can help in the identification of the exact point when the process mean shift occurred. In case of runs on an MR chart, as mentioned in Stapenhurst (2005), it is suggested that "14 points below the mean are required before a process change is indicated". Moreover, because of the relative insensitivity of the X chart at identifying out-of-control situations comparative to the means chart, Stapenhurst (2005) suggests using warning limits to increase the chart's sensitivity and mentions some recommendations for four or five consecutive points beyond additional ±1σ warning limits.

Crowder (1987c) presented the ARL values for the combination of X and MR control chart for various shifts in the process mean and standard deviation and showed that the in-control ARL is much less than the corresponding one for the

Shewhart control chart for the mean if traditional three-sigma control limits are used. He suggested combining the use of the three-sigma limits for the X chart with a different computation of the upper control limit for the MR chart in order to get an in-control ARL value close to that of the Shewhart means control charts. More specifically, he suggested using $UCL = D\overline{MR}, \quad 4 \leq D \leq 5$.

The MR control chart is debatable in literature, since some researchers recommend using it while others oppose. Amin and Ethridge (1998), for example, suggest using X and MR charts together for better detection of shifts than when using only the X chart. On the other hand, for example, Roes et al. (1993) and Rigdon et al. (1994), support the opinion that the MR chart cannot really provide useful information about a shift in the process variability but also shows shifts in the process mean that are presented in the X chart and, therefore, MR chart is not necessary. Moreover, they recommend using the X chart for monitoring both the process mean and process variability, since an increase in variability will cause points on the X chart to be plotted at a greater distance from the central line and if the variability shift is large enough there will be points beyond the control limits of the X chart as well. Montgomery (2009) supports the uselessness of combining X and MR charts but does not discourage it and suggests being careful with the interpretation when using both charts and depending mainly on the X chart.

2.12.5 Shewhart Attribute Control Charts

When the available data are attributes data, before choosing the control chart to use for monitoring them, we need to distinguish between defects and defective items. This is important, because different control charts are used for each of those categories of data. Besides, an item with a defect is not necessarily defective (minor defect(s) not affecting the performance) and a defective item can have one or more defects. When we are interested in monitoring the number of defective items, the control charts are constructed based on the binomial distribution, while control charts for monitoring the number of defects are constructed based on the Poisson distribution. The control charts for monitoring the defectives are the p-charts and the np-charts, while the control charts for the number of defects are the c-charts and u-charts. One of the major differences between the variables control charts and the attributes control charts is that when dealing with variables data (individual observations or not) we need two different charts for monitoring the process mean and process variability while control charts for attributes respond to both mean shifts and variance shifts.

The p-charts (used for monitoring the proportion of nonconforming or defective items) and the np-charts (for monitoring the number of nonconforming items in a sample of size n) have similar control limits especially for constant sample size (in which case the two control charts result in the same conclusions about the process) simply due to the fact that the number of items is equal to the product of the proportion multiplied by the sample size. The control limits of the p-charts are constructed using the binomial distribution with parameters the sample size n and the proportion p. Therefore, according to equation (2.1) the control limits and central line of this chart are given by

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$
$$CL = p$$
$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$$

Similarly the control limits and central line for the np chart are given by

$$UCL = np + 3\sqrt{np(1-p)}$$
$$CL = np$$
$$LCL = np - 3\sqrt{np(1-p)}$$

The sample size required for the construction of the p-chart is suggested by Duncan (1986) to be large enough to give a 50% probability of detecting a process shift of a specific size. According to Montgomery (2009) the required sample size n for the detection of a shift of size δ is given by $n = \left(\frac{L}{\delta}\right)^2 p(1-p)$ for the L-sigma Shewhart control limits. For a small fraction of nonconforming items, the value of

n is chosen so as to have a positive LCL which means that $n > \frac{(1-p)}{p} L^2$. Another problem with the p-charts is the possibility of very small values of UCLs. In a situation like that the control chart will signal with any nonconforming unit in a sample, thus increasing the false alarm rate of the chart. A larger sample size can give the solution to this problem, too. The false alarm rate can also be quite different from the desired due to the discrete nature of the Binomial distribution. Lucas et al. (2010) suggested the addition of a value equal to $1/n$ to the computed value of the UCL of the chart in order to make the false alarm rate of the chart smaller. Another performance-related problem of the p-chart is the bias in its run length performance. A solution was presented by Acosta-Mejia (1999) who proposed a run length unbiased p-chart. Research presenting the problems of the p-charts includes Goh (1987), Xie et al. (1999), Chan et al. (2003a) and Goh and Xie (2003). Xie and Goh (1993) and Schwertman and Ryan (1997) dealt with p-charts with probability limits. Ryan and Schwertman (1997) dealt with the sensitivity of the p-chart and compared the use of probability limits with the traditional 3-sigma limits in terms of out-of-control ARL proving that these charts performed better and then proposed a p-chart with adjusted control limits using the approximation of the Binomial distribution by a Poisson distribution. Ryan and Schwertman (1999), later on, presented a more flexible dual control chart method. Nelson (1997) dealt with supplementary runs rules to increase the sensitivity of the np charts, while Vaughan (1992, 1993) had discussed the issue of np control charts with variable sampling interval. Ho and Quinino (2013) presented an attribute control chart for monitoring variability and compared its performance to that of R and S^2 charts.

For the case of the number of defects charts, the choice between c-charts and u-charts is based mostly on the opportunity for the monitored event. If the opportunity remains the same, the c-charts are used. Otherwise, u-charts are chosen. The u-charts are also chosen in the case of more than one inspection units in our sample. Both control charts assume infinitely large number of opportunities or potential locations of defects (or nonconformities) and small probability of occurrence of a defect at any location in order for the Poisson distribution to fit well. This is the reason that, when monitoring data with low number of rejects compared to the potential number of rejects, the results are similar either we use p- and np- charts or c- and u- charts [Stapenhurst (2005)]. The c-charts for the number of defects or nonconformities are constructed as follows:

$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$

The type I error risk with these control limits is not equally allocated above and below the control limits due to the fact that the Poisson distribution is skewed and, therefore, probability limits are suggested for this chart especially for small c value.

The u-chart for the average number of defects (or nonconformities) per inspection unit is constructed as follows:

$$UCL = u + 3\sqrt{\frac{u}{n}}$$
$$CL = u$$
$$LCL = u - 3\sqrt{\frac{u}{n}}$$

When having to choose between using control charts for the number of defects or fraction of nonconforming items, Montgomery (2009) suggests the control charts for the number of defects or nonconformities, because they are more informative due to the fact that usually there are several different types of defects and analysis of defects by type gives more information about their possible causes, which is very helpful during corrective actions in case of an out-of-control signal.

When monitoring a process based on the output defects, sometimes it is possible to find many types of defects of different amount of importance. This is very usual, for example, in the production of computers, cars, automated equipments and big appliances. In such cases an item with some less serious defects (for example in appearance) which do not affect its performance may not be defective or nonconforming. The defects are, therefore, classified into four categories depending on their severity (the effect they have on the performance) and the number of demerits in an inspection unit is the weighted sum of the number of defects in each class in that particular inspection unit. The weights which are usually used in practice are 100 for Class A (very serious defects), 50 for Class B (serious defects), 10 for Class C (moderately serious defects) and 1 for Class D (minor defects). The classes are assumed to be independent and Poisson distributed (with reasonably large parameter values) and all n inspection units are assumed to be of the same size. Then the number of demerits per unit $u_i = \frac{D}{n}$ (where D is the total number of demerits in all n inspection units) can be monitored with a D chart constructed as follows:

$$UCL = \overline{u} + 3\hat{\sigma}_u$$
$$CL = \overline{u}$$
$$LCL = \overline{u} - 3\hat{\sigma}_u$$

where $\overline{u} = 100\overline{u}_A + 50\overline{u}_B + 10\overline{u}_C + \overline{u}_D$ and $\hat{\sigma}_u = \left(\frac{100^2\overline{u}_A + 50^2\overline{u}_B + 10^2\overline{u}_C + \overline{u}_D}{n} \right)^{1/2}$. The average number of defects in each class per inspection unit is obtained from data drawn from an in-control process. The properties of the D chart were studied by Jones et al. (1999) who suggested the use of probability limits which lead to a chart with superior performance than that of the chart with the three-sigma limits which was presented above. Type II errors of demerit control charts were investigated by Chimka and Arispe (2007), while Chimka and Arispe (2006) proposed a demerit control charts for Poisson distributed defects. One of the most recent applications of demerit control charts was proposed for the textile sector by Yılmaz and Yanık (2020).

Control charts for attributes were first proposed by Shewhart (1926, 1927). They have been used in many areas ever since, from health care [Woodall (2006), Albers (2009)] to animal sciences [Vries and Teneau (2010)]. They have been studied, improved, extended or altered in order to achieve better performance. Early reviews on attribute control charts were presented by Woodall (1997), Woodall et al. (1997), Mohammed et al. (2003) and Jones-Farmer (2008). Other more recent reviews include Szarka and Woodall (2011), Jahromi et al. (2012) and Saghir and Lin (2015a).

Dorris (1977) investigated the effects of inspection errors on the performance of c charts. Soffer (1981) addressed a transformed p chart for monitoring data with variable sample size. Sculli and Woo (1982) dealt with the design of np charts. Nelson (1983a) proposed an early-warning test for Shewhart p-charts. Suich (1988) studied the c chart with inspection error. Chan and Xiao (1990) introduced weighted attribute control charts for variable sample size. Padgett and Spurrier (1990) developed Shewhart-type control charts for monitoring percentiles of strength distributions. Rocke (1990) presented adjusted p- and u- charts for monitoring data with varying sample sizes. Bonnett (1993) addressed the issue of determining the appropriate sample size for p charts. Nayebpour and Woodall (1993) investigated Taguchi's online attributes control charts. Winterbottom (1993) proposed adjustments for improving the control limits of attributes control charts. Grayson et al. (1995) studied the performance of u charts with control limits based on the average sample size. Chen (1998) introduced some adjustments for improving the p charts. Braun (1999) investigated the performance of the p- and c- charts with estimated control limits. Wu et al. (2001) presented np charts with fractional control limits. Jolayemi (2002) addressed the statistical design of np charts with multiple control regions. Chan et al. (2003b) presented a continuity adjustment for the np-chart and the c-chart for attributes by adding a Uniform(0,1) distributed random observation to the conventional sample statistic in order to make its distribution continuous and constructing the control limits given the type I risk. They also provided comparison and guidelines for the selection of the proper control chart among the proposed continuity adjustment control chart, the traditional Shewhart control chart and the control chart based on the exact distribution of the unadjusted statistic. Khoo (2003) increased the sensitivity of control charts for fraction nonconforming. Wu and Luo (2003) discussed the three-triplet np charts. Khoo (2004a) introduced a moving average control chart for monitoring the fraction nonconforming. Khoo (2004d) investigated the performance of the moving average control chart for Poisson distribution compared to the c chart for monitoring nonconformities. Kittlitz (2006) investigated the c chart. Tsai et al. (2006) proposed square root transformation-based attribute control charts. Wu et al. (2006) developed an np chart with curtailment which doubled the detection effectiveness of the conventional np chart. Hart et al. (2007) considered p charts with small subgroup sizes. Wu and Wang (2007) addressed an np chart with double inspections. Chakraborti and Human (2008) dealt with the performance of c-charts in Phase II applications. Morris and Riddle (2008) investigated the sample size required for detecting quality improvements with p charts. Sim and Lim (2008) discussed attribute charts for monitoring zero-inflated processes. Wu et al. (2009d) introduced a new type of np chart for monitoring the mean of a variable based on an attribute inspection using warning limits instead of specification limits for the classification of inspected units as conforming or nonconforming. They proved that, although it is less effective than the $\bar{X}$ chart when using the same sample size and sampling frequency, it can become more effective than the $\bar{X}$ chart in terms of ATS and extra quadratic loss when optimizing the warning limits and using a greater sample size and/or sampling frequency (which is possible with this chart since it does not require any computation due to attribute inspection which is less costly) and allows operators to take corrective actions before actually producing any defective items. Abooie and Aminnayeri (2010) studied the np chart with variable limits. Shu and Wu (2010) addressed p charts for monitoring imprecise fraction nonconforming. Duclos and Voirin (2010) used the p-chart for healthcare-related process improvement. Perez et al. (2010a,b) dealt with the optimization of DS u charts. Ho and Costa (2011) considered monitoring a wandering mean with an np chart. Ho et al. (2011) introduced an alternative np chart in the presence of non-constant misclassification errors with a similar performance (in terms of ARL values) to a traditional np chart without classification errors, using monitoring statistics which are based on the results of independent repeated classifications with classification errors during the inspection process. They found that there are many possible combinations of the sample size and the number of repeated classifications that give an ARL value similar to that of a control chart without misclassification errors. Therefore, they discussed the optimal choice for this combination by minimizing a cost function for the required ARL value which included the cost for a new unit and the cost of repeated classifications. Chen and Song (2012) studied the effects of sample sizes on the performance of p charts during Phase I and Phase II. Park (2013) introduced an improved p chart based on the Wilson interval. An economic alternative to the c chart was proposed by Black and Chimka (2014). Lupo (2014) used the Taguchi loss function to design c charts. Aslam et al. (2015c) introduced a mixed chart for monitoring process quality using attribute data combined with variable data and three pairs of control limits and proved the proposed chart's superiority over the traditional np chart in terms of quick detection of processs shifts. Tiplica (2015) studied the performance of c charts with estimated parameter. Chakraborty and Khurshid (2016) investigated the effect of misclassification due to measurement error on the power of control chart for proportions. Hernández and Garcia (2016) dealt with risk estimation in np charts. Mohammad et al. (2016) developed improved p-charts. Morais (2016) proposed an ARL-unbiased np control chart. Paulino et al. (2016a) introduced an ARL-unbiased c chart, while Paulino et al. (2016b) developed ARL-unbiased c charts for monitoring autocorrelated Poisson data. Wu et al. (2016) presented c- and np-charts with run rules for monitoring processes with estimated parameters. Zhao and Driscoll (2016) discussed c charts with bootstrap adjusted control limits. Faraz et al. (2017) studied the performance of the np-chart. Lee and Khoo (2017) proposed an np chart combining double sampling and variable sampling interval. Aslam et al. (2018a) designed an attribute control chart for two-stage process while Erginel et al. (2018) dealt with attribute control charts with fuzzy sets. Argoti and García (2017, 2018) studied the ARL-bias in Shewhart p-charts. Altuntas et al. (2018) introduced the standardized u-chart which combines the service quality scale and used it for monitoring patient dissatisfaction in hospitals. Argoti and Carrión-García (2018) proposed quasi ARL-unbiased p-charts.

Before we proceed, it should be noted that attributes control charts are based on the assumption of specific underlying distributions which are not always valid. If those assumptions are valid then the attributes control charts are preferable. If the assumptions are not valid, however, attributes control limits will not be accurate and, therefore, individual control charts which do not depend on any assumption are more appropriate. Moreover, as Stapenhurst (2005) states, two usual suggestions are to use individual control charts instead of attribute ones whenever the average of the plotted data is greater than 1 or 5. Stapenhurst (2005) suggests using both attributes and individual control charts when in doubt and if any inconsistencies are found then careful thought is required in order to understand the reason for those inconsistencies and, therefore, better understand the process being monitored.

2.13 Control Charts Dealing with Rare Events and Low Rates of Defects and Time-Between-Events (TBE) Control Charts

When the rate of defects in a process is very low, for example at the level of parts per million, then there will be a lot of samples containing zero defects and c- and u- control charts will be ineffective. A solution to this problem would be to use a control chart for the time between consecutive occurrences. If the monitored defects or events are assumed to be Poisson distributed, then the distribution of the time between them will be the exponential distribution. Therefore, the control charts for the time between events (TBE) will be constructed based on the exponential distribution, which is very skewed and the control chat will be very asymmetric. In order to solve this problem, Nelson (1994) proposed transforming the exponential random variable to a Weibull random variable, because the Weibull distribution is well approximated by the Normal distribution. If X is an exponentially distributed random variable then, according to Nelson (1994), the appropriate transformation for a good Normal approximation is $X^{1/3.6} = X^{0.2777}$.

Rare event data are quite common in real world situations. Some examples include accidents involving airplanes or trains, serious injuries at work, resignations, breakdowns or natural disasters such as fires. A definition from the viewpoint of control charts as presented by Stapenhurst (2005) is that rare events occur when the process average falls below 1 or the LCL is 0.

As mentioned earlier, low rate events are monitored with individual control charts. If the occurrence rate, however, is very low as is the case with rare data, then the individual control charts present the same problem as the attributes control charts, namely many zero observations and occasionally a non-zero one, which would cause the chart to signal every non-zero value as an out-of-control observation. Therefore, other approaches must be followed for the monitoring of rare events in order to reduce the false alarm rate. The most usual approach is to count the TBE and convert it into a number of events per an appropriate interval (for example per month or year). These results will then be monitored with an X/MR control chart. Other approaches would be to increase the sample size or to combine groups (for example monitor incidents quarterly instead of monthly) in order to avoid monitoring rare events [Stapenhurst (2005)].

One of the first studies of control charts for monitoring cases of zero defects was the one by Calvin (1983). Other research regarding the cases of monitoring law defect rates and/or TBE control charts includes Goh (1987, 1991), Lucas (1989), Lawson and Hathaway (1990), Goh and Xie (1994, 1995), Govindaraju and Lai (1998), McCool and Joyner-Motley (1998), Radaelli (1998), Xie et al. (1998, 2002a), Chan et al. (2003a), Liu et al. (2004), Pan (2004), Steiner and MacKay (2004), Di Bucchianico et al. (2005), Zhang (2006), Liu (2007), Zhang et al. (2007b, 2011b), Yeh et al. (2008), Khoo and Xie (2009), Shamsuzzaman et al. (2009), Zhang (2009), Xie et al. (2010), Albers (2012), He et al. (2012b), Xie (2012), Acosta-Mejia (2013), Qu et al. (2014, 2015a), Woodall and Driscoll (2015), Fang et al. (2016), Ali and Pievatolo (2016, 2018), Ali (2017) and Sanusi and Xie (2017). Bourke (1992) investigated the performance of CUSUM charts for monitoring processes with low count level. Jones and Champ (2002a,b) dealt with Phase I TBE control charts. Ranjan et al. (2003) discussed control chats for monitoring inter-arrival times. Alemi and Neuhauser (2004) applied control charts for TBE to monitoring asthma attacks. Chang and Gan (2007) introduced a modified Shewhart np chart for monitoring high-yield processes with very low defect level (cloze to zero) using runs rules and compared its run length performance with that of other control charts for high-yield processes. They also presented the design procedure for the proposed chart for samples or 100% inspection to facilitate its use in practice. Zhang et al. (2007a) presented control charts for monitoring Gamma distributed TBE. Lai and Govindaraju (2008) addressed the reduction of signal variability in control charts for monitoring high-quality processes. Ozsan (2008) studied the effect of estimation errors on TBE EWMA control charts for high-quality processes. Pehlivan (2008) investigated the robustness of the lower-sided TBE EWMA charts. Sego et al. (2008) conducted a comparison of control charts for monitoring small rates. Wu et al. (2009b,c) presented two charts for simultaneously monitoring the time interval and the magnitude of an event. Wang (2009a) compared p-charts for low defective rate. Gan and Tan (2010) presented risk-adjusted control charts for monitoring the number between failures for patients with heart problems while Gandy et al. (2010) applied risk-adjusted control charts for monitoring time to events. Liu et al. (2010) introduced a probability-type control chart for simultaneously monitoring the frequency (time interval between the occurrences, which was assumed to follow an Exponential distribution) and size of an attribute event (which was assumed to follow a Poisson or truncated Poisson distribution) and proved that the proposed chart was more effective than seperate control charts for the frequency and magnitude particularly for detecting downward shifts (smaller TBE and/or smaller event size) and its effectiveness was more invariable against the types of shifts (frequency shift, magnitude shift or both). Qu et al. (2011) introduced the T&TCUSUM chart, which combines a Shewhart T chart and a TCUSUM chart for monitoring the time interval T between the occurrences of an event or the TBE and was proved to perform better than other charts since it was more sensitive to both small and large shifts. Szarka and Woodall (2011) provided a review of control charts for high quality binary processes. Doğu (2012) applied control charts for monitoring the time between medical errors. Dovoedo and Chakraborti (2012) proposed boxplot-based control charts for monitoring TBE during Phase I. Luo et al. (2012) used CUSUM charts for TBE data for online radiation monitoring. Mastrangelo and Gillan (2012) dealt with the monitoring of relatively low rates of hospital-related infection incidents with g-type control charts for monitoring days between infections and other g-type control chart alternatives and Negative Binomial control charts. Fang et al. (2013) introduced synthetic-type control charts for monitoring TBE. Joekes and Barbosa (2013) introduced control charts for monitoring fraction nonconforming in high quality processes. Bersimis et al. (2014) proposed a compound control chart for monitoring high-quality processes. Kumar and Chakraborti (2015) dealt with Phase I control charts for monitoring TBE. Luong and Htet (2015) constructed control charts for monitoring TBE for nonconforming units in high-quality processes. Qu et al. (2015b) developed a CUSUM chart for monitoring TBE. Ali et al. (2016) provided an overview of some control charts used for monitoring high-quality processes. Kumar and Chakraborti (2016) studied the effect of parameter estimation on Shewhart-type control charts for monitoring TBE. Chakraborty et al. (2017b) presented a generally weighted moving average control chart for monitoring TBE. Fallah and Jafarian (2017) noted the inaccuracy of traditional Shewhart charts (even with adjusted control limits) when monitoring high quality processes with very low fraction nonconforming and proposed an np-chart for high quality processes with adjustments for the control limits obtained from Cornish-Fisher expansions in order to improve the in-control performance. Kumar and Chakraborti (2017) proposed a Bayesian statistically designed Shewhart-type chart for TBE monitoring when the interarrival times are assumed to follow an Exponential distribution. Kumar et al. (2017) dealt with Shewhart-type charts for monitoring TBE. Mao et al. (2017) investigated the performance of Wheeler's control chart for monitoring the rate of rare events. Nezhad and Jafarian-Namin (2017) considered adjusted limits for the control chart for monitoring fraction nonconforming in high-quality processes. Alevizakos et al. (2018) used a double EWMA chart for monitoring TBE. Sanusi and Mukherjee (2019) introduced a control chart for monitoring TBE and event magnitudes simultaneously, combining two plotting statistics (one for the magnitude and one for the TBE) into a single plotting statistic using max-type and distance measures. They illustrated the proposed chart with application to real data on damage caused by outbreak of fire disaster. They also compared the proposed control chart with the control charts proposed by Wu et al. (2009b,c) proving their chart's superiority (especially for detecting moderate to large shifts in the process parameters) and its ability to simultaneously detect upward shifts in the magnitude and downward shifts in the TBE and vice versa, contrarily to the other charts. Sanusi et al. (2020) discussed the Max-EWMA chart for monitoring simultaneously the event magnitude and the TBE.

2.14 CUSUM and EWMA Control Charts

Although Shewhart control charts are easily constructed, they have many drawbacks when their assumptions [such as specific underlying distribution, known parameters, independent and identically distributed data (see Sections 2.15 and 2.17)] are violated, which is often the case in real world applications. Moreover, the increasing need for better quality nowadays requires smaller process shifts to be detected, which, as has already been mentioned, is one of the weaknesses of Shewhart control charts. Therefore, other control charts with better performance have been developed. This section is dedicated to some of them, such as the CUSUM and EWMA charts which have been proved to be the more useful and efficient alternatives to the Shewhart control charts.

Other examples of alternative control charts (which are here omitted) include, among others, the moving average control charts (which are a special case of the EWMA charts when $\lambda = \frac{2}{w+1}$ where w is the moving average window [Mitra (2021)] and are generally less effective than EWMA charts in detecting small process parameters shifts), the cumulative count of conforming charts, the median and mid-range charts and control charts based on other sample statistics, the median moving range charts, difference control charts, standardized charts, synthetic charts, sequential probability ratio charts, cuscore and generalized likelihood ratio charts, Tukey's control charts, Bayesian control charts and fuzzy charts. Control charts have also been constructed based on various sampling schemes and using various economic and economic-statistical criteria for optimal control chart design. Moreover, control charts have been proposed for monitoring short production run processes and processes with censored data. Adaprive control charts have also been developed to cover the cases of variable sampling rate (depending on the position of the plotted statistics) and variable design parameters. The cases of risk-adjusted control charts and control charts for autocorrelated processes and profile monitoring have also been addressed in literature to deal with the occurrence of data independence assumption violation. All of the above, however, are beyond the scope of this thesis and, therefore, will not be covered herein.

Comparisons of Shewhart charts to other charts were conducted by Reynolds and Stoumbos (2004a), proving the overall good performance of CUSUM and EWMA charts. Comparisons of Shewhart and CUSUM charts from the economic point of view were conducted by Goel (1968), Von Collani (1987) and Saniga et al. (2006a,b, 2012), showing the cost advantages of CUSUM charts, which, however, are small considering the simplicity of Shewhart charts.

2.14.1 Cumulative Sum (CUSUM) Control Charts

Shewhart control charts are very effective for monitoring shifts of magnitude larger than 1.5σ to 2σ. For smaller shifts they become less effective. On the other hand, CUSUM control charts are a good alternative when we want to monitor smaller shifts. This is the result of the fact that, contrary to the Shewhart control charts which use only the current values, the CUSUM charts use the information from several sample values. Therefore, CUSUM control charts are very useful when dealing with individual observations. In fact, they are usually used with individual data and less with grouped data [Montgomery (2009)].

CUSUM control charts were first proposed by Page (1954) and studied by many authors ever since, such as Goldsmith and Whitfield (1961), Page (1961), Johnson and Leone (1962), Ewan (1963), Bissell (1969), Goel and Wu (1971), Gardiner et al. (1987), Hawkins (1981, 1992a,b, 1993), Woodall (1983, 1986), Waldmann (1986), Gan (1991a, 1993b), Woodall and Adams (1993), Hawkins and

Olwell (1998), Luceño and Puig-Pey (2000) and Musdalifah et al. (2017). An overview of the developments on CUSUM control charts was presented by Ruggeri et al. (2007a). As presented in Qiu (2014), there is a connection between the sequential probability ratio test and the CUSUM chart. This connection was used to obtain various optimality properties of the CUSUM control charts by researchers such as Lorden (1971), Moustakides (1986), Ritov (1990) and Yashchin (1993).

There are two ways to plot CUSUMs, the tabular (or algorithmic) CUSUM and the V-mask form of the CUSUM. The V-mask was proposed by Barnard (1959) and further studied by Johnson (1961) and Lucas (1973, 1976). Montgomery (2009), however, presented some problems with V-mask and "strongly advised against" using them.

2.14.1.1 CUSUM Chart for Monitoring the Process Mean

The tabular CUSUM chart for monitoring the process mean plots the sample number on the horizontal axis, while on the vertical axis it plots two statistics C^{+} and C^{-} which are called one-sided upper and lower CUSUMs, respectively, and are calculated as follows:

$$C_i^{+} = \max\left[0, x_i - \left(\mu_0 + K\right) + C_{i-1}^{+}\right]$$
$$C_i^{-} = \max\left[0, \left(\mu_0 - K\right) - x_i + C_{i-1}^{-}\right]$$

with the starting values being defined as $C_0^{+} = C_0^{-} = 0$ and K which is called the reference value (or the allowance or the slack value) being chosen halfway between the in-control and the out-of-control mean values or equivalently as one-half of the magnitude of the shift which we want to detect expressed in standard deviation units, which means that $K = \frac{\delta}{2}\sigma = \frac{\left|\mu_1 - \mu_0\right|}{2}$. With this control chart a process is considered as out of control if either of the two statistics plots beyond the decision interval $H=h\sigma$, with h being usually chosen as five times the process standard deviation σ. The choice of this parameter is critical for the control chart, because it affects the chart's performance. A combination of $k=\frac{1}{2}$ (if K=$k\sigma$) and h = 4 or h = 5, usually gives good ARL properties to the chart for monitoring a shift of one standard deviation unit in the process mean. Generally, we choose h so as to obtain a desired value of in-control ARL given the selected value $k=\frac{\delta}{2}$. For a small value of type II error probability, the decision interval is computed as $H=\frac{-\sigma^2\ln(\alpha)}{\mu_1-\mu_0}$ [Mitra (2021)]. Gan (1991a) presented graphs useful for choosing the design parameters for the construction of CUSUM control charts with the minimum out-of-control ARL value for a specific shift magnitude of interest for a given in-control ARL, while Hawkins (1993) presented various optimal combinations of those two parameters for achieving an in-control ARL value equal to 370. A program for the computation of CUSUM ARL was given by Vance (1986). ARL computation was presented by Brook and Evans (1972) based on the Markov chain approach and based on two different approximations by Hawkins (1992a) and Woodall and Adams (1993).

If the chart presents an out-of-control signal, we take the same actions as we would in a corresponding situation with any control chart. We investigate the process in order to find the assignable cause and proceed to corrective actions. Then we reset the CUSUM statistics to zero and continue using the CUSUM chart.

One of the most important advantages of the CUSUM chart is that it can help us identify the time point when the assignable cause occurred by counting backwards from the out-of-control signal until we reach the point when the value of the statistic became non-zero. This way we obtain the first period after the process shift. Another advantage of the CUSUM chart is that we can easily estimate the new process mean, using the counters N^+ and N^- of the number of consecutive periods for which the corresponding CUSUM statistics, C^+ and C^-, had a non-zero value before the chart's signal. This can be achieved using the relationships $\hat{\mu} = \mu_0 + K + \frac{C_i^+}{N^+}$ or $\hat{\mu} = \mu_0 - K - \frac{C_i^-}{N^-}$, depending on whether the upper or lower CUSUM statistic was the one which gave the signal.

2.14.1.2 The Standardized CUSUM Control Chart

An alternative to the CUSUM control chart for the mean is the standardized CUSUM chart. For this chart the observations x_i are first standardized to obtain $y_i = \frac{x_i - \mu_0}{\sigma}$ and then these values are used for the computation of the two CUSUM statistics which will be plotted on the chart, as previously. The two new statistics are computed as follows:

$$C_i^+ = \max\left[0, y_i - k + C_{i-1}^+\right]$$
$$C_i^- = \max\left[0, -k - y_i + C_{i-1}^-\right]$$

This control chart has the advantage that the choices of the two parameters, k and h, of the chart are not scale dependent. This chart leads naturally to the CUSUM chart for monitoring the process variability.

2.14.1.3 CUSUM Chart for Monitoring the Process Variability

The CUSUM charts for monitoring the process variability are constructed based on the method proposed by Hawkins (1981, 1993). The observations are first standardized as before and then used for the computation of a new standardized quantity which, according to Hawkins (1981, 1993) is sensitive to changes in the process variance rather than changes in the process mean, but according to Montgomery (2009) and Mitra (2021), is sensitive to both changes. The new standardized quantity is defined as

$$v_i = \frac{\sqrt{|y_i|} - 0.822}{0.349}$$

and it's in control distribution is approximately N(0,1). Using this quantity the CUSUM statistics which will be plotted on the CUSUM chart for variability are computed as follows:

$$S_i^+ = \max\left[0, v_i - k + S_{i-1}^+\right]$$
$$S_i^- = \max\left[0, -k - v_i + S_{i-1}^-\right]$$

where the initial values of the statistics are defined as $S_0^+ = S_0^- = 0$ and the values of the parameters *k* and *h* are chosen as for the CUSUM chart for the mean. The chart's interpretation is also similar to the one for the CUSUM chart for the mean. The CUSUM charts for mean and variability can be plotted separately or, as suggested by Hawkins (1993), on the same graph. If the CUSUM chart for variability gives an out-of-control signal, then a shift has occurred in the process variability, while if both charts present an out-of-control signal, then a shift has occurred in the process mean.

According to Hawkins and Olwell (1998), the optimal CUSUM control chart for monitoring the process variability is a CUSUM of the sum of squared deviations from a subgroup mean. This approach, however, can be affected by the distribution of the statistic, which should be taken under consideration by the CUSUM control chart instead of trying to transform the statistic to obtain approximate normality. So Hawkins and Olwell (1998) found the optimal values for *k* for the case of the Gamma distribution.

Acosta-Mejia et al. (1999) proposed three CUSUM charts for monitoring the process variability and compared various control charts for monitoring variability including Shewhart charts and their own and other CUSUM charts. They found that the CUSUM chart using the likelihood ratio test for the change point of a normal process variance was the best. An adoptive CUSUM for monitoring the process variability was proposed by Shu et al. (2010) using weights which change as the current estimate of the process variance changes. A CUSUM chart for monitoring the process variability was also introduced by Abbasi et al. (2012).

2.14.1.4 FIR CUSUM and Other CUSUM Chart Improvements

If we want to improve the sensitivity of the CUSUM chart at process start-up, then the Fast Initial Response (FIR) or headstart is used, as proposed by Lucas and Crocier (1982a). According to this method, the starting values of the statistics C_0^+ and C_0^- (for the CUSUM for the process mean) or S_0^+ and S_0^- (for the CUSUM for the process variability) are set to a non-zero value, usually $H/2$ (50% headstart). The advantage of this method is that it decreases the out-of-control ARL values when the process starts at an out-of-control value thus improving the chart's performance, while in case of the process starting at the in-control level the headstart has little effect on the chart's performance as the CUSUM statistics drop to zero quickly. Other FIR CUSUM control charts were proposed by Haq et al. (2014b).

If we want to increase the CUSUM (or FIR CUSUM) chart's sensitivity to larger shifts, for which it is not so efficient, we can combine it with a Shewhart control chart as presented in Lucas (1982). The Shewhart control limits are then set at around 3.5 standard deviations from the in-control process average and there is an out-of-control indication for our process when either or both control charts give an out-of-control signal. Combined Shewhart-CUSUM control charts perform better in detecting sudden jumps in the process mean, but as presented in Bissell (1984b), the improvement over a simple Shewhart mean chart (although significant) is less when the shift is a slow drift. Reynolds and Stoumbos (2005) noted that it is not necessary to use Shewhart control limits with a CUSUM (or EWMA) control chart when the chart is based on squared deviations from the in-control value. Combined Shewhart-CUSUM control charts were applied in monitoring non-manufacturing processes by Westgard et al. (1977) and Blacksell et al. (1994) and their optimization design was investigated by Wu et al. (2008b). Haridy et al. (2013) developed an optimization design for the combined np-CUSUM control chart for monitoring attribute data.

As mentioned for Shewhart control charts, a method for enhancing the performance of the chart is the use of runs-type signaling rules (Section 2.10.4).

Although this method is usually used with Shewhart control charts, it was also applied for the CUSUM control chart by Riaz et al. (2011).

2.14.1.5 CUSUM Charts When Using Rational Subgroups Instead of Individual Observations

All the previous CUSUM control charts were constructed based on their most usual use, for individual observations. If, however, rational subgroups are used, then the CUSUM charts for the mean presented earlier are extended to cover the case of rational subgroups by replacing the individual observations by the sample or subgroup average and replacing σ with $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Contrary to the Shewhart control charts for which it is preferred to use rational subgroups instead of individual observations whenever possible, with CUSUM charts it is better using individual observations instead of rational subgroups whenever there is a choice [Hawkins and Olwell (1998)].

When rational subgroups are used the CUSUM control chart for the process variability, as presented in Chang and Gan (1995) and Hawkins and Olwell (1998), are based on the normality assumption. If the sample variance of the *i*th subgroup is S_i^2 and the in-control and out-of-control variance values are σ_0^2 and σ_1^2, respectively, then the CUSUM statistics plotted on the CUSUM chart for variability are computed as follows:

$$C_i^+ = \max\left(0, C_{i-1}^+ + S_i^2 + k\right)$$
$$C_i^- = \max\left(0, C_{i-1}^- + S_i^2 - k\right)$$

with the initial values of the statistics being defined again as $C_0^+ = C_0^- = 0$ while $k = \frac{2\ln\left(\sigma_0/\sigma_1\right)\sigma_0^2\sigma_1^2}{\sigma_0^2 - \sigma_1^2}$. The value of *H* is chosen so as to obtain a desired in-control ARL value for the specific value of *k*. The FIR method can be applied to this chart, too.

2.14.1.6 Risk-Adjusted (RA) CUSUM Charts and CUSUM Charts with Ranked Set Sampling (RSS)

Usually variables control charts are based on the assumption of independent and identically distributed data. This assumption, however, is violated when monitoring health-care processes. The outcome of a surgery, for example, does not only depend on the surgeon's performance but on the pre-operative severity of illness or risk related with the patient. Therefore, risk-adjusted (RA) control charts are required in health-care applications in order to take into account that severity. RA CUSUM charts were discussed by several authors such as Steiner et al. (2000), Grigg et al. (2003), Grunkemeier et al. (2003), Novick et al. (2006), Biswas and Kalbfleisch (2008), Sego et al. (2009) and Gan et al. (2012).

Ranked set sampling (RSS) has also been used as an alternative to random sampling for improving the performance of control charts. It has been applied in literature for monitoring both process mean and variability. Examples for the case of CUSUM charts include Al-Sabah (2010), Haq et al. (2014a), Abujiya et al. (2015a,b, 2016b,c), Abid et al. (2017) and Abujiya and Lee (2019).

2.14.1.7 Other CUSUM Charts

Besides CUSUM control charts for the process mean and variability, CUSUM control charts for other quantities [such as ranges and standard deviations (when rational subgroups are used), fraction of nonconforming items or number of defects], have been proposed by Iwasiewicz et al. (1985), Lucas (1985), Rendtel (1987), Gan (1993a), Lowry et al. (1995), White et al. (1997) and Duran and Albin (2009). Particularly when monitoring count data with low defect rate, CUSUM control charts for the time between events can be used as in Lucas (1985) and Bourke (1991). These charts can perform well even under moderate departures from the exponential distribution, as presented in Borror et al. (2003). O' Campo and Guyer (1999) used a CUSUM control chart for monitoring rates of perinatal health outcomes. The monitoring of proportions was also addressed by Reynolds and Stoumbos (1999, 2000a,b) and Singh et al. (2002). Zhou et al. (2014)

compared weighted CUSUM charts for monitoring process proportions with varying sample sizes.

Taylor (1968) and Chiu (1974) dealt with the economic design of CUSUM control charts. Jones et al. (2004) discussed the case of CUSUM control charts with estimated parameters. Olteanu and Vining (2009) used likelihood ratio methods for CUSUM charts for the case of censored lifetime data. Olteanu (2010) studied CUSUM control charts for censored reliability data. Castagliola and Maravelakis (2011) presented a CUSUM control chart for monitoring process variability with estimated parameters. Khaliq and Riaz (2016) developed a robust Tukey-CUSUM control chart, based on Tukey control chart under CUSUM framework. Qu et al. (2017) introduced a CUSUM chart for monitoring the intensity ratio of negative events.

Although a single CUSUM is usually used for a specific process shift, Sparks (2000) proposed a CUSUM control chart with the simultaneous use of multiple CUSUM statistics with different values of k to deal with the unknown value of δ. Sparks (2000) suggested that the number of simultaneous statistics (which is usually equal to three) should be found by the range of shifts which we want our control chart to detect. Other CUSUM control charts have also been designed for simultaneous detection of a range of mean shifts by Zhao et al. (2005) and Han et al. (2007), combining two or more individual CUSUM control charts, respectively, designed so as to obtain a good overall performance.

As proven by Moustakides (1986), the CUSUM chart is the optimal control chart for the detection of a process shift of a certain magnitude (for which it was designed) among all control charts with the same in-control ARL. The actual shift occurring in the process, however, will not be exactly of that particular magnitude and, therefore, the designed CUSUM chart will not be the optimal control chart. In order to overcome this problem of unknown size of the process shift, Ryu et al. (2010) proposed a CUSUM control chart which uses a probability distribution for the size of the shift in the process mean.

When a shift occurs in the process mean or variability, this shift is not always a step shift as was the case for the CUSUM control charts presented in the beginnings of section 2.14.1. On the contrary, it can be a gradual shift with or without a parametric pattern. If the shift follows a linear model, then it is called a linear drift. The case of CUSUM control charts for the case of a linear drift has been dealt with by Bissell (1984a,b) and Gan (1992a), while, more recently, Shu et al. (2008) presented a weighted CUSUM control chart for the detection of gradual mean shifts following a parametric model.

The case of monitoring both process mean and process variability simultaneously with the use of CUSUM control charts was also addressed in literature. For example, Yeh et al. (2004) proposed a CUSUM chart for monitoring both process mean and variability based on batch data. Wu et al. (2007b) proposed a CUSUM chart with variable sample sizes and sampling intervals for monitoring both process mean and variance. Cheng and Thaga (2010) proposed a CUSUM chart for quickly detecting both small and large shifts in both process mean and standard deviation and compared it with other single charts like the chart proposed by Chen and Cheng (1998) and the EWMA proposed by Cheng and Xie (1999). Maleki and Salmasnia (2017) combined a CUSUM chart with generalized likelihood ratio for monitoring process mean and variability simultaneously under the presence of measurement errors.

2.14.2 Exponentially Weighted Moving Average (EWMA) Control Charts

The EWMA control charts are also a good alternative to the Shewhart control charts when interested in monitoring small process shifts. As Montgomery (2009) mentions, their performance is approximately equivalent to CUSUM charts' performance [as was proved by Lucas and Saccucci (1990)], but they can be easier to construct and implement since their control limits have a similar form to the Shewhart control charts' limits. Ryan (2011) also supports the similar behaviour of the CUSUM and EWMA control charts but seems to suggest the use of CUSUM rather EWMA charts due to their advantages: First of all, the CUSUM statistics for monitoring the process variability do not depend on the process variance, while the exponentially weighted moving averages do. In order to solve this dependency the standardized averages could be used but this would require the formulas for the construction of the control limits to be changed. Moreover, there is the inertia problem that will be mentioned in section 2.14.2.1 which can make the EWMA control chart slower in process shifts detection than the CUSUM chart. If, however, the process shift occurs at the beginning of the process or close to it then the EWMA control chart is preferable to the CUSUM chart, according to Hawkins and Wu (2014), since it detects the shift faster regardless of the shift size.

EWMA control charts are usually used with individual observations, just like the CUSUM control charts [Montgomery (2009)]. In fact, they are ideal for monitoring individual observations due to their insensitivity to the normality assumption because they use a weighted average of all past and current observations. By construction, the possible negative effect of the past data on the sensitivity of the EWMA control charts is reduced by the exponentially reducing weights given to the past data, as we will see in the next section. This is one of the major differences of EWMA charts from the CUSUM charts which accumulate all the past observations assigning equal weights to all of them and use a restarting mechanism in order to eliminate the possible negative effect of the past observations.

2.14.2.1 EWMA Control Charts for Monitoring the Process Mean

The EWMA chart was first introduced by Roberts (1959) as Geometric Moving Average (GMA) chart and since then its design, enhancements and performance have been studied further by Robinson and Ho (1978), Hunter (1986), Crowder (1987a,b, 1989), Lucas and Saccucci (1990), MacGregor and Harris (1990), Saccucci and Lucas (1990), Han and Tsung (2004), Shamsuzzaman and Wu (2012) and Shu et al. (2014). An overview of EWMA control charts was provided by Ruggeri et al. (2007b). The statistic which is plotted on an EWMA control chart versus the sample number is the exponentially weighted moving average of the observations which is calculated as follows:

$$z_i = \lambda x_i + \left(1 - \lambda\right) z_{i-1} \tag{2-2}$$

with the starting value of the statistic being defined to be equal to the in-control process mean ($z_0 = \mu_0$) and $0 < \lambda \leq 1$ being the smoothing constant representing the weight given to the current sample mean. If $\lambda = 1$, the EWMA control chart becomes an ordinary Shewhart control chart. The choice of the value of λ affects the width of the control chart. Therefore, it is very important for the control chart's performance. But first, let's present the control limits of the EWMA control chart to make the last statement clearer. The control limits of the chart, in case of monitoring individual observations, are computed as follows:

$$
\begin{aligned}
UCL &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \\
CL &= \mu_0 \\
LCL &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}
\end{aligned}
\qquad (2\text{-}3)
$$

The design parameters L and λ of the chart are chosen so as to achieve a desired in-control ARL value, which can also be close to the corresponding value for the CUSUM control chart for detecting small shifts for an appropriate combination of L and λ. As Montgomery (2009) mentions, the usually chosen values of λ which work well in practice are $0.05 \leq \lambda \leq 0.25$, with $\lambda = 0.05$, $\lambda = 0.10$ and $\lambda = 0.20$ being popular choices. In general, smaller values of λ are chosen for the detection of smaller shifts. Small λ values make the EWMA control chart more insensitive to normality. On the other hand, when using small λ values the risk of the so called "inertia effect" is increased. This happens when a shift occurs in the mean in the opposite direction of the EWMA statistic relative to the central line. Then the small value of λ does not give much weight to the present data and, therefore, it takes a while until the EWMA statistic reacts to the shift. As a result, the effectiveness of the EWMA chart to detect the shift decreases. Shu et al. (2007) proved that one-sided EWMA charts (for monitoring shifts in the mean of Normally distributed processes) suffer less inertia in detecting shifts than the corresponding two-sided charts. So the use of one-sided EWMA charts could be a solution to this problem whenever possible (when a specific shift direction is more likely to occur or of more interest). In order to overcome the problem of inertia in the general case, however, Capizzi and Masarotto (2003) proposed the use of an adaptive EWMA control chart. The inertia effect can be serious in case of using an EWMA chart with a small λ value, due to that fact that the EWMA chart uses only one statistic, while CUSUM does not suffer significantly from the inertia effect because it uses two statistics with restarting [Yashchin (1987, 1993)]. For this reason, in order to overcome the problem of inertia, Spliid (2010) proposed using one-sided EWMA control charts with resetting. Woodall and Mahmoud (2005) studied the inertial properties of various control charts and defined the signal resistance of a control chart as the "largest standardized deviation of the sample mean from the in-control value not leading to an immediate out-of-control signal". They showed that, the signal resistance for the EWMA chart is significantly higher than the one for the CUSUM chart. Moreover, unlike the Shewhart chart for which the signal resistance is constant and equal to L, the signal resistance of the EWMA chart is a function of the two design parameters of the EWMA chart and the value of the EWMA statistic itself. The most important thing, however, is that it depends on the value of λ in a way that smaller λ values (which are desired as mentioned previously) result to larger values of the chart's signal resistance. In order to overcome this problem, Woodall and Mahmoud (2005) recommended using Shewhart and EWMA control charts together (an EWMA chart with Shewhart control limits), especially for small λ values. When λ is specifically chosen to be equal to 0.1, according to Jones et al. (2001) and Jones (2002), 400 in-control subgroups are required for the EWMA control chart to have desirable properties when parameters are estimated.

2.14.2.2 EWMA Chart for Monitoring the Process Variability

The case of monitoring the process variability was addressed by MacGregor and Harris (1993) for both correlated and uncorrelated data. The EWMA-based statistic for monitoring the process standard deviation is called the exponentially weighted mean square error (EWMS) and is defined as follows:

$$S_i^2 = \lambda\left(x_i - \mu\right)^2 + \left(1-\lambda\right)S_{i-1}^2$$

and its square root is plotted on an exponentially weighted root mean square (EWRMS) control chart for which control limits are constructed as follows:

$$UCL = \sigma_0 \sqrt{\frac{\chi^2_{v,\alpha/2}}{v}}$$

$$CL = \sigma_0$$

$$LCL = \sigma_0 \sqrt{\frac{\chi^2_{v,1-(\alpha/2)}}{v}}$$

According to MacGregor and Harris (1993), the EWMS statistic is sensitive to both process mean shifts and process variability shifts and it is suggested to replace the mean value μ with an estimate at each time and, therefore, the statistic plotted on the exponentially weighted moving variance EWMV control chart is computed as follows:

$$S_i^2 = \lambda\left(x_i - z_i\right)^2 + \left(1-\lambda\right)S_{i-1}^2$$

Chang and Gan (1994) dealt with optimal design of one-sided EWMA control charts for monitoring the process variability. Shu and Jiang (2008) proposed an EWMA control chart for monitoring increases in process variability following Crowder and Hamilton (1992) who had also dealt with monitoring of the process standard deviation using EWMA. Other research dealing with EWMA control charts for monitoring process variability includes Huwang et al. (2009) who studied the EWMV chart and Huwang et al. (2010).

2.14.2.3 FIR EWMA and Other EWMA Control Chart Improvements

As far as their sensitivity to larger shifts is concerned, EWMA control charts can be improved by combining them with Shewhart control charts with wider than usual 3σ limits, as was the case with CUSUM control charts, too. This way EWMA control charts can become effective in detecting both small and large process shifts. The EWMA control charts can also become quicker in detection of processes which are out of control at start-up by the addition of a FIR or headstart feature as presented earlier with CUSUM control charts. Different FIR approaches for EWMA control charts were proposed by Rhoads et al. (1996) and Steiner

(1999b), with the latter mentioned in literature as more easily implemented in practice.

Lucas and Saccucci (1990) noted that a Shewhart-EWMA control chart can be designed so as to have ARL properties similar to those of a good Shewhart-CUSUM control chart. They also presented the FIR EWMA which, as they stated, is especially useful for cases of choosing a small λ value. Woodal and Maragah (1990) noted that the procedure by Lucas and Saccucci (1990) is based on the assumption of independence of observations over time and will, therefore, not work well under the presence of autocorrelation in the data and made some suggestions on the FIR EWMA.

The FIR EWMA control chart proposed by Lucas and Saccucci (1990) had fixed control limits with a head-start. A FIR EWMA control charts with time-varying control limits and a head-start was introduced by Rhoads et al. (1996). Other FIR EWMA control chart with time-varying control limits and different head-starts were proposed by Steiner (1999b) and Haq et al. (2014b).

Lucas and Saccucci (1990) and Knoth (2005) also studied the FIR EWMA control charts and Van Gilder (1994) described their application at General Motors. Chih-Min et al. (2000) applied Shewhart limits on EWMA charts for monitoring wafer data. Reynolds and Stoumbos (2005) showed the improvement of the EWMA control chart when using Shewhart control limits, but they found the combination of a regular EWMA control chart with an EWMA of squared deviation from the in-control value to be superior than the Shewhart-EWMA control chart. Capizzi and Masarotto (2010) studied the performance of Shewhart-EWMA control charts with estimated parameters.

2.14.1.4 EWMA Control Chart for Grouped Data

In case of monitoring data in rational subgroups, instead of individual observation as before, we replace the individual observations with the sample average and σ with $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Steiner (1998) presented an EWMA control chart for grouped data, when grouped data occur in more than just two groups (conforming, nonconforming), similar to the CUSUM chart for grouped data proposed by Steiner et al. (1996).

2.14.2.4 GWMA Control Charts

Sheu and Lin (2003) introduced a generalization of the EWMA chart, called the Generally Weighted Moving Average (GWMA) control chart, and compared it with the EWMA chart proving that the GWMA chart is more sensitive than the EWMA chart for monitoring small shifts in the process mean. Then they made the chart even more sensitive to small shifts by proposing the composite Shewhart-GWMA chart. Sheu and Chiu (2007) considered a GWMA chart for monitoring Poisson processes, while Chiu (2007) studied GWMA and double GEMA control charts for monitoring Poisson distributed processes. Chiu and Sheu (2008) introduced FIR Poisson GWMA charts. Shey and Shin (2008) discussed monitoring process mean and variance with a GWMA chart based on residuals. Sheu and Hsieh (2009) introduced the double GWMA chart which is an extension of the GWMA chart resulting by imitating the double EWMA control chart and proved (through simulation) the proposed chart's superiority over both the GWMA and the double EWMA charts. Chiu and Lu (2015) studied the steady-state performance of the Poisson double GWMA control chart. Areepong and Sukparungsee (2016) investigated the performance of zero-inflated Binomial GWMA chart. Alevizakos et al. (2018) used a double GWMA chart for monitoring time between events. Chen (2020) discussed the double GWMA control chart for monitoring COM-Poisson distributed processes and proved its superiority over the GWMA and double EWMA charts for the COM-Poisson distribution in detecting small shifts in the process mean or variability or both.

2.14.2.5 Other EWMA Control Charts

Most EWMA control charts are designed for monitoring step shifts. Sometimes, however, process shifts are gradual (called "drifts"). EWMA control charts for drifts were proposed by Gan (1991b) and Tseng et al. (2007). Domangue and Patch (1991), Gan (1995), Chen et al. (2001, 2004), Khoo et al. (2010), Haq et al. (2015a) and Raza et al. (2019) proposed an EWMA control chart for monitoring both process mean and variability. Gan (1989a) investigated the performance of modified EWMA charts for monitoring data from the Binomial distribution. Gan (1990a) used a modified EWMA chart for the Binomial distribution, while Gan (1990b) used modified EWMA charts for monitoring data from the Poisson distribution. Wasserman (1995) proposed an EWMA control chart for short-run process monitoring. Jones et al. (2001) investigated the performance of the EWMA control chart with estimated parameters, showing that the EWMA charts are very affected by estimation and have reliable performance for very large sample sizes (n=2000), which are usually difficult to obtain in practice. A solution for reliable and effective EWMA control chart with estimated parameters could be the self-starting EWMA chart. Such a procedure, based on the self-starting CUSUM proposed by Hawkins (1987), was presented in Qiu (2014). Steiner and MacKay (2001) used EWMA charts for censored data. Jones (2002) addressed EWMA control charts with estimated parameters. Zhang et al. (2003) studied the DEWMA chart for monitoring Poisson processes. Zhang and Chen (2004) discussed EWMA control charts for type I censored data. Kotani et al. (2005) introduced an EWMA control chart for high-yield processes constructed by applying the designing method of the EWMA chart to the CCC-r chart, investigated its performance in terms of the average number of observations to signal (using Markov Chain method) and compared it with the control chart proposed by Ohta and Kusukawa (2004), proving the superiority of the proposed chart. Reynolds and Stoumbos (2006b) used a combination of EWMA charts [including an EWMA of squared deviations such as the one presented in Reynolds and Stoumbos (2005)] to effectively monitor shifts in both process mean and process variability. Grigg and Spiegelhalter (2007) dealt with risk adjusted EWMA chart. Knoth (2007) studied the ARL of the EWMA control charts for monitoring Normal mean and variability simultaneously. Han et al. (2007) designed EWMA control charts for simultaneous detection of a range of mean shifts combining individual EWMA control charts, designed so as to obtain a good overall performance. Sheu et al. (2007) proposed an extended EWMA chart for monitoring data from the Poisson distribution. Kusukawa et al. (2008) developed a synthetic EWMA chart for monitoring high-yield processes. Maravelakis and Castagliola (2009) presented an EWMA chart for monitoring process standard deviation in case of unknown process parameters. Serel (2009) introduced economic design of EWMA control charts based on loss function. Tsai and Lin (2009) dealt with EWMA control chart for monitoring the average of type I censored data. Weiß (2009c) introduced the Markov np chart and the Markov EWMA chart for group inspection of dependent binary observations, using the Markov Binomial distribution which is a generalization of the Binomial distribution useful for the approximation of the dependence structure of the binary observations. The performances of the proposed charts were investigated with exact computations of their ARLs and illustrated with application to real web access data. Capizzi and Masarotto (2010) dealt with parameter estimation for combined Shewhart-EWMA charts. Steiner and Jones (2010) used an updating EWMA control chart for the monitoring of risk adjusted survival time. Abbas et al. (2011) used runs-type signaling rules in order to improve the performance of EWMA control charts. Baik et al. (2011) introduced the G-EWMAG control chart which combines the g-chart with the EWMAG chart (EWMA with attribute data applied to g statistics) and duscussed its optimal design so as to make the proposed chart sensitive to both large and small shifts in high-quality processes. Mavroudis and Nicolas (2011) extended the work by Shu et al. (2007) in order to obtain one-sided EWMA charts for high-yield processes following the Geometric distribution and compared the proposed chart's performance with the corresponding two-sided EWMA chart proposed by Yeh et al. (2008) in terms of average number of items until shift, revealing its superior sensitivity. Noorossana et al. (2011) used EWMA control chart for monitoring rare health events based on the zero-inflated Binomial distribution. Patel and Divecha (2011) and Khan et al. (2017a) proposed modified EWMA control charts for detecting small shifts. Kawamura et al. (2012) combined the EWMA chart with process capability analysis in order to decide the time of process adjustment necessary for achieving process variability reduction when monitoring time-series modeled data. Knoth and Steinmetz (2013) proposed EWMA p charts. Saleh et al. (2013) investigated the performance of the adaptive EWMA chart with estimated parameters. Haq (2014) proposed a mean deviation EWMA control chart for monitoring process variability with ranked set sampling. Qiu (2014) presented one-sided EWMA control charts with the restarting characteristic of the CUSUM charts. Saghir and Lin (2014a) developed a flexible and generalized EWMA chart for monitoring count data. Sukparungsee (2014b) introduced a square root transformation-based EWMA p chart. Akhundjanov and Pascual (2015) used moving range EWMA control charts for monitoring the shape parameter of the Weibull distribution. Areepong (2015b) proposed a modified EWMA chart for monitoring Binomial processes using square root transformation. Azam et al. (2015) used repetitive sampling with a hybrid EWMA chart. Haq et al. (2015b) studied the effect of measurement error on EWMA charts with ranked set sampling (RSS). Raza et al. (2015) investigated the performance of EWMA and DEWMA control charts for censored data. Zaman et al. (2015) discussed mixed CUSUM-EWMA charts for monitoring the process location. Arif et al. (2016) presented an EWMA np chart for monitoring Weibull data. Aslam (2016) used a mixed EWMA-CUSUM chart for monitoring Weibull processes. Atta et al. (2016b) addressed monitoring of the sample range of data from the Weibull distribution with an EWMA chart applying the weighted variance method. Khaliq et al. (2016) and Riaz and Ahmad (2016) introduced Tukey-EWMA control charts. Knoth (2016) compared the steady-state performance of the synthetic control chart, the "2 of L+1 ($L \geq 1$)" runs-rule chart and the EWMA charts with two types of control limits, revealing the superiority of the EWMA chart. Saeed and Kamal (2016) used robust estimators for process variance for EWMA charts. Zaman et al. (2016) proposed a mixed CUSUM-EWMA chart for monitoring process variability. Yang and Arnold (2016) used an ARL-unbiased EWMA-p chart for monitoring process variability. Abujiya et al. (2017) introduced an EWMA chart based on RSS for monitoring process variability. Aslam et al. (2017b) presented a HEWMA-CUSUM chart for monitoring data from the Weibull distribution. Lu and Huang (2017) presented an economic-statistical design of double EWMA chart. Riaz et al.

(2017) proposed the mixed Tukey EWMA-CUSUM chart. Sparks (2017) discussed risk-adjusted EWMA p charts. Cheng and Wang (2018) studied the performance of EWMA median and CUSUM median control charts with measurement errors. Naveed et al. (2018) proposed the extended EWMA control chart and proved its superiority over the EWMA and Shewhart control charts. Raza et al. (2018) presented DEWMA control charts for monitoring censored lifetime data from the Rayleigh distribution. Riaz et al. (2019) introduced a mixed EWMA-CUSUM chart with a regression estimator for monitoring the process mean. Tayyab et al. (2019) proposed EWMA charts with RSS for process mean monitoring. Asif et al. (2020) developed a hybrid EWMA chart and studied the effect of measurement error on its performance. Phanthuna et al. (2021) investigated the performance of the modified EWMA chart for the trend stationary AR(1) model. Taboran et al. (2021) designed the Tukey MA-DEWMA control chart. Lee et al. (2022) proposed two-sided EWMA conditional expected value (CEV) control charts for monitoring multiple censored data, showing that two-sided EWMA CEV chart is more effective than the combination of two one-sided EWMA CEV charts, and studied the performance and optimal design of the proposed chart. Haq and Woodall (2023) studied the effect of estimation error on the conditional false alarm rate of the EWMA chart based on the estimated dynamic probability control limits. Nawaz et al. (2023) developed np-EWMA and np-HEWMA control charts through Monte Carlo simulations. Yu et al. (2023) constructed a semi-parametric EWMA chart for highly type-I right censored lifetime data using a Kolmogorov-Smirnov statistic defined by the differences between the in-control cumulative distribution function and the empirical cumulative distribution function, where the cumulative distribution function was constructed using the Kaplan-Meier estimator and the generalized Pareto distribution to improve the tail estimation. The efficiency of the proposed control chart was illustrated with both simulated and real data.

2.14.3 Adaptive CUSUM and EWMA Control Charts

Control charts which have variable sampling rate (VSR), including variable sample size (VSS), variable sampling interval (VSI) or both variable sample size and sampling interval (VSSI) depending on the position of the plotted statistics or variable design parameters are called adaptive control charts. Although adaptive control charts are more complicated than the non-adaptive ones, they have the advantage of better performance. ARL is no longer effective when dealing with these control charts and different performance measures are used for them.

VSS control charts allow the sample sizes to be variable depending on the current sample's observations and the performances of different charts are compared using ANOS or ANSS (Section 2.7) instead of ARL values. In this case, a large sample size is used when the plotted statistic is closer to the control limits, while a smaller sample size is used when the statistic is plotted closer to the central line of the chart. The VSS control charts can detect process shifts quicker than the traditional fixed sample size control charts.

VSI control charts, on the other hand, allow the sampling interval between consecutive samples to be variable depending on the current sample's observation and the performances of different charts are compared using the ATS instead of ARL values. In this case, if the statistic is plotted closer to the control limits a shorter sample interval is used, while a larger sample interval is used when the statistic is plotted closer to the central line of the chart. The VSI control charts detect process shifts quicker than the traditional fixed sampling interval control charts.

VSI and VSS CUSUM charts were discussed by Reynolds et al. (1990), Ken (1997), Shu and Jiang (2006), Wu et al. (2007b,2009a), Luo et al. (2009) and Huang et al. (2016). VSI EWMA control charts were addressed by Shamma et al. (1991), Saccucci et al. (1992), Reynolds and Stoumbos (2001b), Epprecht et al. (2010) and Lu et al. (2017). Reynolds and Arnold (2001) discussed the EWMA control charts with VSS and VSIs. Tseng et al. (2010) and Su et al. (2011) dealt with adaptive EWMA control charts for processes with drifts. Haq et al. (2018) investigated the performance of an adaptive EWMA chart for monitoring the process mean. Tang et al. (2018) investigated the effect of measurement error on the adaptive EWMA $\overline{X}$ chart, proving that the adaptive EWMA chart performs better than the traditional EWMA chart even under the presence of measurement error. Aytaçoğlu et al. (2023) addressed the design of EWMA control charts with VSS using the conditional false alarm rate.

Besides the above, dynamic sampling has been proposed, according to which the sampling interval for the next sample can vary randomly instead of choosing between just two values (a small and a large one). This random length can be determined by the p-value of the plotted statistic at the current time point. This kind of dynamic sampling was applied to CUSUM control charts by Li et al. (2013) and Li and Qiu (2014).

Adaptive control charts are not only the charts with variable sampling rate, as mentioned at the beginning of this section. Another kind of adaptive control charts includes the charts with other variable design parameters. An example can be found in Sparks (2000) who proposed a CUSUM control chart designed for cases of unknown process shift magnitude δ. This kind of CUSUM chart uses an estimation of δ at each time point and updates the chart's design parameters based on that estimate. The method proposed by Sparks (2000), however, is difficult to be applied to an EWMA control chart due to lack of a relationship between δ and the optimum value of λ. A solution to this problem was proposed by Capizzi and Masarotto (2003) who dealt with the choice of λ adaptively so as to obtain an EWMA chart with reasonable good performance in various cases.

2.14.4 Comparisons of EWMA and CUSUM Control Charts in Relevant Literature

Trevanich and Bourke (1993) developed two EWMA charts for attributes data. The first one was constructed for monitoring the fraction nonconforming using as observations in the EWMA statistic the number of conforming items between successive nonconforming ones. The second EWMA chart was constructed for monitoring TBE which are assumed to follow the exponential distribution. The proposed control charts were compared with CUSUM charts revealing their superiority in detecting quickly small to moderate shifts in count-rate.

Perry and Pignatiello (2003) compared CUSUM and EWMA charts for monitoring Poisson distribution. Yeh et al. (2008) proved through simulation that the Geometric EWMA control chart is more sensitive than previously proposed control charts for high-yield processes including the Geometric CUSUM by Chang and Gan (2001). Mavroudis and Nicolas (2013) discussed one-sided Geometric EWMA charts for high-yield processes, determined their optimal design, investigated the performance in terms of average number of items until shift and used the same performance measure for comparisons with the traditional Geometric CUSUM charts.

Haridy et al. (2017) developed Binomial EWMA charts with curtailment for monitoring the fraction nonconforming under the assumptions of known in-control fraction nonconforming, of Binomially distributed number of nonconforming units and of Rayleigh distributed random shifts of the fraction nonconforming. The proposed control chart was proved to have better overall performance than both the corresponding EWMA chart without curtailment and the CUSUM chart. Moreover, when compared to the CUSUM chart with curtailment proposed by Haridy et al. (2014b), the EWMA chart with curtailment was proved to perform better in most of the cases considered in that study.

2.15 Assumptions for Control Charts and the Cases of Their Violation

The control charts are constructed based on the assumptions of a particular distribution and independence of the data. The assumed distribution is the Normal distribution for the case of variables data and the Binomial or Poisson distribution for attributes data. The assumption of a Binomial or Poisson distributed process implies the inherent assumption of constant distribution's parameter and, therefore, mean over time, which in real applications is not always the case and this is especially obvious with large subgroup sizes. This problem was solved by Laney (2002) who proposed a p-chart which uses all the variation in the data (both within and between subgroups) and combines the concepts of X-chart and z-chart. If the sample size is variable then the p-values are converted to z-scores and plotted on an X chart. Attributes control charts for very large sample sizes were also addressed by Mohammed and Laney (2006) for the case of overdispersed data in health care.

As we have already mentioned, when constructing Shewhart control charts the constant k is usually chosen to be equal to three based on the assumption that the underlying process distribution is the Normal distribution. When we have strong evidence of non-Normality, however, an alternative to fixing the value of k is to chose a specific false alarm rate α and then find the corresponding value of k. These are the so called "probability control limits" mentioned in Section 2.8.1, and can be more effective than the Shewhart control limits in cases of non-Normal distributions, particularly the skewed ones. One such example is the use of 3-sigma control limits for c- and u- control charts. These control charts are based on the Poisson distribution which is right skewed and, therefore, the 3-sigma control limits increase the false alarm rate. The use of probability control limits as the solution to this problem was suggested among others by Ryan and Schwertman (1997). Other approaches have also been proposed for the improvement of the c- and u- control charts based on transforming the data or standardizing them or using an optimization of control limits approach. A review on the empirical evaluation of those methods was presented by Aebtarm and Bouguila (2001). A graphical method for checking attribute control chart assumptions was presented by Jones and Govindaraju (2001).

2.15.1 The case when the Poisson distribution is inappropriate for the data

One basic characteristic of the Poisson distribution is that the mean and variability are equal. If there is a strong indication of different mean and variability in our dataset, then the Poisson distribution is not appropriate. Such cases are when the defects tend to occur in clusters or when there are too many or too few zeros in our data. Gardiner (1987) used various discrete distributions for the detection of small shifts in a near-zero defect environment of integrated circuits. Kaminski et al. (1992) proposed control charts for counts assuming independent and identically distributed observations from the geometric distribution, for monitoring total or average number of events. Xie and Goh (1993) used probability limits instead of L-sigma limits for these two charts, using the Negative Binomial (or Pascal) distribution. Using the exact probability limits, a positive value of the LCL is easy to be achieved in practice.

When dealing with the number of nonconformities, the Poisson distribution assumption may not be valid in some cases. Radaelli (1994) dealt with the case of falsely assuming Poisson distribution for data following the Negative Binomial distribution and presented the reduction of the in-control ARL value of the CUSUM control chart in that case. As Hawkins and Olwell (1998) warn, the ARL values of CUSUM control charts for variables which are assumed to be Poisson distributed are very sensitive to departures from the Poisson distribution. Therefore, before making a Poisson assumption a test for overdispersion should be performed, since the mean and variability of a Poisson distributed random variable are equal.

2.15.2 Control Charts for Over-Dispersed or Under-Dispersed Data

As previously, mentioned, the assumption of Poisson distribution is an assumption of equi-dispersion of the data (the parameter of the Poisson distribution is both its mean and its variance) which is not always the case in real world applications. In cases of over-dispersed or under-dispersed data, other distributions are more appropriate than the Poisson distribution. In case of under-dispersion the Binomial (or Bernoulli) distribution can be used, while in case of over-dispersion the Negative-Binomial distribution (which has the geometric distribution as a special case) is more appropriate. It should be noted that monitoring over-dispersed data with a Poisson-based control chart can lead to increased false alarm rate. Monitoring of over-dispersed data was discussed, for example, by Albers (2009), Albers (2011) and Zhang et al. (2013), while Sellers (2012) dealt with monitoring of both over- and under- dispersed data.

Instead of using either the Binomial or the Negative Binomial distribution (and its special case, the Geometric distribution) for the construction of control charts for under-dispersed or over-dispersed attributes data, a generalized distribution which has both properties of under-dispersion and over-dispersion can be used. He et al. (2006) used the generalized Poisson distribution for the construction of a control chart for monitoring over-dispersed data. Famoye (2007) used the shifted (or zero-truncated) generalized Poisson distribution for the construction of control charts for monitoring the total number of events and the average number of events. Chen et al. (2008) dealt with attributes control charts constructed based on generalized zero-inflated Poisson distribution.

2.16 Control Charts for Individual Observations Data

As mentioned earlier in Section 2.12.4, there are situations when the data come available without subgrouping, such as low-rate produced items. Examples of monitoring such data in accounting include the monitoring of days required to process an invoice or the weekly payroll as presented in Walter et al. (1990). In such cases there is no choice of whether the data for the control charts will consist of individual observations or not. If, however, individual observations can be taken frequently enough for us to be able to group them, we should first consider if the shifts are either permanent or transient shifts of short duration. If the latter case is true, then Reynolds and Stoumbos (2004b) proved that it is better to plot individual observations on the control charts with those observations drawn at equally spaced times within a given time period rather than drawing a sample at the end of it and possibly miss the effect of the transient shift. If, however, the transient shifts are of longer duration or if the transient shifts are not of primary concern, then their research concluded that it would be better to use subgroups for the control charts. Therefore, the individual observations control charts should not be the first option whenever subgrouping is possible. This is in accordance with the fact, mentioned earlier, that control charts are equivalent to hypothesis testing, because the higher the sample size gets the higher the power of a hypothesis test gets, too. Besides, the control limits of the means control chart are narrower than the control limits of the individual observations chart and this makes the means chart more sensitive to shifts in the process average and, thus, preferable. Moreover, subgrouping and, therefore, use of the mean can mitigate the effect of non-normality on the control charts and, consequently, it is preferred to individual observations when the data come from very skewed distributions, since the X charts are more sensitive than the mean charts to non-normality, as well as individual measurement abnormalities. Therefore, whenever we have a choice of using an individual observations control chart or a mean chart for example, it is preferred to choose the mean chart. We could also use both of them in cases, for instance, when the existence of just one large observation causes the mean of a subgroup to exceed the upper control limit of a means chart. The individual observations of that particular subgroup could be plotted on an individual observations chart to reveal the magnitude of the specific large observation in relation to the control limits and the rest of the observations in that subgroup and help further investigation of the cause of that out-of-control signal. Instead of using two different control charts together, Albers and Kallenberg (2008) presented a control chart which combines a chart for individual observations with a chart that signals when a number of consecutive observations are plotted beyond a threshold value. Qiu (2014) also described the case of grouping individual data and then using traditional Shewhart control charts for monitoring the new grouped data. The disadvantage of that approach is that when receiving an out-of-control signal from the chart, the process needs to be checked for assignable causes at all time points belonging to the particular created group which produced the signal.

An alternative to the Shewhart X chart that has been suggested in the literature is the individual moving average chart constructed as follows:

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2\sqrt{n}}$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2\sqrt{n}}$$

where n is the number of observations for computing each moving average and $\overline{MR}/d_2$ is the estimate of variance using moving ranges with the same moving window n. The individual moving average control chart has the advantage of smoothing the data and the same disadvantages as the MR chart, namely that the plotted points (especially moving averages that are less than n periods apart) are correlated even for independent individual observations and, therefore, they can be quite deceiving regarding interpretation of patterns on the control chart [Nelson (1983b)]. As a result, the only out-of-control indication when using those charts can be the presence of points beyond the control limits. Another problem with individual moving average control charts is that any of the first n-1 observations could be an indication of an out-of-control process, but they are not used until the next (nth) observation becomes available. Furthermore, the control limits are wider for the initial $i<n$ periods than they are in the final steady state and they change at each sample point during this initial time, since the control limits during this initial period are computed as:

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2\sqrt{i}}$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2\sqrt{i}}$$

with i=1,2,…,n-1. The trouble of different control limits for each of these first observations can be solved by first using a simple X chart for $i<n$ and then an individual moving average chart for $i \geq n$. Moreover, as the window (n) of the individual moving average increases, the width of the control limits decreases and this means that we need a larger value of n in order to detect a smaller shift. Increasing n, however, increases the bias in the estimate of the variability, too,

under the presence of assignable causes. More specifically, if a single observation is affected by an assignable cause, up to *n* moving ranges are affected by this observation while if there is a sustained shift in the process mean, up to *n*-1 moving ranges will be affected by this shift. According to Wetherill and Brown (1991), the presence of an assignable cause can be revealed by sharply rising curves when plotting the estimate of variability against the number of *n* used in order to obtain that estimate. Therefore, although the individual moving average control chart is more effective than the corresponding Shewhart chart in detecting small shifts, we should always bear in mind both the risk of increasing the bias in variability estimation when increasing the size of the window *n* and the reverse relationship between the magnitude of shift we want to detect quickly and the span *n* of the moving averages we use, because if we use larger *n* in order to detect smaller shifts the risk of late response to large shifts increases. It should also be noted that although the individual moving average control chart is simpler in the construction, individual CUSUM or individual EWMA control charts are more effective in detecting small shifts than the individual moving average control chart. The main reason that individual observations are preferred anyway when using CUSUM or EWMA control charts is the need for less observations for the detection of a particular shift if individual observations are used instead of group data, as presented in Qiu (2014).

If the moving range has some very high values, those values will affect the estimate of the standard deviation (through moving ranges), too, and make the width of the control limits very wide. A solution for that problem suggested by Stapenhurst (2005) is to use a median moving range chart.

In Section 2.12.4 we mentioned that X and MR charts are usually used together. Sullivan and Woodall (1996) showed that the gain when combining X and MR charts is little and suggested a completely different alternative, namely the likelihood ratio test (LRT) approach. The LRT control chart is superior since it uses both past and recent data for the computation of the test statistics contrary to the Shewhart chart which plots only the current observation. Moreover, the statistic in LRT chart can be broken into two components whose relative magnitude can suggest whether the shift occurred in the process mean or variability. Another advantage of this control chart is that the point at which a shift is detected is much closer to the actual time that the shift occurred than the corresponding one when using the combination of X and MR charts. The only disadvantage of LRT chart is that it is less effective in detecting temporary shifts in the process mean or variability. In that case, Sullivan and Woodall (1996) suggest combining the LRT chart with an X chart.

Besides all the above, dealing with individual observations is very usual and has attracted a lot of attention in research literature. For example, control charts for individual observations were studied by Nelson (1982), while the effect of the sample size on estimated limits for the individual control chart was studied by Quesenberry (1993). Finison et al. (1993) applied the individual control charts in healthcare for monitoring days between infections. Reynolds and Stoumbos (2001b) dealt with monitoring process mean and variance when using individual observations and variable sampling intervals.

Hawkins (1981) proposed a CUSUM chart for monitoring the process variability using individual observations, while MacGregor and Harris (1993) proposed an exponentially weighted moving variance chart and an exponentially weighted mean squared deviation chart for monitoring variability with individual observations. Albin et al. (1997) applied Shewhart control limits to EWMA control chart for monitoring individual observations in order to gain the ability to detect both small and large shifts. Hawkins and Olwell (1998) studied the use of CUSUM charts for individual observations from both symmetric and asymmetric distributions. Turner et al. (2001) discussed change-point detection for individual observations in Phase I. Vermat et al. (2003) investigated Shewhart individuals charts for monitoring Normal and non-Normal processes. Kan and Yazici (2005) studied the individuals control charts for non-Normal processes. Kan and Yazici (2006a,b) proposed individuals control charts with asymmetric limits for monitoring data from the Burr and the Weibull distribution. Braun and Park (2008) dealt with the estimation of variance for control charts for individual observations. Yeh et al. (2010) also addressed the monitoring of process variance using individual observations, while Human et al. (2011) investigated the robustness of the EWMA control chart for the case of monitoring individual observations. Li (2012) presented GLR charts for monitoring individual observations from the Poisson distribution. Pascual (2012) studied individual control charts for monitoring Weibull processes. Pascual and Nguyen (2011) addressed moving range control charts for monitoring the shape parameter of the Weibull distribution using individual data. Shao and Hou (2011) proposed an EWMA chart with MLE for estimating the change point when monitoring individual observations from the Gamma distribution. Li (2012) presented a GLR chart for monitoring individual Poisson observations. Pascual (2012) studied individual and moving ratio charts for monitoring Weibull processes. Lee et al. (2013b) discussed the individual control chart with variable limits for monitoring the river pollution. Xin et al. (2015) dealt with one-sided individual control charts for monitoring data from the Lognormal distribution. Wang (2017) presented the MaxEWMA chart for individual Weibull distributed observations. Fatemi Ghomi and Sogandi (2019) proposed a two-sided CUSUM chart based on a log-likelihood ratio for monitoring autocorrelated binary individual observations. Oh and Weiß (2020) studied the individuals control chart with supplementary runs rules under serial dependence. If a CUSUM chart for monitoring the process mean is combined with a CUSUM chart for monitoring the process variability, for the case of individual observations, the two CUSUM charts are usually correlated. The formula for the computation of the in-control ARL is not valid in cases like that, so an algorithm for its computation is presented in Qiu (2014). A recent application of the individuals control chart for monitoring healthcare related processes was presented by Seoh et al. (2021).

As far as non-parametric control charts are concerned, Hackl and Ledolter (1991) proposed an EWMA control chart for individual observations based on the observations' ranks, which provides a significant advantage in case of non-normal situations that are far from normality, while Graham et al. (2011) discussed an EWMA sign chart for location for monitoring individual observations.

2.17 Assumptions for the Control Charts for Individual Observations

The assumptions for the control charts for individual observations are the same as the ones for the control charts for the mean, namely normality and independence. In fact the normality assumption in this case is far more important than it is for the case of monitoring the mean, because even a slight departure from normality can decrease the in-control ARL value very much. On the other hand, the consequences of a violation of the independence assumption depend considerably on whether the variance is assumed to be known or not, because the effect of autocorrelation in case of unknown variance depends on the estimator of variance that we use. For example, although the moving range estimation of the variance should not be used even when the data are independent, the situation becomes much worse when the data are autocorrelated. Cryer and Ryan (1990) showed that $E\left(\overline{MR}/d_2\right)=\sigma\sqrt{1-\rho_1}$, where ρ_1 is the correlation between consecutive observations. This relationship means that if the value of ρ_1 is close to 1, then the control limits of the chart will be very narrow, leading to a mush smaller value of the in-control ARL. Using the sample standard deviation, however, for the estimation of the variance will not be such a serious problem for large sample sizes. Therefore, increasing the sample size used for the estimation of the variance can solve the problem of autocorrelation but it cannot solve the problem of non-normality, since the distribution does not change by increasing the sample size. This is the reason why non-normality is more serious than autocorrelation when monitoring individual observations. The effect of non-normality on the individuals control chart was studied by Borror et al. (1999), while the effect of autocorrelation on the individual observations control chart was studied by Maragah and Woodall (1992). Stoumbos and Reynolds (2000) studied the effect of both non-normality and autocorrelation on the individual control chart. Maravelakis (2003) investigated the effect of non-Normality on EWMA control charts for monitoring process variability. Human et al. (2011) studied the robustness to non-normality of EWMA control charts for individual observations, showing that EWMA control charts are not robust for some non-Normal distributions such as the symmetric bimodal and the contaminated Normal distribution.

If the process presents even moderate departure from normality, then the Shewhart individual observations control charts should not be used. It is suggested that the control limits should be constructed using percentiles of the underlying distribution. Another approach would be to transform the data in order to get approximate normality [Chou et al. (1998a, 1998b)].

Non-normality is important for CUSUM control charts, too. The effect of non-normality on the CUSUM control chart for individual observations was studied by Hawkins and Olwell (1998) who presented some numerical results for a CUSUM chart for monitoring individual observations from both symmetric and skewed distributions and showed that the in-control ARL values can be quite small for some distributions and values of k (shift in standard deviation units) but can be compensated for with a proper choice of the combination of k and h (decision interval). Non-normality is also important when using an EWMA control chart for monitoring individual observations. Some EWMA charts for variability are sensitive to non-normality of individual observations as shown by Maravelakis et al. (2005), such as the EWMA of squared deviations discussed in Reynolds and Stoumbos (2005), which, therefore, Maravelakis et al. (2005) recommended not using in case of non-normality.

2.18 Control Charts for Non-Normal Distributions

One of the assumptions for the construction of the control charts is the underlying data distribution. This distribution is usually assumed to be the Normal one. In most cases in practice, however, the Normality assumption is not valid. If there is strong evidence of Normal assumption violation and/or the assumption about the underlying distribution can not be verified due to lack of adequate data, one solution to monitor the data properly is to use nonparametric (or distribution-free) monitoring methods. Nonparametric control charts do not assume a particular underlying distribution for the data and have the advantage of constant in-control performance regardless the shape of the distribution of the monitored data. Additionally, as proven in the relevant literature, they have good out-of-control performance as compared to the parametric control charts. Furthermore, they are not affected by outliers and sometimes do not require estimation of the process variance for setting up a control chart for the process mean. According to Chakraborti et al. (2004), however, nonparametric control charts perform better than the parametric ones only in certain cases such as monitoring skewed or heavy tailed distributions. Moreover, nonparametric control charts will be less efficient than the parametric ones if the correct underlying data distribution is assumed. In spite of its advantages, the nonparametric case is beyond the scope of this thesis and will, therefore, be omitted herein. In what follows, only the parametric control charts will be addressed.

The case of the violation of the Normality assumption has been studied a lot in literature. One of the first studies on control charts for the non-Normal situation was the one by Gayen (1953). The effect of non-Normal distributions to the so called “tail probabilities” (namely the probabilities outside the traditional 3-sigma limits) has been studied by Schilling and Nelson (1976), showing that even for a significant departure from Normality the sum of the two tail probabilities (considered together) does not differ much from the nominal value. If the individual tail probabilities are investigated separately, however, then, as was proven by Moore (1957) and Schilling and Nelson (1976), the results are different. As was shown by Faddy (1996) and Ryan and Feddy (2000), the ARL values for CUSUM charts and especially Shewhart-CUSUM charts are affected by non-Normality, too. These two studies, however, did not deal with reference value investigation. This was done later by Stoumbos and Reynolds (2004) who proved that it is possible to design a CUSUM chart with appropriate reference values so as to be robust to non-Normality. Robustness of EWMA control charts to non-Normality was studied by Borror et al. (1999) showing that there is a possibility of designing the chart so as to be robust to some distributions, when choosing a small λ value. This, however, requires some knowledge about the shape of the distribution and the magnitude of the expected shift so as to design the chart appropriately and this knowledge may not always be available. When the assumption of the Normal distribution is proven to be invalid, usual control charts are not reliable. This have been verified by several authors, including Lucas and Crocier (1982b), Chan et al. (1988), Jacobs (1990), Hackl and Ledolter (1992), Amin et al. (1995) and Qiu and Li (2011a,b). The effect of non-normality on control charts was studied by Burr (1967), Balakrishnan and Kocherlakota (1986), Rocke (1989), Spedding and Rawlings (1994), Shore (2004), Lin and Chou (2007), Amhemad (2009, 2010), Chen et al. (2017), Moghadam et al. (2018). The effect of non-Normality on the economic design of $\overline{X}$ charts with warning limits was studied by Chou et al. (2004), while the effect of non-Normality on the economic-statistical design of $\overline{X}$ charts with Weibull in-control time was investigated by Chen and Cheng (2007). Chakraborti et al. (2004) studied the robustness of nonparametric control charts using data from various non-Normal (skewed or Normal-like heavy-tailed or light-tailed) distributions, such as two Gamma distributions, the Student's t distribution, the Laplace (or double Exponential) distribution and the Uniform distribution. The robustness of the synthetic control chart to non-Normality was examined by Calzada and Scariano (2001), while the robustness of group runs chart to non-Normality was addressed by Gadre et al. (2005). Horng Shiau and Hsu (2005) studied the robustness of the EWMA chart to non-Normality for autocorrelated processes and Kao and Ho (2007) discussed the robustness of the R chart to non-Normality. Lin and Chou (2011) investigated the robustness to non-Normality of EWMA charts and combined $\overline{X}$-EWMA charts with variable sampling intervals. Lee (2012) studied the robustness of the $\overline{X}$ chart to non-Normality and Saghir and Lin (2014b) dealt with the robustness of the G-chart to non-Normality. Singh and Singh (2014) addressed the robustness of control charts to non-Normality and AR(2) processes. Sukparungsee (2016) investigated the robustness of memory-type charts to skewed processes. Lin et al. (2017) discussed the robustness of the EWMA median control chart to non-Normality.

The actual Type I error probabilities of the Shewhart charts in case of non-Normality has proven to be different than the nominal one resulting in either too many false alarms or inability of the chart to detect real process shifts. Solutions suggested in literature for dealing with non-Normality include the increase of the sample size in order to have approximate Normality of the plotted statistic due to the central limit theorem or the use of an appropriate transformation of the observations in order to achieve Normality (which however is not preferred due to different scale of the distribution and, therefore, invalid inferences), the use of nonparametric control charts and the use of robust control charts, which are preferred because they use the original data (and, therefore, inferences are valid for the original data) and they are not very affected by violation of the distribution assumption and outliers. Shewhart charts were applied to transformed data, for example, by Chou et al. (1998b), Yourstone and Zimmer (1992) and Shore (1994, 2001). Figueiredo and Gomes (2006) proposed robust control charts for monitoring non-Normal data based on Box-Cox transformations. Figueiredo and Gomes (2009) dealt with robust control charts for monitoring industrial processes.

Nagendra and Rai (1971) determined the optimum sample size and sampling interval for control charts for monitoring the mean of non-Normal processes. Lashkari and Rahim (1979) studied the economic design of control charts for the mean of non-Normal distributions taking into account the cost of process shut down. Lashkari and Rahim (1982) presented the economic design of CUSUM charts for monitoring the mean of non-Normal distributions. Rahim and Raouf (1983) and Rahim (1985) studied the economic design of $\overline{X}$ charts for monitoring non-Normal processes with measurement or inspection errors. Rahim (1987) addressed the economic design of CUSUM charts for monitoring the mean of non-Normal processes. Haridy and EI-Shabrawy (1996) presented the economic design of CUSUM chats for monitoring the mean of non-Normal processes. Chou and Cheng (1997) studied control charts for monitoring the range of non-Normal data. Duclos and Pillet (1997) dealt with an optimal control chart for monitoring non-Normal processes. Sim (2000) addressed the S chart for monitoring non-Normal data. Chou et al. (2001a) studied the economic design of $\overline{X}$ charts for monitoring non-Normal correlated data, while Chou et al. (2001b) investigated the economic statistical design of control charts for monitoring the mean of non-Normal processes. Yi et al. (2001) compared the ARL performance of neural network models and $\overline{X}$ charts for monitoring non-Normal processes. Chou et al. (2002) designed $\overline{X}$ charts for monitoring non-Normally distributed correlated data with minimum loss. Chen (2003) investigated the economic-statistical design of $\overline{X}$ charts for monitoring non-Normal processes with variable sampling intervals. Chen (2004) presented the economic design of $\overline{X}$ charts for monitoring non-Normal processes with variable sampling policy. Castagliola and Tsung (2005) dealt with monitoring autocorrelated non-Normal processes. Chou et al. (2005) addressed acceptance control charts for non-Normal data. Lin and Chou (2005) investigated VSS and VSI $\overline{X}$ charts for monitoring non-Normal processes. Çetinyürek (2006) constructed control charts with various estimators for symmetric non-Normal distributions (both long-tailed and short-tailed) and studied their robustness. Yeh and Chen (2006) dealt with the economic design of $\overline{X}$ charts for monitoring non-Normal data with Weibull shock models. Chou and Lin (2007) studied the variable parameter $\overline{X}$ charts for non-Normal processes. Li et al. (2008) presented the economic design of $\overline{X}$ charts for non-Normal data with Gamma (λ, 2) failure models. Torng and Lee (2008) investigated the performance of the Tukey's control chart for non-Normal distributions, showing that this chart is not sensitive to shifts detection when the process exhibits large departures from the Normality assumption. Tsai and Chiang (2008) addressed the design of acceptance control charts for non-Normal data. Chen and Yeh (2009) studied the economic statistical design of $\overline{X}$ charts with non-uniform sampling scheme for monitoring non-Normal processes with Gamma shock. Li et al. (2009) dealt with the restrictions in the economic design of $\overline{X}$ charts for monitoring non-Normal data with Weibull shock model. Torng and Lee (2009) studied the performance of $\overline{X}$ charts with double sampling for monitoring non-Normal processes. Chen and Yeh (2010) investigated the economic design of $\overline{X}$ charts with variable sampling interval for monitoring non-Normal processes with Gamma (λ, 2) failure models. Lin et al. (2010) addressed adaptive $\overline{X}$ charts with sampling at fixed times for monitoring data from non-Normal distributions. Schoonhoven and Does (2010) discussed the $\overline{X}$ charts in case of non-Normality. Torng et al. (2010) investigated the performance of $\overline{X}$ charts with combined double sampling and variable sampling interval for monitoring non-Normal processes. Wang et al. (2010) discussed the economic-statistical design of control charts with a Gamma shock model and correlated data. Yeh and Chen (2010) proposed an economic design of $\overline{X}$ charts for monitoring non-Normal data with Gamma failure models. Chen and Pao (2011) studied the joint economic-statistical design of $\overline{X}$ and R charts for monitoring non-Normal processes. Chen and Yeh (2011) addressed the economic statistical design of $\overline{X}$ charts for monitoring non-Normal processes with Weibull in-control time. Yeh et al. (2011) discussed the economic design of $\overline{X}$ charts for monitoring non-Normal processes with Weibull shock models. Abbasi and Miller (2012) studied the choice of control chart for monitoring process variability for Normal and non-Normal processes. Yin and Chong (2012) investigated the effect of non-Normality on the performance of some DEWMA charts. Niaki et al. (2013a,b, 2014) addressed the economic and economic-statistical design of $\overline{X}$ charts with variable sampling interval for monitoring non-Normal autocorrelated processes. Noorossana et al. (2013) dealt with statistical optimization of VSI $\overline{X}$ charts for monitoring non-Normal processes with the presence of multiple assignable causes. Santiago and Smith (2013b) addressed control charts with runs rules for monitoring non-Normal processes. Aichouni et al. (2014) presented control charts for non-Normal distributed data for the construction industry business. Abbasi et al. (2015) dealt with monitoring process variability with EWMA charts for Normal and non-Normal processes. Caballero-Morales and Rahim (2015) investigated the economic-statistical design of $\overline{X}$ control charts under the effect of non-Normality. Emura and Lin (2015) compared Normal approximation rules for attribute control charts. Panthong and Pongpullponsak (2015) discussed the economic design of fuzzy $\overline{X}$ charts for monitoring non-Normal processes. Patil and Shirke (2015) dealt with the economic design of variable sampling interval moving average charts for monitoring non-Normal processes. Aslam et al. (2016c) studied $\overline{X}$ charts for monitoring non-Normal correlated data with repetitive sampling. Noorossana et al. (2016) investigated the performance of EWMA charts with estimated parameters when monitoring non-Normal distributions. Saeed and Kamal (2016) proposed an EWMA control chart for monitoring the mean of a non-Normal process based on a robust estimator for the process variance. Patil and Shirke (2017) studied the economic design of MA charts for monitoring non-Normal processes. Huberts et al. (2018) investigated the performance of $\overline{X}$ charts for monitoring large datasets from non-Normal distributions. Saeed and Kamal (2019) developed EWMA control charts for monitoring non-Normal processes using repetitive sampling scheme.

2.18.1 Control Charts for Skewed Distributions

Burrows (1962) studied $\bar{X}$ control charts for skewed distributions. One way of handling skewed distributions in control charts is to adjust control limits so as to take the distribution's skewness into consideration. Choobinek and Ballard (1987) adjusted the control limits according to the direction of the distribution's skewness using the weighting variance method in order to obtain two symmetrical distributions instead of a skewed one. Abel (1989) also addressed the control limits for monitoring skewed distributions using weighted variance. Tagaras (1989) considered the economic design of $\overline{X}$ charts with asymmetric control limits. DuBois (1991) studied control charts for skewed distributions and dealt with their application in monitoring health-related processes. Shore (1991) introduced control charts with asymmetric control limits corresponding to the distribution's skewness. Schneider and Kasperski (1994) and Schneider et al. (1995) addressed control charts for data positively skewed and censored from below. Bai and Choi (1995) proposed mean and range control charts for monitoring skewed distributions and presented computations and tables useful for the implementation of the weighted variance chart proposed by Choobineh and Ballard (1987). Choi (1996) studied control charts for monitoring skewed processes. Mandraccia et al. (1996) dealt with the design of control charts for monitoring data from skewed distributions. Wu (1996) introduced an $\overline{X}$ chart with asymmetric control limits for monitoring data from skewed distributions. Woodward (1997a,b) discussed control charts for skewed distributions. Zhang and Qinan (1997) addressed the optimization of joint $\overline{X}$ and S control charts with asymmetric control limits. Castagliola (2000), improving the method by Choobineh and Ballard (1987), used the scaled weighted variance method for taking into account the skewness of distributions when using $\bar{X}$ control charts. Shore (2000) constructed Shewhart-type control charts for attributes taking into account the first three moments of the plotted statistic along with an inflated skewness measure during the computation of the control limits, thus making these charts useful for skewed attributes distributions for which traditional Shewhart charts fail to perform well. Chang and Bai (2001a) dealt with monitoring positively-skewed distributions using weighted standard deviations, while Chang and Bai (2001b) used median control charts for monitoring skewed distributions. Marcellus (2001, 2006) studied $\bar{X}$ charts with asymmetric control limits. Dou and Sa (2002) addressed one-sided control charts for monitoring the mean of positively skewed distributions. Yang (2002) studied the effects of imprecise measurement on economic asymmetric control charts. Chan and Cui (2003) proposed a skewness correction method for constructing $\bar{X}$ and R charts for skewed distributions. Khoo (2004b) dealt with the problems of the $\bar{X}$ chart for monitoring data from skewed distributions. Pongpullponsak et al. (2004, 2007) compared the performance of various methods of constructing control charts for skewed distributions. Samanta and Bhattacherjee (2004) introduced a mode chart and a weighted variance chart for monitoring skewed distributions, compared them with the Shewhart charts and illustrated them with an application to data from a surface mine. Chen and Kuo (2007a,b, 2010) conducted comparisons of the symmetric and asymmetric limits for $\bar{X}$ and R charts. Wang and Xu (2007) addressed control charts for monitoring small shifts in skewed distributions. Khoo et al. (2008) introduced a synthetic control chart for monitoring the mean of skewed distributions combining the weighted variance method by Bai and Choi (1995) with the synthetic chart by Wu and Spedding (2000). Khoo and Atta (2008) developed a weighted variance

EWMA chart for monitoring the mean of skewed distributions. Tsai and Wu (2008) proposed an adjusted weighted standard deviation R chart for monitoring processes following skewed distributions. Brill and Bzik (2009) dealt with control charts for skewed and left-censored data. Castagliola and Khoo (2009) presented the synthetic scaled weighted variance control chart for monitoring the process mean of skewed distributions combining the scaled weighted variance control chart proposed by Castagliola (2000) with the synthetic control chart proposed by Wu and Spedding (2000). Hai-Yu (2009) proposed EWMA control charts for skewed distributions. Khoo et al. (2009) developed $\overline{X}$ and S charts for monitoring data from skewed distributions. Lin and Chou (2009) addressed the economic design of adaptive $\overline{X}$ charts for monitoring skewed distributions. Pongpullponsak et al. (2009) studied the economic design of $\overline{X}$ charts for skewed distributions. Wang (2009b,c) addressed EWMA charts for monitoring skewed distributions. Wang (2009d) dealt with skewness and kurtosis correction for $\overline{X}$ and R charts. Yang and Rahim (2009) considered the minimum loss design of asymmetric $\overline{X}$ and S charts with two independent Weibull shocks. Yazici and Kan (2009) discussed control charts with asymmetric control limits for monitoring data with small samples. Teh and Khoo (2009, 2010, 2012) and Teh et al. (2014) studied the influence of skewed distributions on various weighted moving average-based control charts. Ong and Ooi (2010) investigated the influence of skewed distributions on the performance of statistical and neural network control charts for monitoring the process mean. Yin and Chong (2010) discussed the effect of skewed distributions on the performance of some DEWMA charts. Chen et al. (2011) studied one-sided control charts for monitoring the mean of positively skewed distributions with truncated saddlepoint approximations. Lee (2011) discussed the Tukey's control chart with asymmetrical limits. Kao (2012) introduced a range control chart for skewed distributions using the probability density function of the distribution of the range. Karagöz and Canan (2012) developed control charts for some skewed distributions (Weibull, Gamma and Lognormal). Sukparungsee (2012) investigated the robustness of Tukey's control chart in detecting parameter changes for the case of skewed distributions. Hsieh and Chen (2013) examined the economic design of the VSSI $\overline{X}$ chart for monitoring positively skewed distributions. Lee et al. (2013c) dealt with the economically optimum design of Tukey's control chart with asymmetrical limits for monitoring the mean of skewed distributions. Sukparungsee (2013) considered asymmetric Tukey's control chart robust to skewed and non-skewed processes. Liew et al. (2014) studied the effect of skewness on the performance of EWMA and MA charts. Mekparyup et al. (2014a) discussed adjusted Tukey's control charts and Mekparyup et al. (2014b) investigated the performance of the adjusted Tukey's control charts for monitoring skewed distributions. Karagöz (2015) dealt with robust $\overline{X}$ and R charts for skewed distributions. Khaparde and Rajput (2015) discussed control charts with skewness correction for random queue length. Lukin and Yaschenko (2015) developed parametric bootstrap control charts for monitoring data from skewed distributions. Atta et al. (2016a) proposed a scaled weighted variance control chart for monitoring the standard deviation of skewed distributed processes. Karagöz (2016) addressed robust $\overline{X}$ charts for skewed and contaminated processes. Riaz et al. (2016) studied control charts with skewness correction for monitoring contaminated and non-Normal processes. Kao (2017) developed $\overline{X}$ and R charts for monitoring skewed distributions using weighted variance with left-right tail-weighted ratio. Teoh et al. (2016) investigated the performance of the double sampling $\overline{X}$ chart for monitoring skewed distributions with estimated parameters. Atta et al. (2017) introduced a control chart for monitoring the standard deviation of data from skewed distributions using skewness correction. Yang et al. (2017) proposed a median loss control chart for monitoring quality loss with data from skewed distributions. Iqbal and Hassan (2018) discussed robust control charts for monitoring process variability for skewed distributions. Karagöz (2018) studied control charts with asymmetric control limits for monitoring the range of non-Normal distributions with robust estimator. Noiplab and Mayureesawan (2019) considered modified EWMA chart for monitoring skewed distributions and contaminated processes. Atta et al. (2020)

proposed a skewness correction control chart for monitoring process variability for skewed distributions and illustrated it with application in healthcare.

2.18.2 Control Charts for Specific Non-Normal Distributions, Families and Mixtures

Control charts have been proposed in the relevant literature for various non-normal distributions, families and mixtures. This subsection presents a brief overview of control charts for all of them exept the Pareto and Pareto-related distributions, which the next subsection is specially dedicated to, since they are greately connected to Chapter 9 of this thesis. This subsection also pays special attention to EWMA control charts for the distributions mentioned here, since EWMA control charts are the core of Part II of this essay.

Control charts for Bernoulli distribution have been discussed among others by Steiner et al. (1999), Borror and Champ (2001), Steiner et al. (2001), Weiß and Atzmüller (2010), Rossi et al. (2012, 2014), Lee et al. (2013a), Dexter et al. (2014), Martínez-Rego et al. (2015), Noskievičová et al. (2015), Zhang and Woodall (2015,2017a,b), Aminnayeri and Sogandi (2016), and Fatemi Ghomi and Sogandi (2019). Control charts for Binomial distribution have been addressed by many researchers including Quesenberry (1991a,1995a), Bourke (2001a), Morais and Pacheco (2006), Wu et al. (2008a), Fatahi et al. (2010), Areepong and Sukparungsee (2011), Chakraborty and Khursid (2011a,b), Huang et al. (2012), Haridy et al. (2014b) and Aytaçoğlu and Woodall (2020). Papayanopoulos (1997) introduced control charts for monitoring data from the weighted Binomial distribution. Fatahi et al. (2010) presented control charts for monitoring rare health events with truncated zero-inflated Binomial distribution. Chakraborty and Khursid (2011c) dealt with one-sided CUSUM charts for the zero-truncated Binomial distribution. Ho and Alencar (2013) introduced an overdispersed Binomial distribution including the common correlation between the individual Bernoulli variables, estimated its parameters with the methods of moments and MLE and developed Shewhart-type np and EWMA-type np charts for the proposed distribution and compared their performances with each other and with the conventional np and EWMA charts. Khurshid and Chakraborty (2014) investigated the effect of measurement error on the power of the control chart for monitoring data from the zero-truncated Binomial distribution. Rakitzis et al. (2014, 2016c) dealt with control charts for monitoring data from zero-inflated Binomial distribution.

Control charts for the exponential distribution have been studied for example by Vardeman and Ray (1985), Gan (1989b,1992b,1994,1998), Alwan (2000), Xie et al. (2002b), Scariano and Calzada (2003), Zhang et al. (2005,2006,2011a,2014a), Liu et al. (2006a,b,2007), Busaba et al. (2012a), Sukparungsee (2014a), Sun et al. (2017) and many others. EWMA control charts for the Exponential distributions have been discussed by Gan and Chang (2000), Ozsan et al. (2010), Pehlivan and Testik (2010), Suriyakat et al. (2012), Polunchenko et al. (2014), Aslam et al. (2015a,b,2017a,c,d), Khan et al. (2016), Suriyakat (2016) and Arif et al. (2017). Subba and Kantam (2008) addressed control charts for monitoring the mean of double exponential distribution. Busaba et al. (2012b) examined the performance of CUSUM charts for negative exponential data. Rao (2013) introduced one-sided CUSUM charts for the Erlang-truncated Exponential distribution. Luguterah (2015) developed a CUSUM chart for monitoring the parameters of the Erlang-truncated Exponential distribution. Mukherjee et al. (2015) discussed control charts for simultaneous monitoring of the parameters of a shifted exponential distribution. Narayana Murthy and Akhtar (2017) dealt with the optimization of CUSUM charts for the truncated Hyper-Exponential distribution. Kavitha and Gunasekaran (2020) presented an attribute control chart for Exponentiated Exponential distribution under type-I censoring.

Control charts for the Gamma distribution have been covered by several authors including Gonzalez and Viles (2000,2001), Sim (2003a), Lu and Torng et al. (2009), Chen (2016), Yang et al. (2016) and Khan et al. (2017b). Regula (1975) dealt with optimal CUSUM charts for the detection of a change in distribution for the Gamma family. Tsai (2008) developed EWMA charts for monitoring type-I censored data from the Gamma distribution. Ali et al. (2023) discussed one-sided

EWMA charts for the detection of upward or downward shifts in the mean of a process following a truncated Gamma distribution.

Control charts for the Geometric distribution have been addressed among others by Calvin (1983), Kaminsky et al. (1992), Xie et al. (2000a,b), Bourke (2001b,2018), Yang et al. (2002a,b), Zhang et al. (2004,2013), Hong and Lee (2015) and Morais (2017). Control charts have also been considered for the Geometric Poisson distribution [for example Chen (1999), Chen et al. (2005) and Saghir et al. (2015)]. Chen et al. (2006) used Geometric Poisson EWMA charts for the detection of small quality level shifts. Chen (2012) dealt with Geometric Poisson EWMA charts for compound Poisson processes.

Topalidou and Psarakis (2009) provided a review of control charts for the Multinomial distribution. More recently, Lee et al. (2017) discussed the Generalized Likelihood Ratio (GLR) chart for monitoring the Multinomial distribution, while Lee and Woodall (2018) noted the advantages of GLR charts as far as change point detection is concerned, since these charts offer estimates of the process change-point and shift size for post-signal diagnosis for a wide range of shifts of process parameters. The GLR statistic, however, can sometimes be undefined when monitoring count processes, such as those following Binomial, Bernoulli, Poisson and Multinomial distributions. For cases like these, Lee and Woodall (2018) introduced a modified GLR statistic so as to be well defined in every situation.

Control charts for the Negative Binomial distribution were approached by several authors among which Xie and Goh (1993), Yun and Youlin (1996), Schwertman (2005), Albers (2008,2010b), Sparks et al. (2010a), Willem (2010), Cheng and Yu (2015), Albarracin et al. (2017) and many others. Control charts were also investigated for the case of zero-truncated Negative Binomial distribution for example by Khurshid and Chakraborty (2013,2016), Chakraborty et al. (2017a) and Khurshid (2017). EWMA control charts for the Negative Binomial distribution were studied by Sparks et al. (2010b,2011), Yu et al. (2011) and Saghir and Lin (2015c).

Control charts for the case of monitoring data from the Poisson distribution have been discussed among others by Quesenberry (1991b,1992,1995b), Kim et al. (1992), Kenett and Pollak (1996), Singh and Sayyed (2001), Herberts and Jensen (2004), Perry et al. (2007a,b), Weiß (2007,2009a), Weiß and Testik (2009,2011), Ryan and Woodall (2010), Zhao et al. (2015a,b), Abbasi (2017), Pollard et al. (2018) and Mou et al. (2023). EWMA control charts for the case of Poisson distribution were considered by Borror et al. (1998), Testik et al. (2006) EWMA control charts for autocorrelated Poisson processes were addressed by Weiß (2009b), Weiß (2011) and Zhang et al. (2014b). Sparks et al. (2009) discussed EWMA charts for the detection of unusual increases in Poisson counts, while Shu et al. (2012) considered EWMA charts for detecting increases in Poisson rate. Perry and Pignatiello (2011) dealt with estimation of the time of a step change in CUSUM and EWMA charts for the Poisson distribution. Abujiya (2017) used combined Shewhart and EWMA charts for monitoring Poisson data. EWMA control charts for the Poisson distribution were studied by Zhou et al. (2012), Abujiya et al. (2013,2016a) and Zhou et al. (2016).

Famoye (1994) proposed control charts for shifted generalized Poisson distribution. White and Keats (1996) investigated the performance of the Poisson CUSUM chart, while White et al. (1997) compared the use of Poisson CUSUM and c charts for monitoring defect data. He et al. (2003) considered the estimation error in control charts for zero-inflated Poisson distribution, while Fatahi et al. (2012) used an EWMA chart for monitoring rare health events with zero-inflated Poisson distribution. Bhattacharjee and Das (2010) discussed the use of the generalized Poisson II distribution for the construction of control charts for monitoring the number of defects per unit instead of using the traditional Poisson distribution. Control charts for monitoring Poisson rates were covered for example by He et al. (2014a), Han et al. (2010) and Assareh et al. (2016). Richards et al. (2015) studied control charts for nonhomogenous Poisson processes. Control charts for zero-inflated Poisson distribution were addressed among others by Xie et al. (2001), He et al. (2012a,2014c), He and Li (2012), Katemee and Mayureesawan (2012), Areepong (2015a) and Mukherjee and Rakitzis (2019).

Control charts were also studied for the case of the COM-Poisson distribution among others by Sellers (2012), Saghir et al. (2013), Saghir and Lin (2014c) and Aslam et al. (2016a). EWMA control charts in particular were presented for the COM-Poisson distribution by Aslam et al. (2016b,2016e,2017e), Alevizakos and Koukouvinos (2019) and Leong and Tan (2015). Balamurali and Kalyanasundaram (2013) used CUSUM control charts for monitoring data from a truncated Poisson distribution. Chakraborty and Khurshid (2013a) and Chakraborty and Khurshid (2013b) investigated the effect of measurement error on the power of control chart for the ratio of two Poisson distributions and the zero-truncated Poisson distribution, respectively. Katemee and Mayureesawan (2013) studied CUSUM charts for monitoring data from the zero-inflated generalized Poisson distribution. Rakitzis et al. (2016a) introduced CUSUM charts for monitoring data from geometrically inflated Poisson distribution and applied them to monitoring data related to infectious disease, while Rakitzis et al. (2016b) discussed monitoring of general inflated Poisson processes. Rakitzis et al. (2016d) presented control charts for monitoring zero-inflated correlated Poisson data. Areepong (2018) dealt with a MA control chart for monitoring autocorrelated zero-inflated Poisson processes.

Control charts for the Lognormal distribution have been studied for example by Morrison (1958), Joffe and Sichel (1968), Maravelakis et al. (1999), Shibo et al. (2008) and Huang et al. (2016b,2017). Areepong and Sukparungsee (2010) used the EWMA for monitoring Lognormal distributed processes.

Control charts for the Skew-Normal distribution have been addressed among others by Tsai (2007), Figueiredo and Gomes (2013a,b) and Li et al. (2014,2019). Control charts have also been developed for the case of the truncated Normal distribution, such as for example by Rai (1966), Cox (2009), Chakraborty and Khurshid (2015a,b) and other researchers. Control charts for the Inverse Gaussian distribution have been investigated among others by Edgeman (1989a,b,1996), Nabar and Bilgi (1994), Hawkins and Olwell (1997), Sim (2001,2003b), Lio and Park (2008) and Guo et al. (2014). Johnson (1963) proposed CUSUM charts for the Folded Normal distribution, while Rao et al. (2015) developed control charts for the Half Normal distribution and Rao et al. (2018) introduced control charts for the two-piece Normal distribution (useful for asymmetric data) using both single and repetitive sampling and compared the proposed charts' efficiency through simulation.

One of the distributions for which a vast amount of research has been done regarding control charts is the Weibull distribution. Examples include Johnson (1966), Nelson (1979), Ramalhoto and Morais (1995), Nichols and Padgett (2005), Erto and Pallotta (2006,2007,2008), Erto et al. (2008,2015,2018), Huang and Pascual (2011a,b), Chen (2014), Chan et al. (2015), Erto (2015), Wang et al. (2015), Wang et al. (2017a,2018b), Khan et al. (2017c,2018a,b), Zhang et al. (2017) and Pascual and Park (2018). EWMA control charts for the Weibull distribution have been discussed by Ramalhoto and Morais (1996), Ramalhoto and Morais (1999), Zhang (2004), Xie et al. (2008), Pascual (2010) and Black et al. (2011). Pascual et al. (2017) used EWMA charts for monitoring Weibull quantiles. Wang and Cheng (2017a) discussed a likelihood ratio test-based EWMA chart for monitoring the mean and variance of Weibull distributed processes. Wang and Cheng (2017b) proposed EWMA charts for monitoring a Weibull process with subgroups. Wang et al. (2018a) developed a Bayesian EWMA chart for monitoring Weibull percentiles with or without type II censoring.

Control charts have also been constructed for the Birnbaum-Saunders distribution [e.g. Lio and Park (2008), Leiva et al. (2011,2015), Khan et al. (2018c)], the Burr distributions (of various types) [e.g. Yourstone and Zimmer (1992), Chou et al. (2000), Chen and Yeh (2006), Lio et al. (2014), Chen and Chou (2017), Malela-Majika et al. (2018)], the Power Function distribution [Zaka et al. (2021a)], the Reflected Power Function distribution [Zaka et al. (2021b)], the Weighted Power Function distribution [Jabeen and Zaka (2021)], the Transmuted Power function and Survival Weighted Power Function distributions [Zaka et al. (2022)], the Rayleigh distribution [e.g. Raza and Riaz (2013), Raza and Butt (2016), Tyagi and Singh (2016)] and the Inverse Rayleigh distribution [e.g. Ali and Riaz (2014), Nanthakumar and Kavitha (2017)]. Sindhu et al. (2016) dealt with Bayesian cumulative quantity control charts for monitoring a mixture of Rayleigh distribution. Control charts have also been proposed for the Dagum distribution [Gadde et al. (2019)], the Erlang distribution [Knoth (1998a,b)], the Katz family of distributions [Fang (2003)], the Generalized Lambda distribution [Fournier et al. (2006), Das (2012)], the Gompertz distribution [Adewara et al. (2020)], the Log-Logistic distribution [Kantam and Rao (2006), Kantam et al. (2006), Mehmood and Awais (2021)], the Half Logistic distribution [Rao and Kantam (2012)], the Maxwell distribution [Hossain et al. (2017)] and the Inverse Maxwell distribution [Omar et al. (2021)].

Sim and Wong (2003) discussed R charts for monitoring data from the exponential, Laplace and Logistic distributions. Haynes et al. (2008) developed control charts with probability limits for non-Normal distributed data using g-and-k distributions, investigated their performance, the effect of non-Normality on the control limits and the robustness to non-Normality (error in confidence resulting from incorrect assumption of Normality) and illustrated them with real data applications using Bayesian and non-Bayesian estimation for the parameters of the distribution. Chattinnawat (2009) proposed a control chart for monitoring demerits when the process follows a Trinomial distribution. Srinivasa Rao et al. (2010) dealt with the economic statistical design of control chart for a quality characteristic following a Johnson distribution and process in-control times following generalized Pareto distribution and investigated the sensitivity of the design regarding the parameters and costs. Sant'Anna and ten Caten (2012) proposed Beta control charts for monitoring fraction data. Boyapati et al. (2015) constructed control charts for the new Weibull-Pareto distribution using percentiles of various sample statistics such as mean, median, midrange, range and standard deviation and evaluated the power of the proposed control charts in comparison with those using the traditional Shewhart control limits. Rao and Kumar (2015) introduced control charts for monitoring Exponential-Gamma processes. Saghir and Lin (2015b) extended the work by Saghir and Lin (2014b) and Riaz and Saghirr (2007) and developed a control chart with probability limits for monitoring the process variability based on Gini's mean difference for the Exponential, $t_{(5)}$, Logistic and Laplace distributions and compared the performance of the proposed control charts with the 3σ-limits control charts discussed in Saghir and Lin (2014b) and the traditional R and S charts. They also designed the corresponding $\bar{X}$ chart for the process mean related to the Gini-chart for variability, following Schoonhoven and Does (2010). Ahangar and Chimka (2016) proposed an attribute control chart for monitoring count data processes optimally designed so as to minimize the total cost of a linear function of Type I and Type II errors and applied it to the Poisson, Geometric and Negative Binomial distributions. Rao et al. (2016) proposed skewness corrected control charts for monitoring the mean and range of data from the Inverse Rayleigh and Inverse Half Logistic distributions. Raza and Siddiqi (2016) and Raza et al. (2016) presented EWMA and DEWMA charts for monitoring censored data from the Poisson-Exponential distribution. Rao (2018) introduced a control chart for the Exponentiated Half Logistic distribution. Rosaiah et al. (2018) developed an attribute control chart for monitoring truncated life test data from the exponentiated Fréchet distribution. Shafqat et al. (2018) investigated and compared the performances of Shewhart-type attribute control charts under truncated life test for the Burr X, Burr XII, inverse Gaussian and Exponential lifetime-truncated distributions, revealing the superiority of the inverted Gaussian distribution over the others. Shruthiand Deepa (2018) discussed control charts for failure times following the Exponentiated Gamma, Exponentiated Lomax, Beta Weibull and Log Logistic distributions under truncated life test.

Aslam et al. (2019b) introduced the median absolute deviation control chart for monitoring process capability indices for Weibull, Gamma and Lognormal distributions. Lee Ho et al. (2019) presented control charts with probability limits for monitoring rates and proportions for data from the Beta, the Simplex and the Unit Gamma distributions. Elrazik (2020) constructed attribute control chart for the new Weibull Pareto distribution under truncated life tests and used the ARL to evaluate its performance and compare it with the inverse Gaussian. Naseri et al. (2020) showed that, when applying a control chart on the deviation of the actual from the nominal size of each part of a short-run process, differences between the control limits of various deviations can generate a heavy-tailed distribution. Therefore, they suggested the use of a corrected numbers method obtained from

Phase I and applied in Phase II and illustrated the proposed method with an example from the Cauchy distribution. Demertzi and Psarakis (2024) discussed control charts for the two-parameter Lindley distribution by Shanker et al. (2013a) using the skewness correction by Chan and Cui (2003) as part of this essay. All the details on these charts will be presented in sections 7.2-7.8 below.

2.18.3 Control Charts for the Pareto and Pareto-Related Distributions

Petcharat et al. (2012) investigated the performance of CUSUM charts by fitting Pareto distribution with hyperexponential. Prasad et al. (2013) studied the performance of Pareto type II control charts for software reliability. Kumari et al. (2014) constructed control chart for the Pareto-II distribution using an order statistic for monitoring software failures and improving software reliability and compared through control charts this distribution with the Half Logistic distribution taking into account time domain data based on non homogenous Poisson process. The parameters were estimated by the MLE method. Guo and Wang (2015) discussed control charts for monitoring separately each of the parameters of the Pareto distribution based on ordered statistics and investigated the effect of estimating the parameters on the performance of the charts. Aslam et al. (2016d) proposed a control chart for time truncated life tests when the data follow the Pareto-II distribution with known or unknown shape parameter and investigated its performance through simulation. Nasiru (2016) introduced one-sided CUSUM charts for monitoring the shape parameter of the Pareto distribution. Baba and Maahi (2017) and Baba and Luguterah (2018) used CUSUM control charts for monitoring shifts in the parameters of the Pareto distribution. Jeyadurga et al. (2017) developed an np chart with repetitive group sampling for monitoring truncated life test data, under the assumption of Pareto-II distributed lifetime. Shei and Tuahiru (2017) considered a CUSUM chart for monitoring the parameters of the Pareto distribution. Bizunet and Wang (2018) proposed a likelihood ratio based double EWMA chart for monitoring the shape parameter of the inflated Pareto distribution discussed in Figueiredo et al. (2015). Burkhalter

(2020) discussed bootstrap control charts based on MLE, modified moment method and least squares estimation for monitoring generalized Pareto percentiles. Burkhalter and Lio (2021) constructed bootstrap control charts for the generalized Pareto distribution percentiles using the estimation methods of least squared error and maximum likelihood and a modified moment method and compared the performances of the proposed bootstrap charts and the Shewhart-type control charts through Monte Carlo simulation revealing the superiority of the bootstrap control chart based on the maximum likelihood estimator over all the other control charts.

2.19 Conclusion

This chapter has presented some parts of the literature on parametric SPC charts beginning with Shewhart control charts and including other major control charts proposed as alternatives or enhancements. Advantages and problems of existing control charts have been mentioned. Control charts for non-normal distributions and individual observations have been presented in special sections of this chapter since they constitute the motivation for the next chapters of this thesis.

CHAPTER 3

OVERVIEW OF LINDLEY DISTRIBUTION

3.1 Introduction

The Lindley distribution is an asymmetric one-parameter continuous distribution with right asymmetry which has some nice properties to be used in lifetime data analysis such as closed forms for the survival and hazard functions and good flexibility of fit. It was introduced by Lindley (1958, 1965) in the context of Bayesian statistics as a counter example of fiducial distributions (distributions which are opposite to known distributions) to illustrate the difference between fiducial distribution and posterior distribution.

The statistical properties of the distribution itself remained relatively unstudied until a publication by Ghitany et al. (2008) and a study by Hussain (2006), but since then, the Lindley distribution has been generalized, extended, mixed, modified (transmuted, transformed), discretized and used to describe the lifetime of a process or device and to model many types of real-world data such as waiting times of customers in queues until receiving service [e.g. Al-Mutairi et al. (2013)], human mistakes and various accidents [e.g. Ghitany and Al-Mutairi (2009)], failures and repair times of airborne systems and communications [e.g. Abdi et al. (2019)], stress-strength reliability [e.g. Al-Mutairi et al. (2015), Hassan (2017a,b), Joukar et al. (2020)], engineering, life testing and survival analysis [e.g. Al-Babtain et al. (2015), Shanker and Shukla (2016), Shanker et al. (2016a, 2017, 2019), Dey and Nassar (2020)]. It can be used in a wide variety of fields, including medicine, biology, genetics, epidemiology, finance and actuarial sciences, ecology, sociology and demography, agriculture, reliability and engineering, hydrology, etc. and has been generalized so as to model a wide spectrum of phenomena including cancer patient survival, carbon retained by plant leaves, stress-strength reliability and miscellaneous lifetime data, survival times and group mortality data and many other fields which are more likely out of the scope of interest of statistical process control. A recent extensive work on Lindley distribution with many lifetime applications can be found in Sharon Varghese (2018), while a review of some of Lindley distribution's generalizations can be seen in Tomy (2018). What follows below is a review of the Lindley distribution. More specifically, section 3.2 presents the definition and some useful information for the Lindley distribution, section 3.3 deals with the studies on the classical one-parameter Lindley distribution and section 3.4 is dedicated to a specific two-parameter extension of the Lindley distribution by Shanker et al. (2013) for which control charts are going to be constructed in Chapter 7.

3.2 Useful Information for the Lindley Distribution

The Lindley distribution is an asymmetric continuous distribution with right asymmetry which has some nice properties to be used in lifetime data analysis such as closed forms for the survival and hazard functions and good flexibility of fit.

The one-parameter Lindley distribution was introduced by Lindley (1958 and 1965) in the context of Bayesian statistics as a counter example of fiducial distributions (distributions which are opposite to known distributions) to illustrate the difference between fiducial distribution and posterior distribution.

The probability distribution function (p.d.f.) of the one-parameter Lindley distribution is given by

$$f(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}, \quad x>0, \quad \theta>0 \tag{3-1}$$

while its cumulative distribution function (c.d.f.) is given by

$$F(x;\theta)=1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}, \quad x>0, \quad \theta>0 \tag{3-2}$$

Figures 3-1 and 3-2 show the probability density of the Lindley distribution for various values of the distribution's parameter.

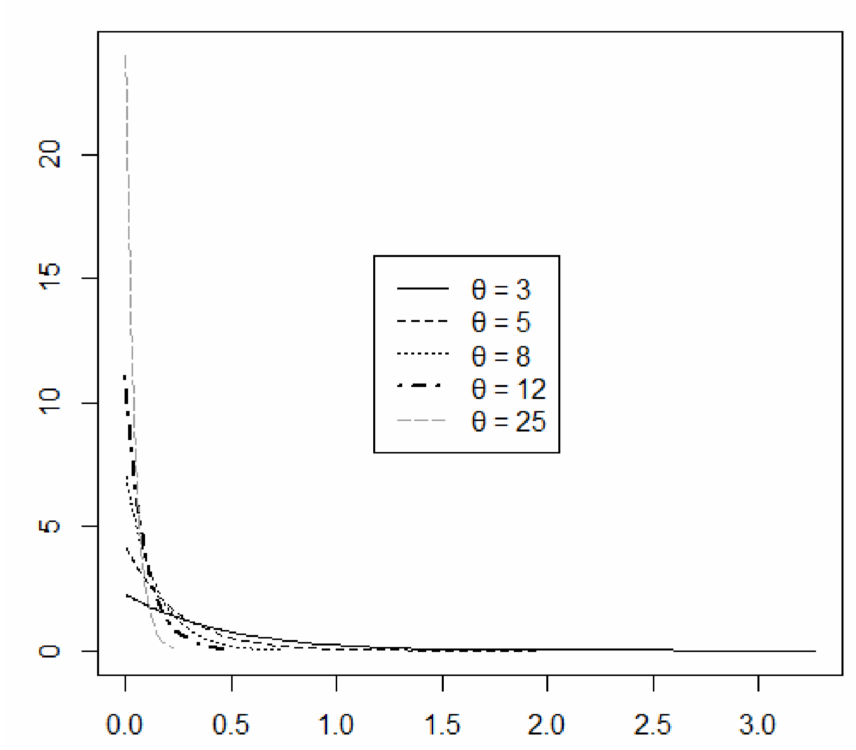

Figure 3 - 1: Probability plot of the Lindley distribution for various values of its parameter

The first four moments about origin of the Lindley distribution are given by

$$E(X) = \mu_1' = \frac{\theta + 2}{\theta(\theta + 1)} \tag{3-3}$$

$\mu_2' = \frac{2(\theta+3)}{\theta^2(\theta+1)}$, $\mu_3' = \frac{6(\theta+4)}{\theta^3(\theta+1)}$ and $\mu_4' = \frac{24(\theta+5)}{\theta^4(\theta+1)}$. The central moments of the Lindley distribution are given by

$$V(X) = \mu_2 = \frac{\theta^2 + 4\theta + 2}{\theta^2(\theta+1)^2} \tag{3-4}$$

$\mu_3 = \frac{2\left(\theta^3+6\theta^2+6\theta+2\right)}{\theta^3\left(\theta+1\right)^3}$ and $\mu_4 = \frac{3\left(3\theta^4+24\theta^3+44\theta^2+32\theta+8\right)}{\theta^4\left(\theta+1\right)^4}$. The coefficient of variation of the Lindley distribution is $\sqrt{\frac{\theta^2+4\theta+2}{\theta+2}}$. The coefficient of skewness $\left(\sqrt{\beta_1}\right)$ of the Lindley distribution is given by

$$\text{sk=E}\left[\left(\frac{\text{X}-\mu}{\sigma}\right)^3\right] = \frac{2\left(\theta^3+6\theta^2+6\theta+2\right)}{\left(\theta^2+4\theta+2\right)^{3/2}} \quad (3\text{-}5)$$

while its coefficient of kurtosis $\left(\beta_2\right)$ is $\frac{3\left(3\theta^4+24\theta^3+44\theta^2+32\theta+8\right)}{\left(\theta^2+4\theta+2\right)^2}$.

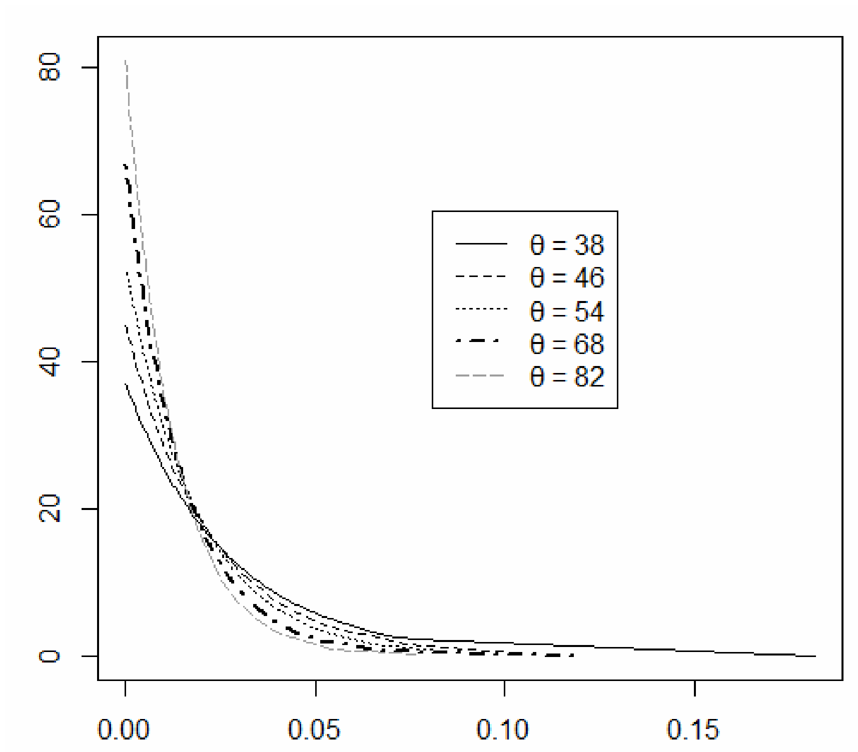

Figure 3 - 2: Probability plot of the Lindley distribution for various values of its parameter

3.3 One-Parameter Lindley Distribution

Hussain (2006) studied the Lindley distribution with respect to its statistical and sampling properties, dealt with parameter estimation and applied the distribution to stress-strength reliability of both a single component and a system of two identical components connected either in parallel or in series. Ghitany et al. (2008) studied various properties of the Lindley distribution and applied it to waiting times before service of bank customers.

Jodrá (2010) presented the computer generation of random variables following the Lindley distribution based on the fact that the quantile functions of both the aforementioned distributions can be written in closed form using the Lambert W function. Based on that, Mazucheli et al. (2016) introduced an R language package for the Lindley distribution and many other generalizations and modifications of the Lindley distribution.

Krishna and Kumar (2011) studied the one-parameter Lindley distribution as a useful reliability model, investigated its properties and reliability measures and dealt with the estimation of the distribution's parameter and other reliability features using both the classical maximum likelihood method and the Bayesian approach. Mazucheli and Achcar (2011) proposed the Lindley distribution as the distribution of competing risks for data sets of death or failure of individuals. Okwuokenye (2012) dealt with the size and power of tests of hypotheses on parameters when modelling time-to-event data with the Lindley distribution for the case of both complete and incomplete data with or without covariates. Covariate information was integrated using the Cox's proportional hazard model with the Lindley distribution as the time dependent component. Ali (2013) investigated the properties of Lindley distribution under different loss functions using the Bayesian approach. Ali et al. (2013) investigated the mathematical properties of the Lindley distribution by means of Bayesian approach under various loss functions and presented a real-life application to waiting time data at the bank comparing the results in view of the posterior risk. Gupta and Singh (2013) investigated and compared the classical and Bayesian analysis of the hybrid censored lifetime data assuming that the data follows the Lindley distribution. Athar et al. (2014)

determined some recurrence relations between moments of progressively Type-II right censored order statistics from the Lindley distribution. Saran et al. (2014) presented the L-moments and TL-moments of the Lindley distribution, used them for parameter estimation and presented the recurrence relations for higher moments of order statistics for the untruncated Lindley distribution or the doubly truncated Lindley distribution. Singh et al. (2014) proposed the upper, lower and double truncated versions of the Lindley distribution and estimated their parameters. Zaninetti (2019) presented the truncated Lindley distribution with scale and double truncation and estimated its parameters. Saran et al. (2015) presented recurrence relations for the moments of generalized order statistics from the Lindley distribution. Shanker et al. (2015) compared the Lindley distribution to the Exponential distribution when used for modeling lifetime data. Bakouch and Popović (2016) dealt with a stationary first-order autoregressive process with Lindley marginal distribution and estimated its parameters with tree different methods. El-Din et al. (2016a) dealt with optimal plans of constant-stress accelerated life tests for failure data from the Lindley distribution and illustrated their analysis with real data sets which they also used for comparison purposes between the Lindley distribution and the exponential distribution. They also presented the optimal proportion of test units allocated to each stress level based on two optimality criteria which they compared with each other and with the traditional optimal plan with two different methods. El-Din et al. (2016b) dealt with point and interval estimation for the parameter of the Lindley distribution in step-stress accelerated life testing with progressive first failure censoring. Kwon and Kim (2016) studied a comparative software development cost model based on the hazard function of the Lindley distribution. Metiri et al. (2016) dealt with Bayesian estimation for the Lindley distribution under Linear-exponential (Linex) loss function using informative and non-informative priors. Okwuokenye and Peace (2016) performed a comparison of the inverse transform and the composition methods for simulating data from the Lindley distribution and compared some statistical properties of the estimates of the distribution’s parameters based on the data they generated using those two methods. Shanker and

Fesshaye (2016a) studied the properties and parameter estimation of (among others) the Lindley distribution and compared it to other distributions commonly used for modeling lifetime data. Shanker and Fesshaye (2016b) studied the relationships of Lindley distribution and other lifetime data distributions and their distributional properties and parameter estimation. Shanker et al. (2016b) applied the Lindley distribution among other distributions for modeling lifetime data from various fields such as medical science and engineering. Sultan and Al-Thubyani (2016) presented the exact explicit expressions for the higher order moments of order statistics from the Lindley distribution and used them to find the best linear unbiased estimates of the distribution's parameters based on Type-II right-censored samples. Asgharzadeh et al. (2017) studied the estimation of the parameter of the Lindley distribution with a Bayesian and two classical approach methods based on Type II censored data. Ayesha (2017) proposed a size biased Lindley, while Messaadia and Zeghdoudi (2018) introduced a distribution obtained by means of biased technique under Lindley distribution studied its properties and dealt with parameter estimation. Joshi et al. (2017) studied a single change point model for a sudden change in the hazard rate of Lindley distribution under right censoring of survival data and estimated the parameters of the change point model. Ahsanullah et al. (2017) presented two characterizations of the Lindley distribution based on relations between left and right truncated moments and failure rate and reverse failure rate functions, respectively. Kilany (2017) presented a characterization of the Lindley distribution based on truncated moments of order statistics, as well as a simulation study which illustrates the usefulness of the characterization results for practitioners who want to verify that the data in hand come from the specific distribution. Pak (2017) dealt with parameter estimation for the Lindley distribution with both the classical maximum likelihood method and the Bayesian one for the case of having fuzzy data. Akgül et al. (2018) dealt with point and interval estimation of stress-strength reliability based on ranked set sampling when stress and strength are random variables following the Lindley distribution and compared through simulation the performances of their proposed methods with the corresponding ones based on simple random sampling. Asgharzadeh et al. (2018) dealt with the estimation of the parameter of the Lindley distribution and the prediction of unobserved records based on record statistics from the Lindley distribution with both the Frequentist and the Bayesian approach. Gómez-Déniz (2018) introduced a generalization of the exponential distribution which can be derived as the natural conjugate prior distribution of the one-parameter Lindley distribution. This distribution was used by Gómez-Déniz and Calderín-Ojeda (2016) who derived a two-parameter discrete distribution as a mixture of the Poisson distribution by mixing its parameter with the generalized exponential distribution proposed by Gómez-Déniz (2018). Irshad and Maya (2018) presented suitable U-statistics from a sample of any size for the estimation of the parameters of the Lindley distribution without the evaluation of moments of order statistics. Maiti and Mukherjee (2018) dealt with the estimation of the probability density function and the cumulative density function of the Lindley distribution with two different methods. Sharon Varghese (2018) dealt with the application of the Lindley lifetime distribution with special reference to accelerated life testing. This article also studied the properties of the distribution and presented a method for discrimination between the Exponential distribution and the Lindley distribution and one for calculation of the minimum sample size needed for this discrimination. Moreover, this paper presented a step-stress accelerated life testing model for the Lindley distribution under Type I censoring and dealt with its parameter estimation, extended the aforementioned model in the case of competing risk and dealt with the estimation of its parameters. Sharon Varghese (2018) studied the Morgenstern type bivariate extension of the Lindley distribution, too, and estimated its parameters.

Besides Sharon Varghese (2018), other papers dealing with selection between Lindley and other distributions are the following: Raqab et al. (2017) dealt with model selection between Lindley distribution, Weibull distribution and Gamma distribution for modeling positively skewed lifetime data, evaluated the closeness of the Lindley distribution to the other two distributions with three different methods and calculated the probability of correct selection between those three distributions through Monte Carlo simulation for various values of parameters and sample size. Sen et al. (2018) addressed the issue of selecting either Lindley or xgamma distribution with unknown parameter for a particular data set. The xgamma distribution has p.d.f. which is similar to the p.d.f. of the Lindley distribution and properties analogous to the Lindley distribution but its random variables are stochastically larger that the ones from the Lindley distribution. These two distributions are both useful for analyzing skewed non-negative data and in modeling time-to-event data sets. The study in Sen et al. (2018) presented the minimum necessary sample size for selecting one of those two distributions. Vaidyanathan and Sharon Varghese (2019) dealt with discrimination between the Exponential distribution and the Lindley distribution and presented a method based on the ratio of the maximum likelihoods and obtained the asymptotic distribution of the test statistic as well as the minimum sample size needed for this discrimination.

Nie and Gui (2019) dealt with parameter estimation for the case of progressive type-II censored data with Binomial removals when the product's lifetime under a single risk follows the Lindley distribution. This parameter estimation was obtained by both the maximum likelihood and the Bayesian method. Prasad et al. (2019) dealt with reliability analysis of symmetrical columns with eccentric loading from the Lindley distribution, presented the hazard rates and mean time to failure and studied the relationship between reliability and the scale parameter of the distribution. Hafez et al. (2020) studied the Lindley distribution under step-stress accelerated life tests when having progressive type II censored samples and dealt with parameter estimation with both the maximum likelihood and the Bayesian method under symmetric loss function. Khan et al. (2020) derived the formulas for the single and product moments of the Lindley distribution based on generalized order statistics including progressive type-II censoring. Khan et al. (2020) used their results for obtaining the best linear unbiased estimators for the location and scale parameters of the Lindley distribution. Krishna and Goel (2020) addressed the issue of sample inference for the case of two independent processes following the Lindley distribution under joint type-II censoring scheme for the two samples simultaneously and presented the joint density for the two Lindley-distributed populations. They dealt with parameter estimation with both the maximum likelihood and the Bayesian method. Panda (2020) derived the exact formulas for the single and product moments of order statistics for the Lindley distribution in the presence of multiple outliers in the data and investigated the robustness of the sample moments in the presence of outliers. Safari et al. (2020) obtained a robust and efficient estimator for the parameter of the Lindley distribution based on the probability integral transform statistic in order to avoid the sensitivity of the most commonly used maximum likelihood estimator to the presence of outliers. Athar et al. (2023) provided characterizations of the Lindley distribution based on doubly truncated moments.

3.4 The Two-Parameter Lindley Distribution by Shanker et al. (2013)

This special section is dedicated to the two-parameter Lindley distribution proposed by Shanker et al. (2013), because this distribution will be used later in Chapter 7 and, therefore, more details are required to be offered for this distribution. The two-parameter Lindley distribution proposed by Shanker et al. (2013) is an asymmetric continuous distribution with right skewness. The graphical representation of the distribution's probability density function for some values of the distribution's parameters can be seen in Figure 3-3, where it is obvious that the two-parameter Lindley distribution is positively skewed and its shape changes as the values of the process parameters change. The probability density function of the two-parameter Lindley distribution is given by

$$f\left(x;\theta,r\right)=\frac{\theta^{2}}{\theta+r}\left(1+rx\right)e^{-\theta x},\quad x>0,\ \theta>0,\ r>-\theta \tag{3-6}$$

with θ being the scale parameter. The cumulative distribution function is given by

$$F\left(x;\theta,r\right)=1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x},\quad x>0,\quad \theta>0,\quad r>-\theta \tag{3-7}$$

The moments of the two-parameter Lindley distribution in (3-6) are computed using the following formulas:

$$E(X) = \frac{\theta + 2r}{\theta(\theta + r)}, \tag{3-8}$$

and

$$V(X) = \frac{\theta^2 + 4\theta r + 2r^2}{\theta^2(\theta + r)^2}. \tag{3-9}$$

The coefficient of skewness of the two-parameter Lindley distribution in (3-6) is given by

$$\text{sk=E}\left[\left(\frac{\text{X} - \mu}{\sigma}\right)^3\right] = \frac{2\left(\theta^3 + 6\theta^2 r + 6\theta r^2 + 2r^3\right)}{\left(\theta^2 + 4\theta r + 2r^2\right)^{3/2}} \tag{3-10}$$

It should be noted that he original one-parameter Lindley distribution is just a special case of the two-parameter Lindley distribution when r=1, in which case all five equations (3-6)-(3-10) reduce to the corresponding ones for the one-parameter Lindley distribution.

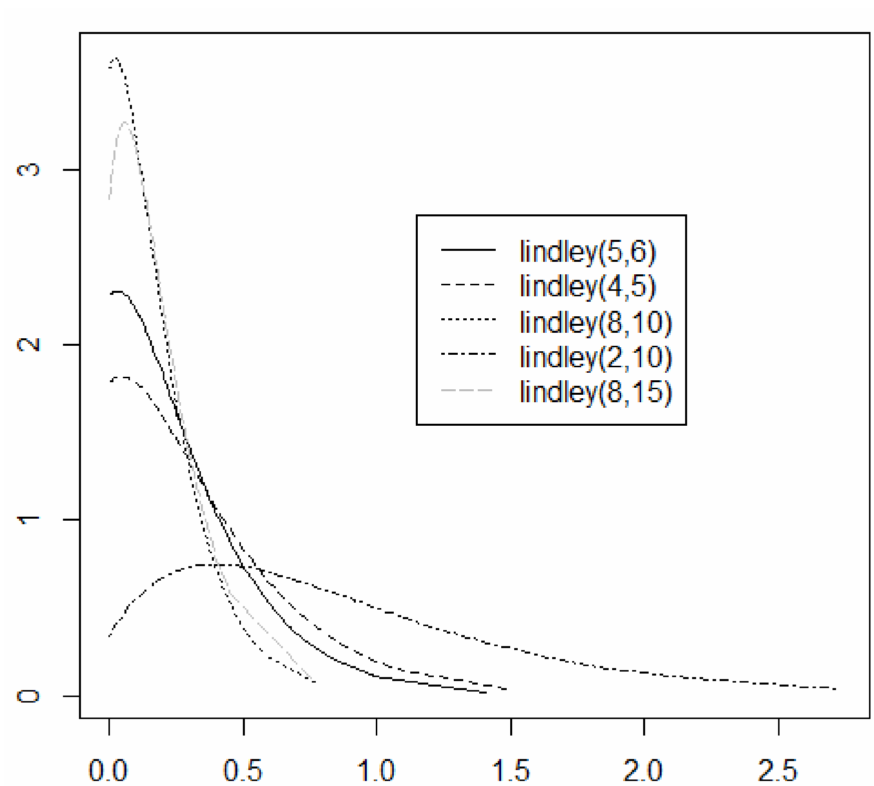

Figure 3 - 3: Probability density function of the two-parameter Lindley distribution for various values of the parameters.

3.5 Conclusion

Lindley distribution receives increasing attention in research and has many applications in various fields. In this chapter, an attempt has been made to briefly review the work done in the field of Lindley distribution. Special subsections have been dedicated to the original one-parameter Lindley distribution and the two-parameter Lindley distribution proposed by Shanker et al. (2013), since they are going to be used in Chapters 6 and 7, respectively, for the construction of control charts for individual observations from these two distributions.

CHAPTER 4

OVERVIEW OF LOGARITHMIC DISTRIBUTION

4.1 Introduction

The Logarithmic distribution is an asymmetric one-parameter discrete distribution with right asymmetry. It was first introduced by Fisher et al. (1943) and was obtained as the limit of a zero-truncated negative Binomial distribution in connection with an investigation of the frequency distribution of number of species of animals obtained from random samples. The distribution's properties were discussed in particular by Anscombe (1950) and Patil (1962). Logarithmic distribution was also further studied by Ahuja (1968). More details about this distribution can be found in Chapter 7 of the book by Johnson et al. (2005). Recurrence relations, random number generation and computational algortihm for the probabilities were presented in Chapter 8 of the book by Krishnamoorthy (2006).

Logarithmic distribution has many applications in biology and ecology (Khang and Ong (2005), Williams (1944), Darwin (1960), Boswell and Patil (1970), etc.) and biology (Corbet (1941), Williams (1947), etc.), since it can be used to describe and model the number of individuals per species or the number of species per genus. It is also applied in purchase studies (Williamson and Bretherton (1964), Chatfield et al. (1966), etc.) and other economic applications, since it can be used for fitting the number of products requested per order from a retailer, which makes it a very useful distribution particularly for companies selling products by phone or mail when they want to check whether the quantities demanded per order changes after a period of time or not. The Logarithmic distribution can also be used in various fields, such as population growth and human ecology (Clark et al. (1964), etc.), computer science, information systems, electrical and electronic engineering, telecommunications, nanoscience and nanotechnology (Kyriakoussis and Papadopoulos (1990), etc.), soil science (Jones and Mollison (1948)), meteorology and atmospheric sciences (Rambhadran (1954), Williams (1952), etc.), climatology (Agnese et al. (2014), etc.), physics and physical chemistry (Ostojic and Sasic (2006), Ross (1978), etc.), applied chemistry, food science and technology (Parvathy et al. (2007), etc.), and other scientific areas. A lot of extensions, mixtures, modifications and generalizations of the Logarithmic distribution can be found in the literature with lots of applications in various fields of our everyday lives, including survival and reliability analysis (Taketomi et al. (2022), etc.), number of publications (Famoye (1997), etc.), risk theory (Hansen and Willekens (1990), etc.), digital software testing and verification to describe the distribution of the total number of observed failures (Şahinoğlu (2003), etc.), biological, medical and ecological applications (Papageorgiou and David (1995), Mishra and Shanker (2002), Wani et al. (2016), etc.), hydrology (Lawal et al. (1997), etc.) and many other areas. What follows in the next sections is an attempt to provide a review of the Logarithmic distribution. More specifically, section 4.2 presents the definition and useful information for the Logarithmic distribution, section 4.3 deals with the literature on investigation of the Logarithmic distribution and estimation of its parameters and section 4.4 provides the literature on applications of the Logarithmic distribution.

4.2 Useful Information for the Logarithmic distribution

The Logarithmic distribution is an asymmetric continuous distribution with right skewness. The graphical representation of the distribution's probability mass function for some randomly chosen values of the distribution's parameter can be seen in Figure 1, where it is obvious that the Logarithmic distribution is positively skewed and its shape changes as the value of the process parameter changes.

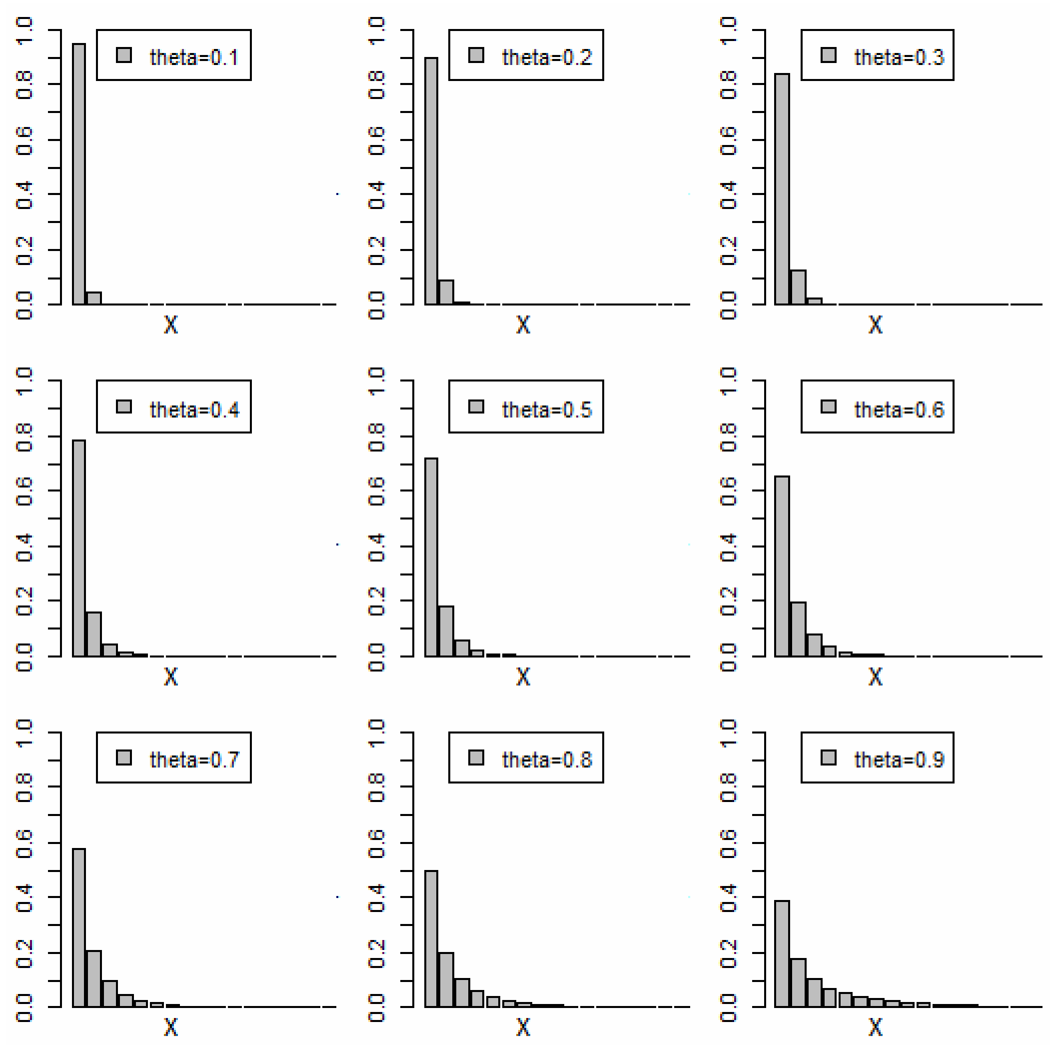

Figure 4 - 1: Probability mass function of the Logarithmic distribution for various values of the parameter.

The probability mass function of the Logarithmic distribution is given by

$$P(X=x)=-\frac{1}{\ln(1-\theta)}\frac{\theta^x}{x}, \quad 0 < \theta < 1, \quad x = 1, 2, \ldots \qquad (4\text{-}1)$$

The cumulative distribution function is given by

$$P(X\le x)=1+\frac{1}{\ln(1-\theta)}\sum_{u=x+1}^{\infty}\frac{\theta^u}{u}=-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}, \quad 0 < \theta < 1, \quad x = 1, 2, \ldots$$

(4-2)

The moments of the Logarithmic distribution in (4-1) are computed using the following formulas:

$$E(X)=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} \qquad (4\text{-}3)$$

and

$$V(X)=-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right) \qquad (4\text{-}4)$$

The coefficient of skewness of the Logarithmic distribution in (4-1) is given by

$$sk=\mathrm{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]=\frac{b\theta\left(1+\theta-3b\theta+2b^2\theta^2\right)}{\left[b\theta(1-b\theta)\right]^{3/2}}, \text{ where } b=-\frac{1}{\ln(1-\theta)} \qquad (4\text{-}5)$$

4.3 Studying the Logarithmic distribution and Estimation of Its Parameters

Levin (1966) investigated the structure and statistics of the Logarithmic Series distribution. Engen (1974) investigated and compared various estimation methods for the Logarithmic Series distribution. Böhning (1983) dealt with MLE of the parameter of the Logarithmic Series distribution. Shanmugam and Singh (1984) provided a characterization for the Logarithmic Series distribution and based on that they proposed a statistic for testing whether a random sample follows a Logarithmic Series distribution. The usefulness of the proposed statistic over the usual goodness-of-fit test was discussed and illustrated with a numerical example. Panaretos and Xekalaki (1986) introduced a Logarithmic Series distribution as a limiting form of the distribution resulting from inverse sampling scheme. Wani and Lo (1986) presented three characterizations of the Logarithmic distribution and other members of the class of Power Series distributions, two of which can be used to choose between the five member distributions of the Power Series family of distributions. Devroye (1987) developed a short algorithm for generating random integers form the Logarithmic Series distribution. Aki and Hirano (1989) discussed the MLE of the parameter of the Logarithmic Series distribution of order k based on independent observations and the asymptotic properties of estimator using the method of moments. Famoye and Consul (1989) dealt with confidence interval estimation of the parameter for the Logarithmic Series distribution considering both small and large sample sizes. Kyriakoussis and Papadopoulos (1990) studied the Logarithmic Series distribution as a failure model from the Bayesian point of view and provided Bayes estimators for the location parameter and reliability function. Kyriakoussis and Papageorgiou (1991) provided characterizations for the distributions of two random variables following the Logarithmic Series distribution based on the regression function of one of those random variables over the other and the conditional distribution of the second random variable given the first one. Papp and Izsák (1997) investigated the relationship between the Lognormal and Logarithmic Series distributions and bimodality through simulations and numerical examples based on the truncated Lognormal and Logarithmic Series distributions. Adamidis (1999) introduced a bivariate distribution defined by a pair of independent random variables following the Logarithmic Series distribution and a related Exponential distribution truncated to (0,1) and used it to derive an EM algorithm which gives the M-step in closed form without the need for additional iterative processes. This algorithm was used for estimating the parameters of the Negative Binomial distribution using the result by Quenouille (1949) that the Negative Binomial distribution can be viewed as a Poisson sum of Logarithmic Series distributed variables. Hall and Temido (2007) investigated the limiting distribution (after appropriate normalization) of the maximum term of integer-valued stationary MA and max-AR models for marginal distributions with a quasi-stable limiting behaviour such as, among others, the Logarithmic distribution. Ameli et al. (2014) addressed the discrete likelihood ratio order for the Power Series distribution family (which includes the Logarithmic Series distribution as a special case) as well as the discrete version of the proportional likelihood ratio as an extension of the likelihood ratio order. Ahmad (2016) obtained the Bayes estimators of functions of parameters of the size-biased Logarithmic Series distribution under squared error loss function and weighted square error loss function. Nasiri and Esfandyarifar (2016) dealt with E-Bayesian parameter estimation (expectation of Bayesian estimation) for the Logarithmic Series distribution. Eryilmaz (2017) computed the optimal number of units and replacement time minimizing the mean cost rate for a parallel system having a random number of units from a Power Series class of distributions including distributions such as the such as modified or truncated Poisson and Logarithmic distributions. Mayster and Tchorbadjieff (2019) investigated the transition probability and Lévy measure of a Lévy process with representative random variable from the Logarithmic Series distribution, as well as the Lévy measure of a subordinated Logarithmic Lévy process directed by a Poisson process and compared the properties of the processes under study. Alshkaki (2020) provided characterizations of the Logarithmic Series distribution based on linear differential equation for the probability generating function. Chattamvelli and Shanmugam (2020) dealt with the Logarithmic Series distribution in chapter 8 and presented a theorem to find moments of Logarithmic distribution using moments of zero-truncated geometric distribution. Kirtland et al. (2020) used a moment preserving finitization called the Negative Taylor Series Finitization method for the Power Series family of discrete distributions along with the method of aliasing in order to improve infinitely supported discrete random variate generation speed with certain limitations and illustrated their proposed method with an application to the Logarithmic Series distribution. They also compared various algorithms for random variates generation from a Logarithmic distribution to the aliasing method of random variate generation from a Negative Taylor Series Finitization version of the Logarithmic distribution in terms of accuracy and speed of all these methods.

4.4 Literature on Applications of the Logarithmic distribution

Kendall (1948) discussed some modes of population growth leading to Fisher's Logarithmic Series distribution. Bond (1952) applied the Logarithmic Series distribution to studies of plants. Williams (1952) described sequences of wet and dry days with Logarithmic Series distributions. Cooke (1953) used the Logarithmic Series distribution to model the duration of wet and dry spells at Moncton, New Brunswick. Roessler (1965) suggested the Logarithmic Series distribution as a model for the number of individuals per species of fish population in Biscayne Bay, Florida. Kobayashi (1966) discussed the use of the Logarithmic distribution for describing the distribution of eggs laid per visit of cabbage butterfly. Holgate (1969) studied the Logarithmic Series distribution as a model describing a random species in a sample in studies of distribution of species abundance in a population. Paster et al. (1974) fitted the Logarithmic distribution to trace elements of the Skaergaard layered series, which is the classic example of a layered silicate intrusion, for six rocks and twelve mineral separates analyzed by neutron activation. Besides using the Logarithmic distribution for trace element partitioning, they also used it for describing the behaviour of the elements during solidification of the layered series. Watterson (1974) presented the Logarithmic distribution as a model for the species abundance distributions used to describe evolving populations of selectively neutral genotypes and provided statistical inference methods and measures of diversity for this distribution. Dunn and Hardy (1980) applied the Logarithmic Series distribution to modelling the number of transient ischemic attacks per cluster with a cluster of transient ischemic attacks being the transient ischemic attacks occurring during a single period of abnormal arterial activity. Coleman (1981) used the Logarithmic Series distribution to describe the number of individuals from a particular species belonging to a collection of individuals from several species living in a region. Berger and

Goossens (1983) and Goossens and Berger (1984) investigated the Logarithmic Series distribution for modelling the sequences of dry and wet days in studies of rainfall persistence at Belgian stations. Rao (1984) considered probability problems in epidemiology (useful for public health officials for ensuring that only a given proportion of the community is infected with the disease) when assuming that the number of infected individuals in the community follows a Binomial distribution and the total community size follows the Logarithmic Series distribution. Andreassen and Hoque (1986) showed that the Logarithmic Series distribution can adequately describe the distribution of accident frequencies and developed a new test in order to evaluate that adequacy by subdividing the data by the functional classes of the intersecting roads, proving that the Logarithmic Series distribution described well the distributions of accident frequencies in all road classes. Chatfield (1986) discussed the use of the Logarithmic Series distribution for describing distributions of purchase noting, however, that the fit is not always very good for some heavily-bought products. Barker and Smith (1987) used the Logarithmic Series distribution to model the number of insect species per sample in the Prairie Provinces. Wright (1988) fitted the Logarithmic Series distribution to abundance species data and discussed their relationship to the species-area relations. Branson (1991, 2000) discussed the Logarithmic Series distribution for the abundance of families of a particular size when modelling inhomogeneous birth-death and birth-death-immigration processes. Mekjian (1991) presented application of the Logarithmic Series distribution in the physical and biological sciences. Mason et al. (1997) showed that the relative abundance of families and species of spiders followed the Logarithmic Series distribution. Lavenda (2000) used the Logarithmic Series distribution to describe quantum noise contribution. Angeja et al. (2004) studied packet arrival and loss for wireless indoor communications environments and used the Logarithmic Series distribution to model the burst lengths of received and lost real time packets. Lonardi et al. (2007) showed that the number of longest matches in a Lempel-Ziv’77 data compression scheme follows a Logarithmic Series distribution with mean equal to the inverse of the source entropy (plus some fluctuations). Agterberg and Liu

(2008) used the Logarithmic Series distribution to describe fossil events in stratigraphic study for the North Sea Basin. Ferreira and Petrere (2008) discussed some aspects of the Logarithmic Series distribution for describing species abundance models in order to contribute to the analysis of the empirical patterns of species abundance and indicate the resources which are important in the structuring of biological communities. Wilson (2008a) applied the Logarithmic Series distribution to the species proportions in seafloor samples from an area off south-east Trinidad. Wilson (2008b) used the Logarithmic Series distribution to describe the epiphytal population structure in shallow water in two bays around Nevis, NE Caribbean Sea. Carling (2009) presented the use of Logarithmic distribution to describe tidal current velocities in studies of current speed with height above the bed from a sandy intertidal zone in South Wales, UK. De Aguiar et al. (2009) studied the global patterns of biodiversity and showed that the tail of the distributions of species abundance can be approximated by the Logarithmic Series distribution. Neumann (2009) applied the Logarithmic Series distribution to the generation of behavior-based recommendations for market baskets found in e-commerce, library environments or social network sites in order to show which co-purchases or co-inspections of products reveal an underlying relationship between those items. Cheli et al. (2010) used the Logarithmic Series distribution to describe the distribution of abundance data for both the family and the species of ground-residing arthropods in Península Valdés in Patagonia, Argentina. Dolgonosov et al. (2010) showed that the statistical distributions of phytoplankton cell concentration follow the Logarithmic distribution during the vegetation period and this was demonstrated with various empirical data that confirmed the theoretical forecasts and provided the possibility of predicting the probabilities of various phytoplankton concentration values of a large range, including large values, which "are of greatest hazard in terms of water quality, water treatment processes, and aquatic ecosystem well-being". Chowdhury and Beecham (2013) fitted the Logarithmic Series distribution to the dry and wet periods while studying rainfall events and inter-event periods with data on daily rainfall sequences for Adelaide and Melbourne in Australia. Bertoli-Barsotti and Lando (2015) considered the Logarithmic distribution for describing the distribution of individual authors' papers' citations and compared it with other distributions fitted to the same data such as the Pareto and Geometric distributions. Doumas and Papanicolaou (2018) used the Logarithmic distribution to describe coupon probabilities for the coupon collector's siblings problem. Visintin et al. (2022) described the mosquito abudance distribution at the southern coast of Mar Chiquita Lake, Argentina, by the Logarithmic Series distribution. Saila et al. (2023) provided an overview of the application of the Logarithmic Series distribution to the temporal and spatial changes assessment of the composition of exploited tropical multispecies fish communities within the Samar Sea in Philippines.

4.5 Conclusion

Logarithmic distribution has received an increasing attention in research especially lately and has many applications in various fields as presented earlier in this chapter. Here an attempt has been made to present a review of most of the literature on the Logarithmic distribution and its applications. Useful information for the distribution has been presented in a special section for easy access, since it will be useful for the constrction of control charts for the distribution in Chapter 8.

CHAPTER 5

OVERVIEW OF PARETO DISTRIBUTION

5.1 Introduction

The Pareto distribution was introduced by Pareto (1964) to assess the allocation of wealth among individuals and describe the distribution of income on the basis that a high proportion of the people in a society have low income and/or a small portion of the wealth of that society, while only a few people have very high incomes and/or a huge amount of that wealth. The Pareto distribution as the distribution of income was further studied by Creedy (1977), while Faber et al. (1985) studied a model leading to the Pareto wealth distribution. In economics, its threshold parameter is some minimum income, and the large value of the shape parameter means the high equality of the allocation of income, which indicates that the shifts in the Pareto distribution means the changes of the allocation of wealth among individuals. More recently, Pareto distribution for describing income and wealth was discussed by Nirei and Aoki (2016) and Abd Raof et al. (2022). Other financial applications have been addressed, for example, in Ball (2003), Fernandes et al. (2008) and Jones (2015).

Since the Pareto distribution is a heavy tailed distribution, it has many applications in various fields where quantities are distributed according to certain statistical distributions with very long right tails, such as modeling income above a theoretical value and the distribution of insurance claims above a threshold value. Newman (2005) discussed applications of the Pareto distribution in physics, biology, earth and planetary sciences, economics and finance, computer science, demography and the social sciences. For instance, the distributions of the sizes of cities, firms, earthquakes, forest fires, solar flares, moon craters and people's personal fortunes all appear to follow the Pareto distribution. Chattamvelli et al.

(2021) mention the following applications: “luminosity of stars and other celestial objects in astronomy, size of various sorts (like firm sizes or headcount in management, size of stored files in computing, size of cities within large countries in sociology, extreme ocean wave heights in ocean engineering, size (area) of aegean islands in geography, species size and abundance in zoology, blackout sizes and restoration times of power grids in power transmission engineering, size or area of a region destroyed by natural calamities like forest fires, oil spills in seas in environmental science, oil-and-gas field-size and reserves distribution in petroleum engineering), frequencies (like frequency of occurrence of family names in a country or in telephone directories, frequency of comet visits in astronomy, frequency of replenishment of perishable items in inventory systems), vibrational amplitudes in mechanical engineering, data faults, or error clusters in communications engineering, position errors in global positioning systems (GPS) and sonar-based rescue and repair missions, durations (like time to complete medical procedures or surgical operations, quarantine periods, duration between major calamities like earthquakes or tsunamis, time to fix bugs in very large and complex software systems, etc.), and costs of commodities (like boats and yachts, air planes, and so on). It is also used for size-frequency modeling studies in aquatic and environmental sciences, epidemiology, microbiology, and semiconductor defects modeling.”

Pareto distribution is used in bibliometrics to describe word frequency rankings and ranking scientists by number of publications, in geology, geochemistry and geophysics, metallurgy, limnology and oceanography, ecology and environmental sciences, physics, sports, biosciences, computer sciences, telecommunications, engineering, astronomy and astrophysics, actuarial science, insurance and risk management, archaeology and software testing. It has been used to describe metal deposits, natural resources, weather forecasting, wildfires, blackouts, terrorism, words, surnames and web links. It has been applied to studies of spatial behavior and structure of cities, size distribution of cities and distribution of urban population, urban luminosity and nighttime light intensity, queuing systems, mortality after diagnosis of a disease, bank sizes and bank’s operational risk, accident occurrence, number of films produced and the sum of box office revenue earned by a movie producer, software failures or reliability, aircraft systems, survival and lifetime data, failures and service times, traffic, pollution, wildfire sizes and absorption capacity in studies of flood forecasting. It has also been used to describe firm size, size distribution of trade unions, global extend and size distribution of surface water areas, planktonic and phytoplanktonic size distributions, low-flow frequencies in rivers, temperatures, distribution of earthquake seismic moment, earthquake slip distribution and energy released by earthquakes, rainfall depth and duration, duration of drought, occurrence of strong mine tremors, biomass size distribution, waiting time of solar flares and coronal mass ejections, distribution and size of water particles, density of polyamide clusters on the surface of liquids, data from radar systems and radar sea clutter, COVID-19 infectivity and other epidemics and many other applications. Husband (1975) and Husband and Schofield (1976) also used the Pareto distribution for management salary structuring. Bhaskar and Dillard (1983) presented an objective method for assigning weights to questions on examinations using cognitive science and applied the Pareto distribution to assign the relative weights. Holman (1983) used Pareto distributions with different scale parameters to describe the survivorship curves for genera and families on their respective time scales. Fujimoto et al. (2001) applied the Pareto distribution to the tail part of packet transmission delay for streaming applications. Alsbih et al. (2011) used different Pareto distributions to describe indicators of the Internet traffic patterns with data from a German digital cable TV based Internet provider. Benavides et al. (2011, 2012) fitted the Pareto distribution to data related to personal social contact networks such as device-device proximity, duration, and location. Karimova et al. (2011) used a Pareto type distribution for the probability density of recurrence intervals for failures on satellites of various types as presented by the US National Geophysical Data Center. Karpischek et al. (2012) fitted the Pareto distribution to user requests in usage analysis of a mobile bargain finder application. Engler et al. (2019) fitted the Pareto distribution to the number of cattle on farm above a certain threshold in studies of factors affecting the farm size and stocking rate in

Namibian commercial cattle farming. Taketomi et al. (2022) reviewed the use of Pareto-I, Pareto-II and Pareto-IV distributions for survival and reliability analysis and illustrated them with a real dataset. Abdullah et al. (2023) presented the Pareto distribution in studies of quality of inbound and outbound internet application services on the Local Area Network campus Metro-E network.

The Pareto distribution has also been extensively used in the analysis of extreme events [Pickands (1975)] in the fields of hydrolody, climatology and other environmental studies (dealing, for example, with rainfall, water levels and sea surface or air temperatures), natural hazards such as (tsunamis, floods and earthquakes) and many more fields outside the scope of interest of statistical process control. Regarding this aspect, however, there are a lot of interesting applications of the Pareto distribution in the literature. For example, Pareto distribution has been used to describe service time in queuing system [Harris (1967,1968), Aalto and Ayesta (2007)] as well as interarrival times in queuing systems [Rodriguez-Dagnino (2004)] and has been a useful model for survival populations associated with business lifetimes [Nigm et al. (2003), Hong et al. (2007,2008,2009)], reliability studies, lifetime data analysis and life testing problems and experiments [e.g. Nigm and Hamdy (1987), Soliman (2000), Wu et al. (2007a), Amin (2008), Mahmoudi (2011)]. It has also been applied in computer science and communications to model among others error clustering in communication circuits and hard disk drive error rates [Nadarajah and Kotz (2008)], data traffic [Bae et al. (1999), Silva and Mateus (2002, 2003), Ghani (2011), Ghani and Iradat (2011)], memory traffic [Tudor and Teo (2013)], flow lengths [Addie and Yevdokimov (2008)], network delays [Jeske and Chakravartty (2006)], file sizes [Kang et al. (2008)], downloads and page views [Liu et al. (2013)], network packet inter-arrival time distribution [Garsva et al. (2014)] and inter-arrival times and occurrence of errors in data transmission over telephone circuits [Berger and Mandelbrot (1963), Sussman (1963), Richters (1965)]. Moreover, it has been used in population studies [Dimitrov et al. (1998)], process safety performance evaluation [Henselwood (2009)], mechanics, metallurgy and engineering [Zagorski and Wnek (2007), Castillo et al. (2004)] and studies of tensile strength [Reed and Jorgensen (2004)], ozone levels [Villasenor-Alva and Gonzalez-Estrada (2010), Eastoe and Tawn (2009)], high concentrations in short-range atmospheric dispersion [Mole et al. (1995)], industrial accidents [Maguire et al. (1952)], etc. Recently, Pareto distribution has also been used in quality control [Prasad et al. (2013)] and control charts for monitoring the distribution's parameters [Nasiru (2016), Aslam et al. (2016d), Baba and Maahi (2017), Baba and Luguterah (2018)]. More details on control charts for the Pareto distribution can be found in Section 2.18.3 herein.

Pareto-related distributions, as well as the non univariate case, however, are beyond the scope of this thesis. What follows is an attempt to present a brief review of the vast literature on the Pareto distribution. More specifically, the structure of this chapter is as follows: Section 5.2 presents useful information for the Pareto I distribution, which will be used for the construction of control charts for the distribution in Chapter 9. Section 5.3 provides a brief review of the literature on Pareto and Pareto-related distributions and their applications. Section 5.4 focuses on further investigation of the Pareto distribution in the relevant literature.

5.2 Useful Information for the Pareto Distribution

Pareto distribution is an asymmetric continuous power law probability distribution. The graphical representation of the distribution's probability density function for some values of the distribution's parameters is shown in Figure 5-1, where it is obvious that the Pareto distribution is positively skewed and its shape changes as the values of the process parameters change. The probability density function of the Pareto distribution is given by

$$f_X(x) = dr^d x^{-(d+1)}, \quad r, d > 0, x \geq r, \tag{5-1}$$

where r is the scale parameter (also called threshold parameter or cutoff value) and d is the shape parameter (also called tail index, Pareto index, or Pareto exponent. It is also called income inequality parameter in economics and finance.). This version of the Pareto distribution is more properly known as Pareto distribution of the first kind. Its cumulative distribution function is given by

$$F_X(x) = 1 - \left(\frac{r}{x}\right)^d, \quad r, d > 0, x \geq r \tag{5-2}$$

The moments of the Pareto distribution in (5-1) are computed using the following formulas:

$$E(X) = dr(d-1)^{-1}, \quad d > 1, \tag{5-3}$$

and

$$V(X) = dr^2(d-1)^{-2}(d-2)^{-1}, \quad d > 2. \tag{5-4}$$

The coefficient of skewness of the Pareto distribution is given by

$$\text{sk=E}\left[\left(\frac{\text{X}-\mu}{\sigma}\right)^3\right] = \frac{2(d+1)}{d-3}\sqrt{\frac{d-2}{d}}, \text{ d} > 3 \tag{5-5}$$

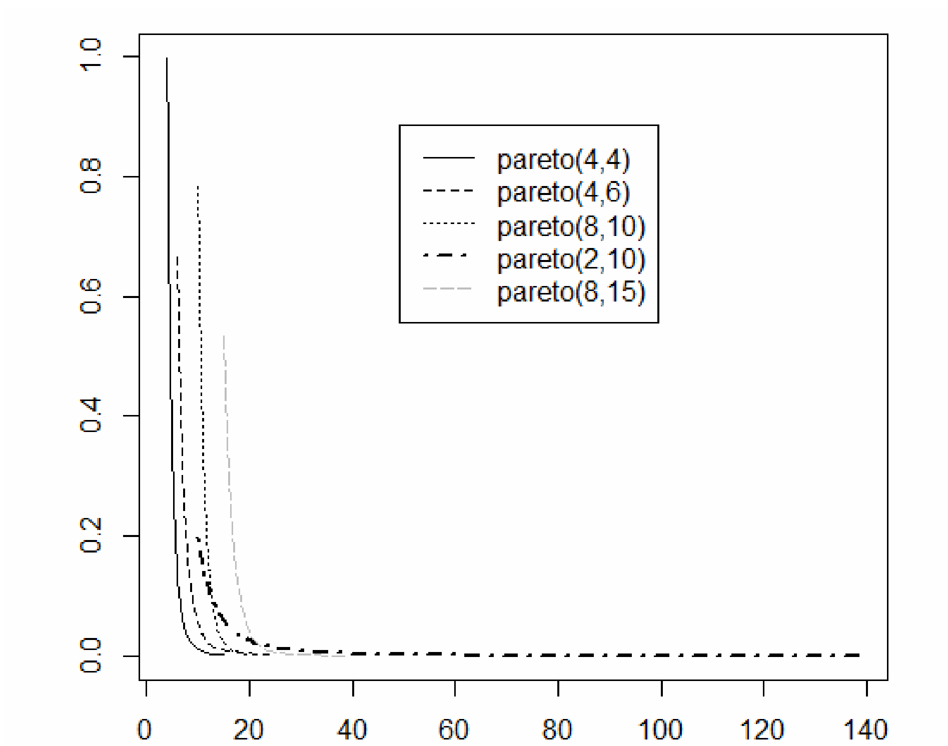

Figure 5 - 1: Probability density function of the Pareto distribution for various values of the parameters

5.3 Brief Overview of the Literature on the Pareto Distribution

Before we proceed, it should be noted that a lot of research has been done on the Pareto and Pareto-related distributions and there have been presented four types of the Pareto distribution. The first one is defined later in equation (5-2) and it is the type we will deal with in Chapter 9. The second type is also known as the Lomax distribution [Lomax (1954)] and is defined as

$$F(x) = 1 - \frac{C^a}{(x+C)^a}, \quad x \geq 0 .$$

It has been used for reliability modeling and life testing in engineering as well as in survival analysis and in the biological sciences, and has been applied to the sizes of computer files on servers. The Pareto distribution of the third kind [which was further studied by Bottazzi (2022)] has a cumulative distribution function given by

$$F(x) = 1 - \frac{Ce^{-bx}}{(x+C)^a}, \quad x > 0 .$$

The cumulative distribution function of the Pareto distribution of the fourth kind is defined by

$$F(x) = \left[1 + \left(\frac{x-\mu}{\sigma}\right)^{1/\gamma}\right]^{-\alpha}, \quad x > \mu, \quad \alpha, \gamma, \sigma > 0$$

Harris (1968) showed that the Pareto distribution can result from the mixture of an exponential distribution with the inverse of its parameter following a Gamma distribution and with origin at zero. Hürlimann (2003) studied the Pareto distribution as an exponential transform. Kopperer (2003) discussed the genesis of Pareto distributions, definitive Pareto-formulae, Pareto distributions' synthetic generation and a method for fine-fitting of Pareto curves and presented a visualization of the interconnections between Normal, Lognormal and Pareto distributions. Arnold (2014) studied univariate and multivariate Pareto distributions by representing them in terms of independent components following the Gamma distribution, while multivariate Pareto distribution was also discussed in Kotz et al. (2005).

Pareto distribution has received huge attention in the literature. Many researchers have dealt with goodness-of-fit tests [e.g. Marlin (1984), Porter et al. (1992), Rizzo (2009), Obradović (2015), Obradović et al. (2015), Allison et al. (2022) and Ndwandwe et al. (2023a,b)] and stress-strength reliability studies for it [including for example Dargahi-Noubary (1988), Nadarajah (2003), Nadarajah and Kotz (2003), Odat (2010), Gunasekera (2015), Juvairiyya and Anilkumar (2019) and Mahapatra et al. (2021)]. A lot of research has also been dedicated to Bayesian methods for the Pareto distribution. Some examples include Arnold and Press (1983, 1989), Soliman (2000, 2001), Mousa (2001), Ali Mousa (2003), Ahmadi and Doostparast (2006), Jeevanand and Abdul-Sathar (2006), Amin (2008), Balakrishnan and Shafay (2012), Mahajan et al. (2015), Renjini et al. (2016), Patel and Patel (2019), Shukla et al. (2020), Savita and Kumar (2022), Shafay (2022), Andrade and Rathie (2023) and many others.

A huge amount of literature has also been dedicated to various (Bayesian and non-Bayesian, as well as non-parametric) methods of estimation and prediction of parameters, quantiles and other related to the Pareto distribution quantities based on either complete or censored data. Examples of these include Quandt (1966), Moore and Harter (1967, 1969), Kulldorff and Vännman (1973), Ashour et al. (1994), Dunsmore and Amin (1998), Bickel (2003), Wu (2003,2010), Wu et al. (2004,2012), Ahmadi et al. (2009), Bhatti et al. (2018), Brazauskas and Upretee (2019) and Hussain et al. (2021). Examples of literature dealing with both classical and Bayesian methods for estimation and prediction include Raqab et al. (2007), Asgharzadeh et al. (2014), Prakash (2021), Hassan et al. (2023) and Sobhanan and Sathar (2023). Hossain and Zimmer (2000) compared estimation methods for the Pareto-I distribution's parameters for the case of censored data with two different type of censoring (type II censoring and multiple random censoring). Rahman and Pearson (2003) also compared various estimation methods for the two-parameter Pareto distribution.

A great deal of research has also addressed estimation of the tail index of the Pareto distribution, which represents the degree of fatness of the tail distribution and is an important component of extreme value theory since it dominates the asymptotic distribution of extreme values such as the sample maximum. Examples of this kind of research include Reiss (1987), Beirlant et al. (1996, 2006), Brazauskas and Serfling (2000,2001), Wagner and Marsh (2004), Gardes and Girard (2008), Ghosh (2017) and Ocran et al. (2022). Mora (2011) compared through simulation various methods of estimating the tail index of Pareto type distributions and applied them to Danish Fire data, while Fedotenkov (2021) reviewed Pareto tail index estimators, concentrating on univariate estimators for non-truncated data and presented their analytical expressions along with non-technical explanations of the methods. They also presented the estimators' strengths and weaknesses and compared lots of estimators through Monte Carlo simulation.

Beirlant et al. (2018) reviewed the available tail estimators of the extreme value index and introduced a bias reduced estimator for Pareto-type distributed censored data. They showed the usefulness of shrinkage estimation in keeping the MSE under control, developed a bootstrap algorithm for deriving confidence intervals, compared the proposed estimators with other estimators in the literature and illustrated the usefulness of the new estimators through a real long-tailed and heavy censored car insurance portfolio. Nicolau et al. (2023) discussed the estimation of the conditional tail index of Pareto and Pareto-type distributions in a time series framework and illustrated their study with the analysis of stock returns' tail risk dynamics.

Besides the vast amount of literature on the Pareto distribution itself and its applications (in all of its forms) a great deal of research has been done on its discretization, extensions, mixtures, modifications and generalizations and their applications. Fang et al. (2012), for example, discussed the double Pareto Lognormal distribution and presented an overview of complex networks and natural phenomena described by the double Pareto Lognormal distribution, such as the number of friends in social networks, the number of downloads on the Internet,

Internet file sizes, stockmarket returns, wealth in human societies, human settlement sizes, oil field reserves and areas burnt from forest wildfire. Pareto-related distributions have been used in studies of sea level, river discharge, precipitation, wind speed, wave height, temperature maxima and minima, avalanche activity, earthquake magnitude and seismic moment, wildfire sizes, floods, storms, magnitude and frequency of landslides following a rainstorm, tsunamis and other natural disasters, metals deposits, electricity demand, banking systems, carbon dioxide emissions, surface ozone and nitrogen dioxide concentrations, injuries or fatal accidents, blood pressure or cholesterol measurements, traffic, growth rates (such as annual gross domestic product, stock prices, foreign currency exchange rates and company sizes), city and firm sizes, oil and gas fields, article citations and number of publications, sports performance and records, financial and market risks, bank operational risk and radar background clutter information for object recognition. They have also been applied in climatology, hydrology and atmospheric science, meteorology, environmental sciences, limnology and oceanology, ecology, geology and geophysics, breaking strength and other fatigue life and reliability studies, survival and lifetime data, failures and service times, waiting times, demography, COVID-19 infectivity and other epidemics, medicine, genetics, health care, biology and bioinformatics, pharmaceutics and pharmacokinetics, economics and finance, insurance and actuarial sciences, telecommunications, computer science, network traffic, energy, physics and chemistry, food industry, astronomy and astrophysics, engineering, archaeology and many other fields.

A vast amount of literature has been dedicated to the generalized Pareto distribution, dealing with applications or with various methods of estimation of parameters, quantiles and other quantities [e.g. Davis and Feldstein (1979), Hosking et al. (1987), Singh and Guo (1995a,b), Fitzgerald (1996), Castillo and Hadi (1997), Salvadori (2003, 2021), Juárez et al. (2004), Madi and Raqab (2008), You et al. (2010), Zhang (2010), Guégan and Zhao (2014), He et al. (2014b), Askari et al. (2016), Chen et al. (2019), From and Ratnasingam (2022), Martín et al. (2022)], prediction [e.g. Rosbjerg et al. (1992), Raqab et al. (2018)] or reparameterization [e.g. Jonathan and Ewans (2010), Hunter et al. (2017)], reliability [e.g. Rezaei et al. (2010), Chacko and Mathew (2021)] and moments [e.g. Balakrishnan and Ahsanullah (1994), Mahmoud et al. (2005), Kim (2010), Kumar et al. (2023)] of the generalized Pareto distribution and several other related studies. Many researchers have also addressed the issue of choosing or estimating the appropriate threshold value for the generalized Pareto distribution [e.g. Tancredi et al. (2006), Coelho et al. (2008), Miranda (2014), Beirlant et al. (2022), Benito et al. (2023)] or focused on censored data from the generalized Pareto distribution [e.g. Lin and Wang (2000), Hu and Gui (2018), Pham et al. (2018), Sauer et al. (2020), Kumar et al. (2023)].

A review of quantile estimation methods for the generalized Pareto distribution was provided by Jocković (2012) along with their application in finance for estimating the value at risk. de Zea Bermudez and Kotz (2010a,b) reviewed the methods for estimating the parameters of the generalized Pareto distribution concentrating on the methods with simple and easy application in hydrological and other practical situations, as well as robust methods and Bayesian methods easily applied to real data. Kang and Song (2017) compared (through simulation) six estimation methods for the parameters and quantiles of the generalized Pareto distribution combined with the peaks-over-threshold method. Pels et al. (2020) compared the performances of twenty-one estimation methods for the generalized Pareto distribution with the peaks-over-threshold method. Gamet and Jalbert (2022) presented extensions of the generalized Pareto distribution with positive and finite density at the threshold and proved that these extensions produce better upper tail index estimates for low thresholds and they are also suitable for high thresholds because then they reduce to the generalized Pareto distribution.

The sum, product and ratio of two variables one of which follows a Pareto or Pareto-related distribution and the other one follows some other distribution (or sum of Pareto related distributions) have also been studied by various authors, such as Nadarajah and Kibria (2006), Nadarajah (2010) and Hamedani et al. (2022), for example. Many researchers have dealt with censored data from the

Pareto distribution, including but not limited to Bilikam and Moore (1978), Akritas (1988), Crato (2000), Fernández (2007, 2008), Shafay (2016) and Mahmoud et al. (2021).

Several reasearchers have also provided various characterizations of the Pareto distribution, such as for instance Samanta (1972), Fakhry (1996), Ahmad (2001), Wu and Lee (2001), Xekalaki and Dimaki (2005), Ahsanullah and Shakil (2012), Kumar and Singh (2018), Tzavelas (2019), Jin (2023) and many others. Characterizations for the generalized Pareto distribution were provided among many others by Falk (1990), Asadi and Ebrahimi (2000), Dimaki and Xekalaki (2006), Tavangar and Asadi (2012) and Kumar and Singh (2023).

Many researchers have compared Pareto and Lognormal distributions for various applications. Examples include Fisk (1961), Attanasi and Charpentier (2002) and Fazio and Modica (2015). Several studies have also considered order statistics from the Pareto distribution, such as the ones by Malik (1966,1967,1970), Kabe (1972), Kamps (1995), Kamps and Cramer (2001), Athar et al. (2008), Adler (2011), Ling and Fang (2019) and Abd Elgawad et al. (2021). Sampling plans for Pareto distributions were discussed among others by Aslam et al. (2011), Mughal and Ismail (2013), Sathya Narayanan and Rajarathinam (2013), Mughal et al. (2015a,b,c,d,2016), Aslam et al. (2019a), Zain and Aziz (2019) and Saranya et al. (2022).

Besides the univariate case, there is also a great amount of literature dealing with bivariate and multivariate Pareto distributions including but not limited to Hutchinson (1979), Arnold (1983,1990,2015), Jeevanand (1997), Nadarajah and Kotz (2005), Zografos and Nadarajah (2005), Navarro et al. (2007,2008), Tsoukalas and Agrafiotis (2013), Sankaran and Kundu (2014), Paul et al. (2018) and Michael and Dang (2022). Examples of literature on bivariate and multivariate generalized Pareto distribution include Falk and Reiss (2001), Rootzén and Tajvidi (2006), Salvadori and De Michele (2006), Aulbach et al. (2012a,b), Park et al. (2019) and Li and Tang (2022).

5.4 Further investigation of the Pareto Distribution

Hagstroem (1960) discussed the properties, convolutions and risk theory for the case of the Pareto distribution. Malik (1970) studied the distribution of product statistics from the Pareto distribution. Wallis et al. (1974) obtained the distribution functions for the mean, standard deviation and coefficient of skewness of the Pareto type I distribution for small samples using the Monte Carlo method. Thomas (1976) derived the reciprocal moments of a linear combination of exponential variates and used the resulting formula to obtain the moments of quantile and other similar estimators for the shape parameter of a Pareto distribution and proved that, although these estimators are more biased and less precise than the Monte Carlo estimates of the moments, they are "potentially useful in linear models and in studying models of the variation in the rate of births in a pure birth process". Thorin (1977) proved that the Pareto distribution belongs to a subclass of the class of infinitely divisible distributions by showing that it can be viewed as a generalized T-convolution. Goovaerts et al. (1977) presented a set of sufficient conditions that should be met for a distribution function to be a generalized T-convolution, generalizing the results for the Pareto distribution by Thorin (1977). Alvo (1978) addressed the sequential estimation of the parameter of a Uniform distribution using the Pareto distribution as a prior distribution for the parameter. Lorah and Stark (1978) used the Mellin transform with its convolution and exponentiation properties in order to derive the distribution of some functions of Pareto variables and provided expressions for products, quotients, and sums of products of Pareto variables including the distribution of the geometric mean and the product of minimum values of Pareto variables.

Goovaerts and de Pril (1980) and Seal (1980) studied survival probabilities based on Pareto claim distributions. Berg (1981) provided a new short proof of the result in Thorin (1977) that the Pareto distribution belongs to the class of generalized T-convolutions. Dyer (1981) obtained the structural distribution function of the strong Pareto law using the structural density function of the parameters of a Pareto distribution, computed its fractiles for special cases, presented the results through graphs from which structural one-sided probability bounds may be found and showed that these graphs may be used to find structural tolerance bounds for the Pareto distribution as well. Jasso (1982) studied a measure of inequality defined as the ratio of the geometric mean to the arithmetic mean for the Pareto distribution. Aggarwal and Singh (1984) presented exactly optimum boundaries for optimum stratification with proportional allocation for a class of Pareto distributions arising from the representation of the Lorenz curve in Wang and Aggarwal (1984). Dharmadhikari and Gupta (1984) presented the relationship between the Power Function distribution and the Pareto distribution. Wang and Aggarwal (1984) discussed optimum determination of strata boundaries for a positively skewed stratification variable following a Pareto type distribution and extended the method in order to include the case when stratification and estimation variables are different but related by a simple regression model. Engelhardt et al. (1986) addressed the Pareto-II distribution, expressed as a two-parameter mixed (or compound) Exponential failure distribution, estimated its parameters with MLE method, discussed small-sample means and variances, presented hypotheses tests for each parameter considering the other as an unknown nuisance parameter and constructed confidence limits. Ocana et al. (1986) discussed the ECOGEN simulation language algorithm and underlying theory for random deviate generators for the Pareto distribution. Teugels and Van Assche (1986) discussed the exact calculation of the decision boundaries for sequential probability ratio tests for simple hypotheses and alternatives in the case of a Pareto distribution. Ahsanullah and Houchens (1989) studied record values for Pareto distributions. Berrebi and Silber (1989) obtained a measure of the sharpness or kurtosis of the Pareto distribution from the Gini Index of Income Inequality by dividing the population into two subgroups of equal size.

Hwang and Hu (1990) presented exact expressions of the asymptotic expected deficiency of the maximum likelihood estimator relative to the uniformly minimum variance unbiased estimator for a given one-parameter estimable function for the case of the Pareto distribution. Ahsanullah (1991) investigated the distributional properties of record values for a sequence of i.i.d. random variables following the Lomax (Pareto II) distribution and derived moments up to the second order and estimators of the distribution's parameters based on a series of observed record values. Mahmoud and Maswadah (1992) discussed the stractural densities of the parameters of the two-parameter Pareto distribution based on complete and censored samples and the corresponding shortest confidence intervals of the parameters. Wagner and Geyer (1995) presented a maximum entropy method for inverting Laplace transforms of density functions of positive random variables following the Pareto distribution. Asmussen and Klüppelberg (1996) dealt with random walk or Lévy processes with heavy-tailed upwards jumps following the Pareto distribution. Balakirsky (1996) proved that the number of computations in the first incorrect path in the code tree of sequential decoding for discrete memoryless multiple-access channels follows a Pareto distribution with its parameter being estimated similarly to the parameter for systems of information transmission with one source. Chen (1996) presented a method for exact joint confidence region for the parameters of Pareto distribution, which can be used for both complete and type-II censored samples. Drees and Reiss (1996) considered the mean residual life function (MRLF) for the Pareto distribution and proved that the empirical MRLF is an innacurate estimator of the true MRLF of a Pareto distribution with its shape parameter being close to 1. As a result they studied alternatives such as the median and trimmed mean residual life functions and investigated their asymptotic properties for large age values. Adamidis and Loukas (1998) introduced a two-parameter lifetime distribution (the Exponential-Geometric distribution) with decreasing failure rate and presented its relationship with the Pareto II distribution. Jeevanand and Nair (1998) proposed a method for determining the number of outliers in Pareto samples, using the predictive interval approach.

Abate and Whitt (1999) used numerically inverted Laplace transforms for deriving the probability density function and cumulative density function of the Pareto distribution which they used to describe the distribution of service time in queues. Feuerverger and Hall (1999) developed two semiparametric methods for describing departures from a Pareto distribution when estimating a tail exponent by fitting the distribution to extreme observations. Those two methods were based on approximate likelihood and least squares with the latter being more robust to departures from usual extreme-value approximations but leading to estimators with greater variance. The proposed methods were proved to reduce bias compared to the assumption of an exact Pareto distribution beyond a threshold and were illustrated with application to extreme data regarding community sizes. Pawlas and Szynal (1999) provided recurrence relations for single and product moments of *k*-th record values from the Pareto distribution. Sengupta and Nanda (1999) dealt with the class of log-concave distributions and the subclass of concave distributions for reliability studies (because most common lifetime distributions, including the Pareto distribution, are log-concave while the remaining life of maintained and old units tend to have a concave distribution), investigated the properties of these two classes as well as their closure under various reliability operations and presented sharp reliability bounds for nonmaintained and maintained units having life distribution belonging to these classes.

Jurečková (2000) developed a test of the Pareto-type tail of the distribution of errors in the linear regression model, based on the extreme regression quantiles. Badía et al. (2001) derived the optimum inspection policy in terms of minimizing cost per unit of time for an infinite time interval when the time to failure follows the Pareto distribution. Manas (2001) discussed the function relating percentile ranks to density ordinates in continuous distributions, which also provides a likelihood based estimation method which asymptotically yields the frequency moment estimators and illustrated it with various distributions including the Pareto distribution. Gerchak and He (2002) dealt with the probability of a specific random variable taking the smallest value among a set of random variables for the case of the Pareto distribution. Hall et al. (2002) investigated the effect of extrapolation on coverage accuracy of prediction intervals computed from Pareto-type data and proved that, in a way which can be defined theoretically and confirmed numerically, it is possible to make predictions exponentially far into the future without serious errors. Marazzi (2002) presented bootstrap methods for testing equality of robust means in the one-, two-, and multi-sample problems for asymmetrically distributed data with unequal shapes and applied them to various distributions including the Pareto distribution. Abdel-All et al. (2003) studied the geometrical properties of the Pareto distribution, defined its parameter space using the Fisher's matrix and described the relationship between the differential geometry and the statistics for the Pareto distribution. Brazauskas (2003) provided the exact form of information matrix for Pareto-IV and related distributions. Landsman and Makov (2003) developed a sequential quasi-credibility formula for the scale dispersion family which includes the Pareto distribution. André (2005) addressed limit theorems for weighted sums of the ratios of randomly selected pairs of adjacent order statistics from the Pareto distribution with a prior distribution on choosing each of these possible pairs. Zaliapin et al. (2005) discussed five approximation methods for the sums of independent random variables with common Pareto distribution and focused on the median and the upper and lower quantiles of the distribution of the sums. The proposed methods were illustrated with application to the approximation of the observed cumulative seismic moment in California.

Balakrishnan and Stepanov (2006) presented the Fisher information contained in record values as well as in record values and record times for the Pareto distribution and the Fisher information in record statistics obtained from a new inverse sampling plan and introduced some new estimators based on records and weak records. Cuadras et al. (2006) expanded a Pareto distributed random variable as a series of principal components, conducted a comparison with the exponential distribution and presented an inequality regarding a function and its derivative and the asymptotic distribution of some statistics related to Rao's quadratic entropy. Gay (2006) discussed tail-ratios of the Pareto distribution with application to insurance, proving that the consecutive ratios of the largest Pareto claims are independent and that the minimum-variance unbiased maximum likelihood estimator for the Pareto tail-index is equivalent to Hill's estimator. The analysis was illustrated with both simulated and real data. Kaiser and Brazauskas (2006) investigated the performance of interval estimators of various actuarial risk measures and constructed confidence intervals for them with various methods (MLE, trimmed means-based estimation and empirical and bootstrap nonparametric methods). The average lengths and coverage proportions of the intervals were compared through Monte Carlo simulation for both clean and contaminated data. For the case of clean data several distributions were used, including the Pareto distribution, while for the contaminated data case, the clean Pareto-distributed data were mixed with a small fraction of outliers. The intervals resulting from a sufficiently robust estimator designed for the specific distribution were proved to have satisfactory performance under both data conditions. Singh (2006) constructed simultaneous confidence intervals for the successive ratios of scale parameters of Pareto distributions when assuming that scale parameters satisfy a simple ordering (as is the case, for example, when the populations are the outcome of successive runs of a production process).

Huang et al. (2007) introduced a randomized quasi-Monte Carlo method for estimating the mean and variance of the Pareto distribution. They developed a randomized quasi-random number generator of random samples from the Pareto distribution, such that the sample mean and sample variance estimators become more efficient. The generator's efficiency was investigated through simulation and compared with a usually used generator in terms of mean square errors. The study also presented comparison of the results of the Kolmogorov-Smirnov goodness-of-fit tests using these two sample generators. Jones (2007) studied a class of distributions, which includes the Pareto distribution as a special case, with its members' density function and distribution function defined by a specific relationship. The study presented the family's symmetry, modality, tail behaviour, order statistics, shape properties based on the mode, L-moments and transformations between members of the family. Ladoucette (2007) investigated the asymptotic behaviour of the moments of the ratio of the random sum of squares to the square of the random sum for a sequence of independent and identically distributed positive random variables of Pareto-type.

Agarwal and Pant (2008) obtained the expectations of the trimmed mean and the winsorised mean for the Pareto distribution and L-moments which are expectations of linear combinations of order statistics. Klüppelberg and Resnick (2008) presented a transformation of a multivariate distribution leading to the

Pareto distribution for the marginals and discussed the use of the resulting distribution (which they called the Pareto copula). Nadarajah and Ali (2008) presented the distribution of the sum, product and ratio of two independent Pareto distributed random variables useful for hydrological problems and applied them to extreme rainfall data from Florida. Ramsay (2008) presented the distribution of sums of i.i.d. Pareto distributed random variables with arbitrary shape parameter. Sarabia and Sarabia (2008) presented the Leimkuhler curve of the classical Pareto and Lomax distributions.

Asmussen (2009) addressed importance sampling for failure probabilities in computing and data transmission with a Pareto distributed conditional limit of the ideal time that a job needs to be restarted after a failure given that the total time of this job exceeds a specific value. Balakrishnan et al. (2009a) discussed the issue of reconstructing past records from the known values of future records when the underlying distribution is the Pareto distribution deriving and comparing several reconstructors and illustrated the proposed method with application to a real data set of the record values of average July temperatures in Neuenburg, Switzerland. Benguigui and Blumenfeld-Lieberthal (2009) developed and studied a framework for classifying income distributions with the help of a positive index, a special value of which corresponds to Pareto distribution. Kim and Lee (2009) dealt with testing for a change in the tail index of stationary time series data with Pareto-type marginal distribution.

Alfons et al. (2010) compared, through simulation, different robust methods for Pareto tail modelling in order to reduce the influence of outliers in the upper tail of the income distribution in the case of Laeken indicators. Balakrishnan et al. (2010) provided a relation between the Leimkuhler curve and the mean residual life for the Pareto distribution as well as relationships with other reliability concepts. Das et al. (2010) proposed a Pareto regression model with an unknown shape parameter for studying extreme drinking in patients with alcohol dependence using a generalized linear model framework and the log-link to incorporate the covariate information through the scale parameter of the generalized Pareto distribution. They also used a Bayesian method with Ridge prior and Zellner's g-prior for the regression coefficients and proved its superiority over likelihood-based inference through simulation. Dierckx and Teugels (2010) noted that the limit distribution of the absolute excesses of the data over a high threshold is a generalized Pareto distribution and that the relative excesses of the data over a high threshold in case of a positive extreme value index can be described in the limit by a Pareto distribution with this index as parameter. Therefore, in order to deal with change-point detection of extreme values, they focused on testing changes in the value of the extreme value index and/or the scale parameter of the distribution using the likelihood method for independent data. They investigated the asymptotic properties of the proposed test statistics, provided critical values and illustrated their analysis with application to both simulated and real data. Grandits et al. (2010) addressed the compound-Poisson distribution with Pareto-type claims in the case of non i.i.d. claims with the scale and location parameters of the Pareto distribution following a specific trend and studied the effect of this trend (and its misspecification or neglect) on parameter estimation and on the value-at-risk. Jørgensen et al. (2010) proposed a class of extreme generalized linear regression models for analysis of extremes and lifetime data and noted that the set of quadratic and power slope functions characterize distributions such as the Pareto distribution. Therefore, they proved a convergence theorem for slope functions, which is useful for expressing the classical extreme value convergence results in terms of asymptotics for extreme dispersion models. Riabi et al. (2010) presented the β-entropy for Pareto-type and related distributions and some weighted versions of those distributions, order statistics, proportional hazards, proportional reversed hazards, probability weighted moments, upper record and lower record. Stehlík et al. (2010) discussed the exact distribution of the likelihood ratio tests of homogeneity and simple hypothesis on the tail index of a two-parameter Pareto distribution.

Bansal et al. (2011) developed a multi-sample test for Gini indices against simple-ordered alternatives and presented the exact critical points (obtained through simulation) for the case of the Pareto distribution. They also constructed simultaneous one-sided confidence intervals and computed the power of the test.

Benbya and McKelvey (2011) developed Pareto rank/frequency distributions as well as methods for using them at various points on Pareto distributions for obtaining practical knowledge about managerial problems. Blanchet and Shi (2011) discussed the cross entropy method for rare event simulation which requires the selection of a suitable parametric family for the successful application of the method and suggested two properties necessary for such a selection. They presented parametric families for which the proposed properties are satisfied for a large class of heavy-tailed distributions including Pareto and proved the proposed estimators' efficiency. Corbellini and Crosato (2011) discussed a stepwise fitting of the Pareto-II distribution based on the forward search method. According to their method, the observations added at each iteration are decided taking into account the results of the estimation at the previous step (instead of their rank, as is the case with the sequential fitting). Cramer and Bagh (2011) developed minimum and maximum entropy plans for the Pareto distribution using expressions for the entropy and the Kullback-Leibler information for distributions of progressively Type-II censored order statistics. Gerrard and Tsanakas (2011) discussed the computation of failure probabilities in risk analysis for loss distributions such as the Pareto distribution in the presence of parameter uncertainty and obtaining an exact measure of the effect of that parameter uncertainty on failure probability.

Barranco-Chamorro and Jiménez-Gamero (2012) presented asymptotic confidence intervals for quartiles for several Pareto distributions and proved their superiority over asymptotic intervals based on sample quartiles in terms of smaller length with similar coverage probability. Shahi (2012) addressed hypothesis testing for the scale parameter of the Pareto distribution by constructing the test statistics based on ranked set sampling and extreme ranked set sampling and compared their powers with the power of the uniformly most powerful test revealing the superiority of the test based on the extreme ranked set sampling.

Gagolewski (2013) addressed a hypothesis test for the equality of probability distributions based on the difference between Hirsch's h-indices of two i.i.d. random samples of equal length and investigated its performance with application to data from the Pareto distribution. Gunasekera (2013) discussed hypothesis testing and interval estimation of the availability of a series system with several renewable components with Pareto-distributed failure and repair times. Hubert et al. (2013) noted that estimators of the extreme value index of Pareto-type distributions (like the Hill estimator) tend to overestimate it in the presence of outliers. Therefore, they constructed the empirical influence function plot which presents the effect of each datapoint on the Hill estimator, basing the empirical influence function on a new robust GLM estimator (for the extreme value index) which was used to obtain high quantiles of the distribution and marking datapoints exceeding those high quantiles as unusually large. Kostal et al. (2013) introduced the Shannon entropy-based and Fisher information-based dispersion measures for the case of the Pareto distribution, investigated the relationships between them and discussed their properties and applications. Kuş et al. (2013) discussed the optimal decision of the number of test units, the number of inspections and the length of inspection interval under the restriction of prespecified limited budget such that the asymptotic variance of the maximum likelihood estimator of the Pareto parameter is minimum when the life test is progressively group censored. Luo (2013) addressed the issues of parameter estimation for the Pareto distribution with partially missing data, testing equality of two Pareto populations and presenting its limit. Zhang (2013) simplified joint confidence regions for the parameters of the Pareto distribution proposed by Chen (1996) and Wu (2008).

Balbás et al. (2014) developed a method for obtaining coherent risk measures for risks with infinite expectation, such as those characterized by some Pareto distributions, presented extensions of the conditional value at risk and the weighted conditional value at risk and illustrated the proposed method with actuarial applications such as extensions of the expected value premium principle when expected losses are unbounded. Barakat et al. (2014) provided general recurrence relations between the single and product moments for the upper and lower current records based on Pareto and negative Pareto distributions, respectively, as well as asymptotic results for general current records. Saeidi et al. (2014) dealt with hypotheses testing with fuzzy concepts based on records from the Pareto distribution for applications related to weather, sports, economics and life testing and illustrated the analysis with real annual wage data. Tudor (2014) discussed chaos expansion and asymptotic behavior of the Pareto distribution.

Barik (2015) dealt with a linearly constrained probabilistic fuzzy goal programming problem with the right hand side parameters in some constraints following the Pareto distribution with known mean and variance. Gagolewski (2015) constructed Sugeno integral-based confidence intervals for the theoretical h-index of a sequence of i.i.d. random variables following the Pareto distribution and compared them with the ones based on other estimators. Kämpke and Radermacher (2015) discussed the one-parametric version of the Pareto distribution which results as a unique solution of a differential equation for Lorenz curves and the Pareto distribution derived from an iterative process considering every Lorenz curve as a distribution function. They also provided the parameter values of the best fit Pareto distributions for empirical income data and proved that the Pareto distribution is the unique distribution to result from a certain proportionality law and from self-similarity of Lorenz curves. Nakagawa (2015) presented a sufficient condition for a non-negative random variable to follow a Pareto type distribution by investigating the Laplace-Stieltjes transform of the cumulative distribution function. Nguyen and Robert (2015) presented infinite series expansions for convolutions of Pareto distributions with non-integer tail indices, where the Pareto distributions may have different tail indices and different scale parameters. Their series expansion was not asymptotic and, therefore, was used for the computation of quantiles of the distribution of the sum as well as other risk measures such as the tail value at risk.

Beirlant et al. (2016) presented bias reduced estimators for the tail index and tail probabilities of Pareto-type distributions based on randomly right censored data. Jasiulewicz and Kordecki (2016) presented the multiplicative parameters and their properties for distribution with financial and insurance applications, among which the Pareto distribution, and applied them to the modelling of large losses. They illustrated their analysis with application to data from the Warsaw Stock Exchange and data from a bid of treasury bills in Poland. Kamalov and Leung

(2016) discussed the receiver operating characteristic curve graphical tool for analyzing the performance of a binary classifier in the case of Pareto distribution. They also computed the corresponding area under the receiver operating characteristic curve (which is a scalar measure of the classifier's performance) and investigated the optimal threshold for the classifier performance. López-Blázquez and Salamanca-Miño (2016) discussed the distribution of the geometric records of a sequence of i.i.d. observations from a Pareto distribution. Nechval et al. (2016) dealt with lower and upper tolerance limits on order statistics in future samples from the Pareto distribution, useful for describing time to failure in reliability studies. Their method can be applied to cases of having either complete or type-II censored past data.

Ahmadi and Wu (2017) introduced a unified cost structure for joint optimization of inspection frequency and replacement time for parallel systems in reliability engineering with the lifetime of a component following the Pareto distribution. Baker (2017) introduced a method for blunting cusped distributions and applied it to the double-sided asymmetric Pareto distribution. The method was illustrated with an example of fitting the resulting blunted asymmetric Pareto distribution to real data. Banik and Chaudhry (2017) dealt with queue length distributions and performance measures (such as probability of loss for the first, an arbitrary, and the last customer of a batch, mean queue lengths, and mean waiting times) for queuing systems with Pareto service time distribution. Nadarajah et al. (2017) presented conditions for stochastic, hazard rate, likelihood ratio, reversed hazard rate, increasing convex and mean residual life orderings of Pareto distributions with different shape and scale parameters. Shafiei et al. (2017) dealt with interval estimation and hypotheses testing for the generalized Lorenz curve under the Pareto distribution and illustrated the analysis with application to real data representing the median income of the 20 occupations in the United States Census of Population. Shafiq (2017) dealt with classical and Bayesian inference for the Pareto distribution for fuzzy observations of life time. Wu and Lu (2017) used MLE for the lifetime performance index under progressive type I interval censoring for the one-parameter Pareto distribution and investigated the estimator's asymptotic distribution. They also used the estimator to introduce a hypothesis testing algorithmic method (under the assumption of known lower specification limit) which they illustrated with two real data applications for deciding whether the process is capable.

Al-Mosawi and Khan (2018) dealt with the case of independent random samples from populations described by Pareto distributions with the same known shape parameter but different scale parameters and the selection of one of those populations related to the largest value among a set consisting of the smallest observation of each of those samples. The moments of the selected population were estimated under asymmetric scale invariant loss function and risk-unbiasedness and consistency of the estimators for those moments were investigated and their risk and risk-bias were computed. Balkema and Embrechts (2018) compared the performance of several estimators of the regression line in the simple linear regression when the explanatory variable has a Pareto distribution and the error has a symmetric Student distribution or a one-sided Pareto distribution through simulation for various tail indices. Grahovac (2018) presented distributions of different ruin-related quantities and their tail behaviour for Pareto-distributed claim sizes using the Cramér-Lundberg risk model. The study also included investigation of the effect of the Pareto distribution tail index on the tails of the distribution of the ruin-related quantities. Kamlşllk et al. (2018) considered a class generated by intersection of two important subclasses of heavy-tailed distributions (the long-tailed distributions and dominated varying distributions) trying to obtain some results on renewal functions generated by this class. Their main focus was on the Pareto distribution which is a special case of its subclass of heavy-tailed distributions. They derived asymptotic results for the renewal function generated by the Pareto distribution from this class and applied them to renewal reward processes. They illustrated the analysis with an application to an inventory model with demands following the Pareto distribution from this class. Mohd Safari et al. (2018) investigated the presence of outliers in the upper tail of Malaysian income distribution under the assumption that the data follow the Pareto distribution using the generalized boxplot which was chosen after comparing (through simulation) the performances of three types of boxplots. Vernic (2018) studied risk measures and capital allocation for the Pareto distribution depending on parameters with interval or fuzzy uncertainty.

Fader et al. (2019) discussed the differences, similarities and equivalence of the Beta-Geometric and Pareto-II distributions. Jabbari Nooghabi (2019) proposed two statistics for detecting outliers in the Pareto distribution. The power of the proposed statistics was compared with the power of other statistics for outliers detection for the Pareto distribution. The performance of the test was illustrated through application to different insurance claims. Jordanova and Stehlík (2019) discussed logarithms of ratios of two order statistics of a sample of independent observations from Pareto distribution with regularly varying tails and transformed the function so as to derive unbiased, asymptotically efficient, and asymptotically normal estimator for the tail parameter of the Pareto distribution. The proposed estimator was proved, through simulation, to be superior to several other estimators. Sarabia et al. (2019) further investigated the new Pareto-type distribution proposed by Bourguignon et al. (2016) and illustrated their analysis with applications to real income data. Baratnia and Doostparast (2020) compared Pareto distributions with a one-way classification analysis with random effects, provided exact expressions for several characteristics of the Pareto response variable such as marginal distribution and hazard functions, mean, variance and intraclass correlation coefficient, as well as estimations of the proposed model parameters and predictions with the minimum mean square error loss and introduced a method for testing homogeneity of the distributions. Buitendag et al. (2020) discussed confidence intervals for extreme quantiles of Pareto-type distributions and investigated their small-sample properties and usefulness through simulation and real insurance data application. Gouet et al. (2020) discussed δ-records in the linear drift model (defined as observations which are greater than all previous observations, plus a fixed real quantity δ) and illustrated them with application to specific distributions, including the Pareto distribution, and with real data regarding summer temperatures in Spain. Jiang et al. (2020) determined the restricted minimum volume confidence region for the parameters of the Pareto distribution, for both complete and (left, right or doubly) censored data. Jordanova and Stehlík (2020) discussed estimators of the index of regular variation for the Pareto distribution based on central order statistics and presented the conditions which insure unbiasedness, consistency and asymptotic Normality for these estimators. Urzúa (2020) developed a test for Pareto behaviour proving that it is locally optimal if the possible alternative distributions are members of the Pareto IV family and applied it to data on the frequency of unique words in an English text (Moby Dick), the human populations of U.S. cities, the frequency of U.S. family names and the peak gamma-ray intensity of solar flares, proving existence of Pareto behaviour evidence only for the second and fourth dataset.

Eugene et al. (2021) proposed the Gini Shortfall as a risk measure, studied its advantages compared to other risk measures, presented exact formulas for its computation in the case of the Pareto distribution and applied it to real stock data. Lala Bouali et al. (2021) introduced a robust estimator of conditional tail expectation of Pareto-type distribution using the extreme value index estimator. Yoshida (2021) studied an additive model for extremal quantile regression for estimating conditional quantiles in the tail of Pareto-type distributions and investigated the properties of the intermediate-order and extreme-order quantile estimators by combining the asymptotic and extreme value theories. The estimators' perfromance was investigated through simulation and illustrated with real data application.

Bakoban and Aldahlan (2022) used the Pareto distribution as a non-informative prior for Bayesian estimation of the shape parameter of the generalized inverted exponential distribution with quadratic loss function in the case of complete samples. Blanchet et al. (2022) used extreme value theory to obtain optimal thresholds for the cases of the utility distribution being Pareto and correlated Pareto distribution, showing that when the right tails of the utility distribution become heavier, the threshold level becomes higher. Hassan et al. (2022) estimated the extropy (considered to be a complementary dual of the Shannon's entropy) and the cumulative residual extropy of the Pareto distribution using MLE (in the presence of outliers) and Bayesian estimation (based on symmetric and asymmetric loss functions) methods, using MCMC for complex computations. They investigated and compared the estimators' precision through simulation and real data. Josaphat et al. (2022) proposed a copula-based conditional tail moment of target loss related to another loss, called Dependent Tail Value-at-Risk, for the case of the new Pareto-type distribution. They also introduced the Dependent Conditional Tail Variance risk measure, as a special case of copula-based conditional tail central moment of target loss related with another loss, for measuring the variance of the tail of loss distributions and illustrated their analysis through real data application.

5.5 Conclusion

The Pareto distribution, as was made obvious in this chapter, has received huge attention in research and has many applications in various fields. A lot of extensions, mixtures, modifications and generalizations have been proposed and investigated and this chapter presented only a very brief overview of them. A special subsection offered some useful information for the distribution which is going to be useful for the construction of control charts for the Pareto I distribution in Chapter 9.

PART 2

<u>Introduction to Part 2</u>

This part of the thesis contains new contributions to the existing literature on control charts for non-Normal distributions which were presented in Section 2.29 earlier. As it is clear after studying that section, there are still some distributions for which control charts have not yet been constructed at all (e.g. Logarithmic and Lindley-related distributions) or have not been addressed in the proper extend (e.g. Pareto-related distributions). This was the motivation for this thesis and that gap is going to be filled herein.

As pointed out in section 2.12.4 there are many situations in which samples from a process consist of just one observation, such as cases of automated inspection of all manufactured products or multiple measurements on the same unit of a product, cases when the production rate is low or the data comes available relatively slowly (e.g. accounting data), situations where successive observations differ only due to measurement error or errors during the analysis (e.g. chemical processes) and circumstances when quality testing leads to the destruction of the product or the cost measurement is high. In all those instances, control charts for individual observations are really useful. Therefore, in this study, the interest lays on individual observations from the distributions mentioned above [Lindley-related (one-parameter and two-parameter Lindley), Logarithmic and Pareto distributions].

To begin with, the construction of the individual control charts is going to be done in two ways. First, the control limits of the chart will be derived in terms of the probability of type I error or false alarm rate, α, using the distribution of interest (see for example, Chang and Gan (1999) for the case of the modified geometric distribution). Another way of constructing the individual control chart for each of the desired distributions will be based on the Shewhart-type individual control charts using the skewness correction method in Chan and Cui (2003), since the distributions of concern are asymmetric and this method, as also mentioned in Chan and Cui (2003), enhances the performance of the control chart and is better than other methods for considering the distribution's skewness when constructing control charts for asymmetric distributions in terms of Type I risk. The performance of the proposed control charts is investigated and illustrated with both simulated and real datasets. As it will be proved below, the performance of the Shewhart-type control chart with the skewness correction is better than the probability-type control chart, for all the distributions considered. Then EWMA control charts for individual observations from the distributions of concern are constructed and their performance is investigated and illustrated with the same simulated and real data as the previous control charts for the shake of comparison.

Afterwards, Shewhart-type and EWMA control charts for monitoring individual observations from the distributions of concern are improved by using another method for taking into account each distribution's skewness, namely the scaled weighted variance method proposed by Castagliola (2000). Performance investigation and illustration of the proposed control charts through application to the same simulated and real data as the rest of the charts reveals the superiority of using this method for the construction of the charts. Last but not least, suggestions for future research regarding the control charts considered in this part are also provided.

More specifically this second part of the current thesis is organized as follows: Chapter 6 deals with all the aforementioned charts for the case of individual observations from the original one-parameter Lindley distribution. Chapter 7 discusses the corresponding control charts for monitoring individual observations from the two-parameter Lindley distribution which was proposed by Shanker et al. (2013) as an extension to the one-parameter Lindley distribution. Chapter 8 addresses the control charts for individual observations from the Logarithmic distribution, while Chapter 9 covers the case of the Pareto I distribution. Part 2 is completed with chapter 10, which offers conclusions and suggestions for further research.

CHAPTER 6

CONTROL CHARTS FOR INDIVIDUAL OBSERVATIONS FROM THE ONE-PARAMETER LINDLEY DISTRIBUTION

6.1 Introduction

As mentioned in Chapter 3, Lindley distribution is a continuous distribution with various applications some of which are in medicine, genetics, epidemiology, biology, finance and actuarial sciences, ecology, meteorology, sociology, demography, agriculture, hydrology, geosciences, reliability and engineering, life testing and survival analysis, airborne systems and communications, environmental studies and modeling and describing of human mistakes, strikes, accidents, behavioural and emotional or IQ test scores and waiting times of customers in queues until service etc. Due to its variety of applications, it appears to be important that control charts for detecting shifts in a process should be constructed under the assumption that the quality characteristic of interest follows the Lindley distribution.

Here we construct probability-type, as well as Shewhart-type and EWMA control charts (and deal with the optimal choice of its parameters) for individual observations from the one-parameter Lindley distribution, considering two different types of skewness correction for taking into account the distribution's skewness in the construction of the Shewhart-type and EWMA charts. The performance of all the proposed control charts is investigated and illustrated using examples with both simulated and real data (same for all the charts for the shake of comparison). The whole analysis reveals the superiority of using skewness correction for the construction of the control charts against not using it, as well as the superiority of the scaled weighted variance method for taking into consideration the distribution's skewness.

The structure of the present chapter is the following: Section 6.2 presents the construction of probability-type control charts for individual observations from the one-parameter Lindley distribution, while section 6.3 deals with the construction of the corresponding Shewhart-type control charts using the skewness correction method proposed by Chan and Cui (2003). The investigation and comparison of the performances of the proposed control charts of the previous two sections is addressed in section 6.4. Section 6.5 describes the construction of EWMA control charts for one-parameter Lindley-distributed individual observations using the skewness correction, followed by Section 6.6 which discusses the performance investigation for these charts including comparison with the corresponding EWMA control charts without skewness correction. Section 6.7 addresses the optimal design of the control charts proposed in section 6.5. The three types of control charts presented so far (probability-type, Shewhart-type and EWMA charts) for individual observations from the one-parameter Lindley distribution are illustrated with both simulated and real data in section 6.8. Section 6.9 presents the use of another skewness correction method for the construction of the Shewhart-type and EWMA charts for individual one-parameter Lindley observations. This section uses the scaled weighted variance method proposed by Castagliola (2000) and presents the construction (subsections 6.9.1 and 6.9.3) and performance investigation (subsections 6.9.2 and 6.9.4) of the two proposed control charts with this skewness correction method and compares them with those based on the skewness correction method by Chan and Cui (2003) presented in the previous sections. Examples are also presented for illustration of the proposed charts based on the same simulated (subsection 6.9.5) and real data (subsection 6.9.6) as in subsections 6.8.1 and 6.8.2 for comparison purposes. Last but not least, section 6.10 presents conclusions and further research recommendations regarding the control charts discussed in this chapter.

6.2 Probability-Type Control Charts for Individual Observations from the One-Parameter Lindley Distribution

The control limits of the one-parameter Lindley individual probability-type control chart will be derived in terms of the probability of type I error or false alarm rate, α, using our distribution of interest (see for example, Chang and Gan (1999) for the case of the modified geometric distribution). For this procedure we will need the quantile function of the one-parameter Lindley distribution, which is derived in the following subsection.

6.2.1 The Quantile Function of the One-Parameter Lindley Distribution

For the case of using the probability of type I error to obtain the control charts for the one-parameter Lindley distribution we need the distribution's quantile function. Applying the methodology in Theorem 1 of Jodrá's (2010) paper, we can find a formula for the required quantile function in terms of the Lambert's W function [Corless et al. (1996)] as presented here.

The quantile function in general, is given by $Q_X(u) = F_X^{-1}(u)$, with u such as 0<u<1. For the case of the one-parameter Lindley distribution under study, we have:

$$F_X(x) = u \Rightarrow u = 1 - \frac{\theta+1+\theta x}{\theta+1} e^{-\theta x} \Rightarrow$$

$$\Rightarrow -(\theta+1+\theta x) e^{-(\theta+1)-\theta x} = -(1-u)(\theta+1) e^{-(\theta+1)} \Rightarrow$$

$$\Rightarrow W\left(-(1-u)(\theta+1)e^{-(\theta+1)}\right) = -(\theta+1+\theta x) \Rightarrow$$

$$x = -\frac{\theta+1}{\theta} - \frac{1}{\theta} W_{-1}\left(-(1-u)(\theta+1)e^{-(\theta+1)}\right) \tag{6-1}$$

It should be noted that we use the negative brunch of the Lambert's W function in the formula above, considering its properties as presented in Section 2 of Jodrá's (2010) paper.

6.2.2 Control Limits of the Probability-Type One-Parameter Lindley Individual Control Charts

This subsection is dedicated in finding the control limits of the chart in terms of the probability of type I error or false alarm rate, α. In order to do that we need to use the cumulative probability of the one-parameter Lindley distribution as presented in equation (3-7). The construction procedure is as follows.

For a significance level α, we have

$$P(X < LCL) = \frac{\alpha}{2}$$

and

$$P(X < LCL) = 1 - \frac{\theta + 1 + \theta \cdot LCL}{\theta + 1} e^{-\theta \cdot LCL}, \quad LCL > 0, \quad \theta > 0,$$

from which using equation (6-1) we obtain

$$1 - \frac{\theta + 1 + \theta \cdot LCL}{\theta + 1} e^{-\theta \cdot LCL} = \frac{\alpha}{2} \Rightarrow LCL = -\frac{\theta + 1}{\theta} - \frac{1}{\theta} W_{-1}\left(-\left(1 - \frac{\alpha}{2}\right)(\theta + 1) e^{-(\theta+1)}\right),$$

where $W_{-1}(x)$ is the negative branch of the Lambert W function.

Similarly, for the upper control limit, we have

$$P(X > UCL) = \frac{\alpha}{2}$$

and

$$P(X > UCL) = 1 - P(X \le UCL) = \frac{\theta + 1 + \theta UCL}{\theta + 1} e^{-\theta UCL}, \quad \theta > 0,$$

from which, using equation (6-1) once again, we get that

$$\frac{\theta + 1 + \theta \cdot UCL}{\theta + 1} e^{-\theta \cdot UCL} = \frac{\alpha}{2} \Rightarrow UCL = -\frac{\theta + 1}{\theta} - \frac{1}{\theta} W_{-1}\left(-\frac{\alpha}{2}(\theta + 1) e^{-(\theta+1)}\right)$$

Similarly for the central line we obtain

$$CL = -\frac{\theta + 1}{\theta} - \frac{1}{\theta} W_{-1}\left(-0.5(\theta + 1) e^{-(\theta+1)}\right)$$

As a result from all the above, the control limits of the chart in terms of the probability of type I error, α, are as follows.

$$UCL_\alpha = -\frac{\theta+1}{\theta} - \frac{1}{\theta} W_{-1}\left(-\frac{\alpha}{2}(\theta+1)e^{-(\theta+1)} \right)$$

$$CL_\alpha = -\frac{\theta+1}{\theta} - \frac{1}{\theta} W_{-1}\left(-0.5(\theta+1)e^{-(\theta+1)} \right) \quad , \quad \theta > 0 \qquad (6\text{-}2)$$

$$LCL_\alpha = -\frac{\theta+1}{\theta} - \frac{1}{\theta} W_{-1}\left(-\left(1-\frac{\alpha}{2}\right)(\theta+1)e^{-(\theta+1)} \right)$$

6.3 Shewhart-Type Control Charts for Individual Observations Coming from the One-Parameter Lindley Distribution

In this subsection, the construction of the individual one-parameter Lindley control charts is going to be done based on the Shewhart-type individual control charts using the skewness correction as in Chan and Cui (2003). More specifically, following equation (2-1), the construction procedure according to this method is as follows: the central line is placed at the mean of the one-parameter Lindley distribution, which is computed using equation (3-3), while the control limits are placed around the mean at L times its standard deviation (the square root of the quantity computed by equation (3-4)) plus c_4^* times its standard deviation, where $c_4^*(x) = \frac{\frac{4}{3}\left[sk(x)\right]}{1+0.2\left[sk(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient computed from equation (3-5). This means that the skewness correction for the one-parameter Lindley distribution will be

$$c_4^*(x) = \frac{8\left[2(\theta+1)^3 - \theta^3\right]\left(\theta^2+4\theta+2\right)^{3/2}}{3(\theta+4\theta+2)^3 + 0.24\left[2(\theta+1)^3 - \theta^3\right]^2} \qquad (6\text{-}3)$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the one-parameter Lindley control chart are as follows.

$$UCL = \frac{\theta+2}{\theta(\theta+1)} + \left[L + c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}$$

$$CL = \frac{\theta+2}{\theta(\theta+1)} \quad (6\text{-}4)$$

$$LCL = \frac{\theta+2}{\theta(\theta+1)} + \left[-L + c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}$$

6.4 Performance Investigation for the Individual One-Parameter Lindley Control Charts

As a performance measure of the charts we constructed above, we can use either the out-of-control average run length (ARL) value, which is noted by ARL_1, or the in-control ARL, which is noted by ARL_0. ARL_1 is defined as the average number of observations needed in order to detect an out-of-control situation given that the process of concern is indeed in an out-of-control state (presence of an assignable cause), while ARL_0 is defined as the average number of observations needed in order to have an indication of an out-of-control situation given that the process of concern is actually in an in-control state (case of false alarms). This means that we prefer a control chart with a large value of ARL_0 and a small value of ARL_1.

Using the aforementioned definitions for the computation of the in-control and out-of-control ARLs, we have

$ARL_0 = \frac{1}{\alpha}$ and $ARL_1 = \frac{1}{1-\beta}$ or

$$ARL_0 = \frac{1}{1 - F_{in}(UCL) + F_{in}(LCL)} \quad (6\text{-}5)$$

where $F_{in}(x)$ is the cumulative distribution function of the one-parameter Lindley distribution in equation (3-2) with in-control parameter and control limits as computed with equation (6-2) for the probability-type control charts or equations (6-3) and (6-4) for the Shewhart-type control charts and

$$ARL_1 = \frac{1}{1 - F_{out}\left(UCL\right) + F_{out}\left(LCL\right)} \tag{6-6}$$

where $F_{out}\left(x\right)$ is the cumulative distribution function for the distribution of concern with out-of-control parameter and same control limits as before. For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameter of the distribution with the shifted mean will be computed by solving equations (3-3) and (3-4) in terms of the distribution's parameter. The resulting value is given by $\theta_{new} = \frac{1 - \left(\mu_0 + k\sigma\right) \pm \sqrt{\left(\mu_0 + k\sigma\right)^2 + 6\left(\mu_0 + k\sigma\right) + 1}}{2\left(\mu_0 + k\sigma\right)}$.

Using the above formulas we obtain Table 6-1 and Table 6-2, which show the in-control and out-of-control ARL values for the individual probability-type and individual Shewhart-type control chart, respectively, for the one-parameter Lindley distribution for various values of the parameter θ of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the probability-type control charts we have chosen a significance level equal to the most commonly used value of 0.27%, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

Comparison of Tables 6-1 and 6-2 reveals the improvement in the performance of the chart when the skewness corrected limits are used instead of the probability-based ones. The difference in ARL values between Shewhart-type and probability-type control charts is greater than 5% for all shift sizes.

k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
-3	2.8061	2.8461	2.9546	3.1094	3.1280	3.2871	3.2953	3.4087
-2.8	4.1059	4.2830	4.4378	4.5461	4.6123	4.7946	4.8160	4.9481
-2.6	5.1271	5.3028	5.5064	5.5121	5.5269	5.5936	5.7720	5.8321
-2.4	6.0661	6.1661	6.1885	6.1949	6.3240	6.5082	6.7639	6.8290
-2.2	7.1877	7.2748	7.3254	7.3534	7.4901	7.5180	7.7251	7.8996
-2	9.1215	9.4195	9.5742	9.6405	9.6449	9.6612	9.8802	9.9577
-1.8	12.0417	12.2951	12.4646	12.6126	12.7609	12.7628	12.8977	12.9005
-1.6	13.0072	13.5659	13.6026	13.6059	13.7269	13.7536	13.7626	13.9251
-1.4	15.0724	15.1262	15.3542	15.3724	15.4801	15.5157	15.6912	15.8296
-1.2	19.0140	19.2727	19.3716	19.3913	19.4309	19.6325	19.7468	19.8143
-1	26.0496	26.0255	26.0553	26.2706	26.2945	26.5760	26.7359	26.7804
-0.8	48.7265	48.6767	48.6575	48.6240	48.6091	48.5982	48.5917	48.5788
-0.6	46.6756	46.6448	46.6368	46.6227	46.6164	46.6128	46.6090	46.6035
-0.4	141.3919	141.3967	141.3985	141.4015	141.4029	141.4038	141.4044	141.4055
-0.2	163.7400	163.7783	163.7930	163.8190	163.8306	163.8390	163.8442	163.8543
0	369.8338	369.9099	369.9132	369.9320	370.0328	370.0433	370.0549	370.0905
0.2	192.9531	192.9193	192.9063	192.8831	192.8727	192.8651	192.8605	192.8513
0.4	101.2999	101.2715	101.2604	101.2408	101.2320	101.2256	101.2216	101.2139
0.6	59.1338	59.1253	59.1081	59.0953	59.0895	59.0853	59.0827	59.0776
0.8	38.2053	38.1939	38.1894	38.1815	38.1780	38.1754	38.1738	38.1706
1	26.7509	26.7440	26.7413	26.7365	26.7344	26.7328	26.7319	26.7300
1.2	19.9238	19.9198	19.9183	19.9155	19.9143	19.9134	19.9129	19.9128
1.4	15.5625	15.5605	15.5598	15.5584	15.5578	15.5573	15.5570	15.5565
1.6	12.6166	12.6159	12.6157	12.6152	12.6150	12.6149	12.6148	12.6146
1.8	10.5345	10.5348	10.5349	10.5351	10.5352	10.5353	10.5354	10.5355
2	9.0074	9.0084	9.0087	9.0095	9.0098	9.0101	9.0102	9.0105
2.2	7.8523	7.8538	7.8544	7.8555	7.8560	7.8564	7.8566	7.8571
2.4	6.9557	6.9576	6.9584	6.9598	6.9604	6.9609	6.9612	6.9618
2.6	5.2444	5.2466	5.2475	5.2492	5.2499	5.2505	5.2508	5.2515
2.8	4.6694	4.6719	4.6729	4.6747	4.6756	4.6762	4.6766	4.6774
3	3.3970	3.3997	3.4008	3.4027	3.4036	3.4043	3.4048	3.4056

Table 6 - 1: ARL values for individual probability-type control charts for the one-parameter Lindley distribution, with $\alpha = 0.0027$.

k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
-3	2.4122	2.3160	2.2689	2.2407	2.2314	2.1878	2.1228	2.0599
-2.8	3.9348	3.9308	3.9089	3.8236	3.7908	3.6484	3.6443	3.5434
-2.6	4.5317	4.4057	4.3182	4.2816	4.2484	4.2375	4.0379	4.0123
-2.4	5.9797	5.8287	5.7519	5.6872	5.6304	5.5727	5.3250	5.2126
-2.2	6.9784	6.8731	6.8016	6.6437	6.6319	6.5957	6.4846	6.2193
-2	7.6998	7.5573	7.4442	7.4068	7.2815	7.1788	7.1081	7.0952
-1.8	8.9790	8.6090	8.5991	8.4371	8.3969	8.1612	8.0596	8.0150
-1.6	9.7937	9.7368	9.6481	9.5097	9.3175	9.2842	9.1237	9.0231
-1.4	10.6873	10.5936	10.5048	10.3488	10.3200	10.2641	10.2421	10.2284
-1.2	14.7810	14.5506	14.5062	14.4044	14.2808	14.1727	14.1227	14.0416
-1	19.8408	19.8181	19.7128	19.5390	19.4648	19.4333	19.3372	19.2688
-0.8	26.9348	26.8054	26.7781	26.7126	26.5417	26.2343	26.2302	26.0968
-0.6	40.9304	40.8860	40.8648	40.6436	40.5408	40.3363	40.0693	40.0188
-0.4	68.7363	68.6120	68.5715	68.5544	68.3364	68.2577	68.2121	68.0312
-0.2	138.6431	138.5450	138.5039	138.3306	138.2370	138.2024	138.1522	138.0826
0	370.1248	370.1433	370.1648	370.2079	370.2406	370.2595	370.2848	370.3690
0.2	138.1506	138.1757	138.1898	138.1933	138.2127	138.2148	138.2312	138.2416
0.4	68.3289	68.3424	68.3488	68.3548	68.3593	68.3735	68.3736	68.3751
0.6	40.2888	40.3054	40.3091	40.3148	40.3148	40.3225	40.3245	40.3248
0.8	26.7254	26.7284	26.7302	26.7302	26.7314	26.7345	26.7348	26.7393
1	19.2408	19.2432	19.2444	19.2457	19.2464	19.2484	19.2484	19.2486
1.2	14.7048	14.7063	14.7070	14.7071	14.7080	14.7084	14.7093	14.7100
1.4	10.7531	10.7532	10.7534	10.7540	10.7541	10.7544	10.7548	10.7551
1.6	9.7212	9.7223	9.7230	9.7232	9.7237	9.7239	9.7243	9.7248
1.8	8.2609	8.2628	8.2631	8.2641	8.2643	8.2648	8.2648	8.2648
2	7.1737	7.1751	7.1752	7.1786	7.1786	7.1787	7.1787	7.1788
2.2	6.3403	6.3428	6.3431	6.3455	6.3457	6.3459	6.3460	6.3463
2.4	5.6845	5.6881	5.6884	5.6910	5.6914	5.6916	5.6918	5.6932
2.6	4.1602	4.1630	4.1636	4.1640	4.1644	4.1648	4.1681	4.1684
2.8	3.7314	3.7343	3.7348	3.7370	3.7373	3.7373	3.7377	3.7393
3	2.3759	2.3788	2.3796	2.3821	2.3828	2.3833	2.3836	2.3842

Table 6 - 2: ARL values for individual Shewhart-type control charts for the one-parameter Lindley distribution

Comparison of the ARL values for positive and negative shifts shows that, although the control charts can detect both positive and negative shifts well, there are some slight differences with the values for the negative shifts being a little higher than those for the corresponding positive ones for smaller values of the parameter. This holds for either the probability-type or the Shewhart-type control chart. The only differences (in either direction) that are above 5% concern the shifts corresponding to values of k between 0.2 and 0.8 and 1.6 and 1.8 for the probability-type control charts and values of k between 1.8 and 2.8 for the Shewhart-type control charts for small or large parameter values.

6.5 Construction of the EWMA Control Charts for Individual Observations from the One-Parameter Lindley Distribution

As mentioned in Section 2.14.2, one other control chart useful for monitoring processes besides the Shewhart chart is the EWMA chart. When dealing with individual observations EWMA control charts are a better alternative to the Shewhart-type control charts. Moreover, when we are interested in detected small shifts in the process, EWMA charts are preferable. Therefore, besides the Shewhart-type control charts, it is useful to also construct EWMA control charts for individual observations from the one-parameter Lindley distribution.

We will construct the individual EWMA control chart, as generally, by plotting the exponentially weighted moving average of our observations x_i defined by equation (2-2) with the constant λ reflecting the weight we assign to each of the past values of our observations and smaller values of λ being chosen for the detection of smaller shifts, while the starting value being defined as $z_0 = \mu_0$ when the process target is known or $z_0 = \overline{x}$ when using the average of an initial dataset. The central line and control limits of the EWMA chart will be constructed based on the EWMA control charts (2-3) using the skewness correction as in Chan and Cui (2003), since the distribution of concern is asymmetric and, as also mentioned in Weiß and Atzmüller (2011), this is an easily applied method for taking the distribution's skewness into consideration and leads to a better ARL performance of the resulting control chart. In the next section, where we deal with the performance investigation of the constructed control chart, we will further demonstrate the need for this adjustment considering the asymmetry of the distribution and the improvement in the performance of the chart when using the skewness correction contrary to not using it but using the traditionally used symmetric EWMA control limits instead.

More specifically, the procedure for the construction of the proposed control chart is as follows: in equation (2-3) we will replace L by L plus c_4^*, where $c_4^*(x)=\frac{\frac{4}{3}\left[\text{sk}(x)\right]}{1+0.2\left[\text{sk}(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient. EWMA control charts for individual observations from one-parameter Lindley distribution are constructed using the mean of the one-parameter Lindley distribution, which is computed using equation (3-3), its standard deviation (the square root of the quantity computed by equation (3-4)) and the distribution's skewness coefficient computed from equation (3-5). This means that the skewness correction for the mean of the one-parameter Lindley distribution will be

$$c_4^*(x)=\frac{8\left[2(\theta+1)^3-\theta^3\right]\left(\theta^2+4\theta+2\right)^{3/2}}{3\left(\theta+4\theta+2\right)^3+0.24\left[2(\theta+1)^3-\theta^3\right]^2} \tag{6-7}$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the one-parameter Lindley EWMA control chart are as follows.

$$\begin{aligned} UCL&=\frac{\theta+2}{\theta(\theta+1)}+\left[L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \\ CL&=\frac{\theta+2}{\theta(\theta+1)} \\ LCL&=\frac{\theta+2}{\theta(\theta+1)}+\left[-L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \end{aligned} \tag{6-8}$$

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our one-parameter Lindley distribution.

6.6 Performance Investigation for the EWMA Control Charts for Individual Observations from the One-Parameter Lindley Distribution

Once again, as a performance measure of the charts we constructed above, we can use the in-control and out-of-control ARL. According to Lucas and Saccucchi (1990) the ARL of the EWMA control chart is computed by means of the Markov chain method and discretization of the control statistic. More specifically, the region between the upper and lower control limits is divided into $2m+1$ subintervals. Each subinterval S_j (j=1,2,…,$2m$+1) is taken to be represented by its midpoint s_j and then, if δ is the half size of each subinterval, which means that $\delta = \frac{UCL-LCL}{2(2m+1)}$, then whenever $s_j - \delta < Z_i < s_j + \delta$ the process is in a transient state. Otherwise, the process is in the absorbing state. Therefore, the in-control transition probability from one transient state S_j to another transient state S_k is given by

$$\begin{aligned} p_{kj} &= P\left(Z_i \in S_k \middle| Z_{i-1} \in S_j\right) \\ &= P\left(s_k - \delta < Z_i < s_k + \delta \middle| Z_{i-1} = s_j\right) \\ &= P\left(s_k - \delta < \lambda X_i + (1-\lambda) Z_{i-1} < s_k + \delta \middle| Z_{i-1} = s_j\right) \\ &= P\left(\frac{s_k - \delta - (1-\lambda)s_j}{\lambda} < X_i < \frac{s_k + \delta - (1-\lambda)s_j}{\lambda}\right), \quad j,k = 1,2,\ldots,2m+1 \end{aligned} \tag{6-9}$$

The ith-stage transition probability matrix $\mathbf{P}^i$ is, then, defined as $\mathbf{P}^i = \begin{pmatrix} \mathbf{R}^i & \left(\mathbf{I}-\mathbf{R}^i\right)\mathbf{1} \\ \mathbf{0}^T & 1 \end{pmatrix}$, where $\mathbf{R}$ is the ($2m$+1, $2m$+1) matrix of the transient probabilities p_{kj} mentioned in (6-9) above and $\mathbf{0}^T$=(0,0,…,0), i.e. $\mathbf{0}^T$ is the transpose of $\mathbf{0}$ which is a vector of $2m$+1 zeros. The ith-stage transition probability matrix $\mathbf{P}^i$ contains the probabilities that the control statistic goes from one transient state to another in i steps and is used for the computation of the ARL of the EWMA control chart, which is given by

$$ARL = \mathbf{p}^T \left(\mathbf{I}-\mathbf{R}\right)^{-1} \mathbf{1} \tag{6-10}$$

where $\mathbf{p}=\left(p_{-m},p_{-m+1},\dots p_{m-1},p_m\right)^T$ is the vector of the initial probabilities related to the $2m+1$ transient states.

For the transient probabilities in (6-9) the cumulative distribution function for the one-parameter Lindley distribution, i.e. equation (3-2), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equations (6-8) and (6-7) for $i\to\infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL=\frac{\theta+2r}{\theta(\theta+r)}+\left[L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL=\frac{\theta+2r}{\theta(\theta+r)}+\left[-L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}} \tag{6-11}$$

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1=\mu_0+k\sigma$. Using this relationship, the new parameter of the distribution with the shifted mean will be computed by solving equations (3-3) and (3-4) in terms of its parameter, as for the Shewhart-type control chart.

Using those formulae we get Tables 6-3, 6-4, 6-5, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the one-parameter Lindley distribution for various values of the parameter θ of the distribution of concern and for various values of k which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 6-3 contains the ARL values for λ=0.3 and L=6.932 (combination which gives in-control ARL value close to 370) for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ and L the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=50, above which ARL increases very slightly. Consequently, the suggested value of m for the computation of ARL in the formulae above is m=50. Therefore, Tables 6-4 and 6-5 show the ARL values for m=50 for various values of L and λ for positive and negative shifts, respectively.

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some slight differences in ARL values between those two tables, with most of the differences being in favour of the ARL values for negative shifts. The only differences (in either direction) that are less than 5% concern values of k=0.2 for values of λ greater than 0.08 and values of k between 2 and 2.5 for all values of λ. Moreover, comparing Table 6-4 and Table 6-5 we observe that as the value of λ increases ARL values for negative shifts are smaller than the corresponding ones for the positive shifts for large values of k and large values of the parameter. Large negative shifts present smaller ARL values than the large positive ones for small values of λ. Furthermore, for k=0.2 negative shifts give smaller ARL values than the corresponding positive ones for very small λ values.

The need for using the skewness correction for the construction of the individual EWMA control charts for the one-parameter Lindley distribution is justified by the fact that if we had used the traditional symmetric EWMA control limits without the skewness correction term $c_4^*(x)$ in equation (6-11) above, the ARL performance of the chart would have been worse, as can be seen when comparing the results in Table 6-6 for the case of not using the skewness correction term against the results in Table 6-4 for the case of using it. It should be noted that the ARL values in Table 6-6 have resulted from using the same values for λ and L as the ones in Table 6-4 for the shake of making comparisons between the two tables easier. The differences between the ARL values in Tables 6-4 and 6-6 are almost all higher than 5%. The only values for which the difference is less than 5% concern absolute values of k less than 1 for values of λ equal to 0.05 and equal to or greater than 0.12. Comparison is similar for the case of negative shifts so the corresponding table is omitted for space reasons.

m	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
5	0	371.2500	370.8238	370.6635	370.3867	370.2657	370.1780	370.1255	370.0227
	0.2	124.6383	124.4293	124.3500	124.2122	124.1514	124.1073	124.0807	124.0286
	0.5	42.3069	42.2238	42.1921	42.1367	42.1121	42.0942	42.0834	42.0621
	1	14.9888	14.9634	14.9537	14.9365	14.9288	14.9232	14.9198	14.913
	1.5	8.4665	8.4570	8.4534	8.4469	8.4439	8.4418	8.4405	8.4379
	2	6.0234	6.0200	6.0186	6.0162	6.0151	6.0143	6.0138	6.0128
	2.5	4.8648	4.8640	4.8637	4.8631	4.8628	4.8626	4.8625	4.8622
	3	4.2360	4.2364	4.2366	4.2368	4.2370	4.2371	4.2371	4.2373
10	0	417.8972	417.4598	417.2951	417.0105	416.8858	416.7955	416.7415	416.6355
	0.2	135.5535	135.334	135.2508	135.1058	135.0419	134.9954	134.9675	134.9125
	0.5	44.7216	44.6351	44.6021	44.5443	44.5187	44.5000	44.4887	44.4664
	1	15.5363	15.5103	15.5003	15.4826	15.4747	15.4689	15.4654	15.4585
	1.5	8.7317	8.722	8.7183	8.7116	8.7086	8.7063	8.7050	8.7023
	2	6.2160	6.2123	6.2108	6.2082	6.2071	6.2062	6.2057	6.2046
	2.5	5.0338	5.0327	5.0323	5.0315	5.0311	5.0308	5.0307	5.0304
	3	4.3976	4.3976	4.3976	4.3976	4.3976	4.3977	4.3977	4.3977
20	0	432.6378	432.1973	432.0314	431.7446	431.619	431.5279	431.4734	431.3668
	0.2	138.8909	138.6684	138.584	138.437	138.3721	138.3249	138.2966	138.2408
	0.5	45.4520	45.3646	45.3312	45.2728	45.2468	45.2279	45.2165	45.194
	1	15.7166	15.6905	15.6804	15.6626	15.6547	15.6488	15.6453	15.6383
	1.5	8.8336	8.8238	8.82	8.8133	8.8102	8.808	8.8066	8.8039
	2	6.2996	6.2958	6.2943	6.2916	6.2904	6.2895	6.289	6.2879
	2.5	5.1129	5.1116	5.1111	5.1102	5.1098	5.1095	5.1093	5.1090
	3	4.4766	4.4764	4.4763	4.4762	4.4761	4.4761	4.4761	4.4760
30	0	435.6139	435.1729	435.0067	434.7196	434.5938	434.5026	434.448	434.341
	0.2	139.5612	139.3382	139.2535	139.1061	139.0411	138.9938	138.9654	138.9094
	0.5	45.602	45.5145	45.481	45.4225	45.3965	45.3775	45.3661	45.3435
	1	15.7592	15.733	15.7229	15.7051	15.6972	15.6913	15.6878	15.6808
	1.5	8.8618	8.852	8.8482	8.8414	8.8384	8.8361	8.8348	8.8321
	2	6.3251	6.3212	6.3197	6.317	6.3158	6.3149	6.3144	6.3133
	2.5	5.1382	5.1369	5.1363	5.1354	5.135	5.1346	5.1345	5.1341
	3	4.5025	4.5022	4.5021	4.5019	4.5018	4.5018	4.5018	4.5017
40	0	436.6846	436.2434	436.0771	435.7898	435.6640	435.5727	435.5181	435.4110
	0.2	139.8030	139.5798	139.495	139.3475	139.2824	139.2351	139.2066	139.1506
	0.5	45.6576	45.5700	45.5365	45.4779	45.4519	45.433	45.4215	45.399
	1	15.7768	15.7507	15.7406	15.7228	15.7148	15.709	15.7055	15.6985
	1.5	8.8747	8.8649	8.8611	8.8543	8.8513	8.8491	8.8477	8.8450
	2	6.3373	6.3334	6.3319	6.3292	6.3280	6.3271	6.3265	6.3254
	2.5	5.1506	5.1493	5.1487	5.1477	5.1473	5.1470	5.1468	5.1464
	3	4.5154	4.5150	4.5149	4.5147	4.5146	4.5145	4.5145	4.5145
50	0	437.1871	436.7458	436.5795	436.2921	436.1663	436.075	436.0204	435.9133
	0.2	139.9169	139.6936	139.6088	139.4612	139.3961	139.3487	139.3203	139.2643
	0.5	45.6845	45.5968	45.5634	45.5048	45.4788	45.4598	45.4484	45.4258
	1	15.7862	15.7601	15.7500	15.7322	15.7242	15.7184	15.7149	15.7079
	1.5	8.8821	8.8723	8.8685	8.8617	8.8587	8.8564	8.8551	8.8524
	2	6.3445	6.3406	6.3391	6.3364	6.3352	6.3342	6.3337	6.3326
	2.5	5.1580	5.1566	5.1561	5.1551	5.1546	5.1543	5.1541	5.1537
	3	4.5231	4.5227	4.5225	4.5223	4.5222	4.5222	4.5221	4.5221
80	0	437.4624	437.0211	436.8548	436.5674	436.4415	436.3502	436.2956	436.1885
	0.2	139.9796	139.7562	139.6714	139.5238	139.4587	139.4113	139.3828	139.3268
	0.5	45.6996	45.612	45.5785	45.5199	45.4939	45.475	45.4635	45.4410
	1	15.7920	15.7658	15.7557	15.7380	15.7300	15.7242	15.7207	15.7137
	1.5	8.8868	8.8770	8.8732	8.8664	8.8634	8.8612	8.8598	8.8571
	2	6.3492	6.3453	6.3438	6.3411	6.3399	6.3389	6.3384	6.3373
	2.5	5.1629	5.1615	5.1609	5.1599	5.1595	5.1592	5.1590	5.1586
	3	4.5282	4.5278	4.5276	4.5274	4.5273	4.5272	4.5272	4.5271
100	0	437.6295	437.1880	437.0218	436.7343	436.6084	436.5172	436.4625	436.3554
	0.2	140.0177	139.7944	139.7096	139.562	139.4968	139.4494	139.4209	139.3649
	0.5	45.7091	45.6215	45.588	45.5294	45.5034	45.4844	45.4730	45.4504
	1	15.7959	15.7697	15.7596	15.7418	15.7339	15.7281	15.7245	15.7175
	1.5	8.8901	8.8803	8.8765	8.8697	8.8667	8.8645	8.8631	8.8604
	2	6.3525	6.3487	6.3471	6.3444	6.3432	6.3423	6.3417	6.3406
	2.5	5.1664	5.165	5.1644	5.1634	5.1630	5.1626	5.1624	5.1620
	3	4.5318	4.5314	4.5313	4.5310	4.5309	4.5308	4.5308	4.5307

Table 6 - 3: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (λ=0.3 and L=6.932)

λ, L	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
	0	371.0995	370.9004	370.7822	370.5157	370.3712	370.2547	370.1798	370.0204
	0.2	107.9284	107.9523	107.9475	107.9194	107.8983	107.8793	107.8663	107.8368
	0.4	46.2877	46.3266	46.3353	46.3416	46.3406	46.3382	46.3361	46.3302
λ=0.05	0.6	25.2879	25.3206	25.3297	25.3409	25.3438	25.3451	25.3455	25.3453
	0.8	16.2161	16.2419	16.2497	16.2605	16.264	16.2661	16.2671	16.2685
L=2.246	1	11.614	11.6346	11.6412	11.6506	11.6539	11.656	11.6571	11.659
	1.5	6.7964	6.8092	6.8135	6.8201	6.8227	6.8244	6.8254	6.8271
	2	5.0523	5.0612	5.0642	5.069	5.071	5.0723	5.0731	5.0745
	2.5	4.2462	4.2526	4.2548	4.2585	4.26	4.261	4.2616	4.2628
	3	3.8228	3.8274	3.829	3.8317	3.8328	3.8336	3.8341	3.835
	0	373.4099	372.3355	371.9071	371.1317	370.7762	370.5116	370.35	370.026
	0.2	109.2832	109.0704	108.9817	108.8156	108.7372	108.6779	108.6413	108.5669
	0.4	46.973	46.9225	46.8993	46.8534	46.8307	46.813	46.8019	46.779
λ=0.08	0.6	25.6717	25.6625	25.6567	25.6434	25.6362	25.6302	25.6264	25.6183
	0.8	16.454	16.4571	16.4568	16.4543	16.4523	16.4505	16.4492	16.4463
L=2.862	1	11.774	11.7809	11.7826	11.7841	11.7841	11.7839	11.7836	11.7829
	1.5	6.8727	6.8804	6.8829	6.8865	6.8878	6.8887	6.8891	6.8899
	2	5.0981	5.1044	5.1066	5.12	5.1214	5.1223	5.1228	5.1238
	2.5	4.2778	4.2828	4.2845	4.2874	4.2885	4.2894	4.2898	4.2907
	3	3.8466	3.8503	3.8517	3.8539	3.8549	3.8555	3.8559	3.8567
	0	374.5269	373.027	372.4478	371.4246	370.9664	370.6298	370.4261	370.0221
	0.2	110.2547	109.9196	109.7873	109.5497	109.4414	109.3612	109.3123	109.2146
	0.4	47.4958	47.3975	47.3573	47.2833	47.2488	47.2229	47.207	47.1749
λ=0.10	0.6	25.9707	25.9386	25.9247	25.898	25.8851	25.8752	25.8691	25.8565
	0.8	16.6404	16.631	16.6263	16.6164	16.6113	16.6073	16.6047	16.5994
L=3.216	1	11.8992	11.8986	11.8975	11.8945	11.8927	11.8911	11.8901	11.8879
	1.5	6.931	6.9358	6.9373	6.9392	6.9399	6.9402	6.9404	6.9406
	2	5.1316	5.1366	5.1383	5.1409	5.1419	5.1426	5.143	5.1437
	2.5	4.2995	4.3038	4.3053	4.3077	4.3087	4.3094	4.3098	4.3106
	3	3.8617	3.8651	3.8663	3.8683	3.8691	3.8697	3.8701	3.8707
	0	375.4484	373.5997	372.8966	371.6704	371.128	370.7324	370.4943	370.025
	0.2	111.295	110.8554	110.6859	110.387	110.2532	110.155	110.0956	109.9778
	0.4	48.075	47.9348	47.8797	47.781	47.7362	47.703	47.6829	47.6426
λ=0.12	0.6	26.306	26.2537	26.2325	26.1937	26.1758	26.1624	26.1542	26.1376
	0.8	16.8504	16.8298	16.821	16.8045	16.7966	16.7906	16.7869	16.7794
L=3.697	1	12.0402	12.0328	12.0293	12.0222	12.0187	12.016	12.0142	12.0106
	1.5	6.9959	6.9982	6.9987	6.9992	6.9991	6.999	6.9989	6.9986
	2	5.1681	5.1718	5.1731	5.175	5.1757	5.1762	5.1764	5.1769
	2.5	4.3224	4.326	4.3273	4.3293	4.3301	4.3307	4.3311	4.3317
	3	3.8768	3.8798	3.8809	3.8827	3.8834	3.884	3.8843	3.8849
	0	376.6284	374.3328	373.4712	371.985	371.3348	370.8637	370.5816	370.0287
	0.2	112.9738	112.3968	112.1783	111.7985	111.631	111.5091	111.4359	111.2917
	0.4	49.0396	48.8433	48.7681	48.6362	48.5775	48.5345	48.5086	48.4574
λ=0.15	0.6	26.8721	26.7922	26.7612	26.7061	26.6813	26.663	26.652	26.63
	0.8	17.2072	17.1712	17.1569	17.1414	17.1296	17.1209	17.1055	17.0949
L=4.736	1	12.2802	12.2634	12.2565	12.2439	12.238	12.2336	12.2309	12.2255
	1.5	7.1054	7.1044	7.1037	7.102	7.1012	7.1004	7.0999	7.0989
	2	5.2285	5.2306	5.2313	5.2322	5.2325	5.2327	5.2328	5.2329
	2.5	4.359	4.3617	4.3627	4.3643	4.3649	4.3653	4.3656	4.366
	3	3.8997	3.9023	3.9032	3.9047	3.9054	3.9058	3.9061	3.9067
	0	375.2487	373.4264	372.7435	371.5681	371.0552	370.6843	370.4624	370.0285
	0.2	115.7838	115.256	115.0571	114.7128	114.5617	114.4521	114.3863	114.2574
	0.4	51.0131	50.8123	50.7348	50.6015	50.5427	50.4999	50.4742	50.4235
λ=0.20	0.6	28.1605	28.0688	28.0338	27.9724	27.9452	27.9253	27.9133	27.8896
	0.8	18.0868	18.0403	18.0224	17.9909	17.9768	17.9665	17.9602	17.9478
L=5.984	1	12.9146	12.8894	12.8797	12.8623	12.8544	12.8487	12.8452	12.8382
	1.5	7.4465	7.4408	7.4384	7.4341	7.432	7.4305	7.4296	7.4277
	2	5.4512	5.4505	5.4502	5.4496	5.4492	5.4489	5.4487	5.4483
	2.5	4.5235	4.5246	4.5249	4.5255	4.5257	4.5258	4.5259	4.5261
	3	4.0319	4.0333	4.0338	4.0347	4.0351	4.0354	4.0355	4.0358

Table 6 - 4: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (m=50) for various positive shifts

λ, L	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
λ=0.05 L=2.246	0	371.0995	370.9004	370.7822	370.5157	370.3712	370.2547	370.1798	370.0204
	-0.2	106.2449	97.9415	97.9415	97.9415	97.9414	97.9414	97.9412	97.9412
	-0.4	87.9412	86.2364	71.4430	71.4285	60.5988	55.7609	53.7905	53.8175
	-0.6	52.5720	48.8757	41.7680	41.6584	31.9832	28.7815	28.5025	27.5272
	-0.8	34.0550	32.2641	27.4455	27.0067	26.2012	23.2605	22.1601	22.1768
	-1	14.6412	13.8623	13.7841	13.7740	13.2336	13.0222	12.8986	12.8879
	-1.5	10.8322	10.7291	10.6126	10.5269	10.4180	10.2843	10.1812	10.0417
	-2	5.3964	5.3359	5.2123	5.0882	4.9716	4.8982	4.7215	4.6143
	-2.5	4.3883	4.2468	4.1280	4.0157	3.8724	3.8061	3.6476	3.5140
	-3	3.2376	3.1464	3.0059	2.9082	2.8076	2.7064	2.5885	2.4072
λ=0.08 L=2.862	0	373.4099	372.3355	371.9071	371.1217	370.7762	370.5126	370.3500	370.0260
	-0.2	106.2455	100.6003	100.5999	100.5997	100.5994	100.5990	100.5980	100.5975
	-0.4	90.5961	86.2562	71.4444	71.4339	60.6018	55.7610	53.7905	53.8226
	-0.6	52.5752	48.9307	41.7784	41.6990	31.9875	28.7816	28.5025	27.5297
	-0.8	34.0681	32.2643	27.4844	27.1743	26.2022	23.2607	22.1602	22.1847
	-1	14.6506	13.8797	13.7841	13.7809	13.2380	13.0293	12.8992	12.8901
	-1.5	10.8891	10.7609	10.6337	10.5496	10.4295	10.2951	10.1949	10.0794
	-2	5.4160	5.3375	5.2405	5.0940	4.9751	4.9157	4.8125	4.6164
	-2.5	4.3996	4.2871	4.1286	4.0180	3.8912	3.8254	3.6877	3.5155
	-3	3.2532	3.1536	3.0125	2.9242	2.7943	2.7240	2.5949	2.4456
λ=0.10 L=3.216	0	374.5269	373.0270	372.4478	371.4246	370.9644	370.6298	370.4261	370.0221
	-0.2	112.9750	112.9694	112.9644	112.9619	112.9554	112.9407	112.9323	112.9103
	-0.4	96.2462	86.2680	86.0197	71.4359	60.6032	55.7612	55.7590	54.8231
	-0.6	52.5796	48.9638	48.2818	41.7145	31.9895	28.7816	28.7806	27.5382
	-0.8	34.0776	32.2655	32.2494	27.2372	26.2027	23.2608	23.2589	22.1974
	-1	14.6539	13.8894	13.8382	13.7826	13.2439	13.0328	13.0106	12.8912
	-1.5	10.8942	10.7628	10.6493	10.5809	10.4574	10.3028	10.2098	10.1214
	-2	5.4271	5.3386	5.2449	5.1461	5.0195	4.9173	4.8369	4.6175
	-2.5	4.4087	4.2953	4.1720	4.0442	3.9309	3.8461	3.7309	3.5198
	-3	3.3001	3.1626	3.0153	2.9424	2.7993	2.7465	2.6272	2.4543
λ=0.12 L=3.697	0	375.4484	373.5997	372.8964	371.6704	371.1280	370.7324	370.4943	370.0250
	-0.2	113.9875	113.8585	113.7930	113.6842	113.5353	113.1996	113.0080	112.5085
	-0.4	96.2480	86.2912	86.1225	71.4395	60.6041	55.7614	55.7598	54.8307
	-0.6	52.5897	49.0291	48.5339	41.7417	31.9909	28.7817	28.7810	27.5445
	-0.8	34.0833	32.2679	32.2557	27.3450	26.2031	23.2609	23.2596	22.2045
	-1	14.6560	13.9146	13.8452	13.7829	13.2565	13.0402	13.0142	12.8927
	-1.5	10.8977	10.7708	10.7033	10.5936	10.4644	10.3184	10.2107	10.1221
	-2	5.4802	5.3541	5.2612	5.1672	5.0281	4.9261	4.8703	4.6360
	-2.5	4.4240	4.3143	4.1876	4.0856	3.9546	3.8534	3.7727	3.5504
	-3	3.3148	3.1639	3.0933	2.9459	2.8439	2.7512	2.6355	2.4641
λ=0.15 L=4.736	0	376.6284	374.3328	373.4712	371.9850	371.3348	370.8637	370.5816	370.0287
	-0.2	116.5770	116.5474	116.5361	116.5164	116.5080	116.5017	116.4980	116.4907
	-0.4	96.2490	86.2439	86.1478	71.4412	60.6047	60.5926	55.7601	54.8346
	-0.6	52.5956	50.5644	48.6307	41.7538	31.9917	31.9743	29.7812	28.5512
	-0.8	34.0945	34.0082	32.2581	27.3917	26.2033	26.1988	23.2599	22.2129
	-1	14.6571	14.6140	13.8487	13.7836	13.2634	13.2255	13.0160	12.8945
	-1.5	10.9005	10.7720	10.7042	10.6008	10.5064	10.3830	10.2617	10.1271
	-2	5.5481	5.3804	5.2839	5.1742	5.0378	4.9514	4.8830	4.6512
	-2.5	4.4763	4.3296	4.1912	4.1094	3.9801	3.8542	3.7748	3.5724
	-3	3.3251	3.1649	3.1269	2.9498	2.8601	2.7518	2.6712	2.4838
λ=0.20 L=5.984	0	375.2487	373.4264	372.7435	371.5681	371.0552	370.6843	370.4624	370.0285
	-0.2	119.7358	119.7357	119.7356	119.7356	119.7354	119.7351	119.7350	119.7345
	-0.4	96.2517	86.2446	86.2093	71.4423	60.6058	60.5971	55.7606	54.8430
	-0.6	52.6122	50.5701	48.8003	41.7627	31.9934	31.9807	29.7814	28.5559
	-0.8	35.9632	34.0253	32.2623	27.4254	26.2037	26.2005	23.2603	22.2173
	-1	14.6590	14.6346	13.8544	13.7839	13.2802	13.2309	13.0187	12.8975
	-1.5	10.9422	10.8321	10.7170	10.6124	10.5121	10.3864	10.2680	10.1291
	-2	5.5577	5.3946	5.2901	5.1760	5.0415	4.9586	4.8945	4.7059
	-2.5	4.4821	4.3723	4.2251	4.1225	3.9901	3.8716	3.7925	3.6262
	-3	3.3894	3.2290	3.1412	3.0026	2.8782	2.7584	2.6822	2.5641

Table 6 - 5: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (m=50) for various negative shifts

λ, L	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
λ=0.05 L=2.246	0	369.2722	369.2902	369.2924	369.2895	369.2852	369.2809	369.2787	369.2701
	0.2	109.0139	109.0311	109.0358	109.0412	109.0424	109.0427	109.0427	109.0421
	0.4	48.2060	48.2192	48.2232	48.2289	48.2308	48.2319	48.2324	48.2333
	0.6	26.4868	26.4897	26.4903	26.4903	26.4900	26.4896	26.4893	26.4886
	0.8	17.0489	17.0533	17.0546	17.0563	17.0568	17.0560	17.0571	17.0572
	1	12.1425	12.1470	12.1486	12.1507	12.1515	12.1520	12.1523	12.1528
	1.5	7.7441	7.7484	7.7500	7.7525	7.7435	7.7543	7.7547	7.7555
	2	5.7351	5.7494	5.7510	5.7535	5.7445	5.7553	5.7557	5.7465
	2.5	5.1257	5.1276	5.1283	5.1294	5.1299	5.1303	5.1305	5.1309
	3	3.9584	3.9592	3.9595	3.9601	3.9603	3.9605	3.9605	3.9607
λ=0.08 L=2.862	0	369.6276	369.5008	369.4488	369.3527	369.3079	369.2741	369.2534	369.2115
	0.2	109.3643	109.1398	109.1288	109.1072	109.0965	109.0883	109.0831	109.0725
	0.4	48.5520	48.5492	48.5472	48.5425	48.5397	48.5375	48.5361	48.5329
	0.6	27.2589	27.2691	27.2724	27.2773	27.2291	27.2803	27.2809	27.2820
	0.8	17.3181	17.3262	17.3289	17.3331	17.3247	17.3358	17.3364	17.3374
	1	12.1676	12.1742	12.1765	12.1801	12.1815	12.1824	12.1830	12.1840
	1.5	7.9383	7.9422	7.9435	7.9457	7.9466	7.9472	7.9475	7.9482
	2	5.9484	5.9523	5.9536	5.9558	5.9567	5.9573	5.9576	5.9583
	2.5	5.6756	5.6777	5.6785	5.6798	5.6803	5.6807	5.6810	5.6814
	3	3.8758	3.8769	3.8774	3.8781	3.8784	3.8787	3.8788	3.8791
λ=0.10 L=3.216	0	369.7318	369.5579	369.5845	369.3803	369.4888	369.3214	369.2806	369.2394
	0.2	111.2526	111.4824	111.6542	110.6029	110.5792	110.5616	110.5508	110.5292
	0.4	48.9186	48.8980	48.8896	48.8738	48.8662	48.8604	48.8569	48.8506
	0.6	27.6748	27.6513	27.6523	27.6265	27.6194	27.6142	27.6111	27.6050
	0.8	17.3673	17.3680	17.3678	17.3670	17.3664	17.3658	17.3655	17.3646
	1	12.6058	12.6084	12.6092	12.6100	12.6102	12.6102	12.6102	12.6102
	1.5	8.1455	8.1489	8.1501	8.1320	8.1527	8.1532	8.1535	8.1540
	2	7.1254	7.1288	7.1300	7.1319	7.1326	7.1331	7.1334	7.1339
	2.5	5.7271	5.7304	5.7303	5.7317	5.7323	5.7327	5.7329	5.7334
	3	3.8879	3.8893	3.8899	3.8808	3.8812	3.8815	3.8816	3.8820
λ=0.12 L=3.697	0	369.9686	369.6919	369.6352	369.6918	369.5398	369.4389	369.3622	369.2806
	0.2	112.4326	112.6817	112.9510	111.8612	111.8209	111.7912	111.7733	111.7376
	0.4	48.9367	48.9347	48.9379	48.9377	48.9339	48.9336	48.9374	48.9348
	0.6	27.7068	27.7023	27.7000	27.6951	27.6926	27.6906	27.6993	27.6866
	0.8	17.4378	17.4333	17.4313	17.4271	17.4250	17.4234	17.4224	17.4203
	1	12.6262	12.6260	12.6256	12.6246	12.6250	12.6234	12.6231	12.6223
	1.5	8.3063	8.3091	8.3100	8.3113	8.3118	8.3122	8.3123	8.3127
	2	7.3273	7.3301	7.3309	7.3323	7.3328	7.3332	7.3333	7.3337
	2.5	5.7357	5.7371	5.7368	5.7300	5.7398	5.7384	5.7315	5.7322
	3	3.8903	3.8919	3.8925	3.8936	3.8940	3.8944	3.8946	3.8949
λ=0.15 L=4.736	0	369.9722	369.7047	369.6038	369.4191	369.3525	369.2969	369.2636	369.1983
	0.2	112.9970	112.6961	112.6578	112.5912	112.5618	112.5404	112.5276	112.5024
	0.4	49.0518	48.8959	48.8783	48.8477	48.8341	48.8242	48.8182	48.8065
	0.6	27.8526	27.8379	27.8319	27.8205	27.8151	27.8111	27.8086	27.8035
	0.8	17.5490	17.5361	17.5311	17.5223	17.5184	17.5155	17.5137	17.5102
	1	12.6377	12.6304	12.6275	12.6225	12.6202	12.6185	12.6175	12.6155
	1.5	8.7390	8.7374	8.7367	8.7356	8.7349	8.7345	8.7343	8.7338
	2	7.4289	7.4273	7.4266	7.4254	7.4248	7.4244	7.4242	7.4237
	2.5	5.7589	5.7594	5.7596	5.7599	5.7601	5.7602	5.7602	5.7604
	3	3.9080	3.9094	3.9082	3.9090	3.9080	3.9092	3.9089	3.9093
λ=0.20 L=5.984	0	369.9896	369.9961	369.9607	369.9993	369.9723	369.9527	369.9410	369.9179
	0.2	116.8972	116.8537	116.8372	116.8083	116.8956	116.8863	116.8807	116.8697
	0.4	51.9534	51.9309	51.9223	51.9073	51.9006	51.9957	51.9928	51.9870
	0.6	28.2720	28.2595	28.2546	28.2461	28.2424	28.2396	28.2379	28.2346
	0.8	18.8686	18.8612	18.8584	18.8534	18.8511	18.8495	18.8485	18.8465
	1	13.0206	13.0162	13.0145	13.0115	13.0101	13.0091	13.0085	13.0073
	1.5	8.8853	8.8842	8.8838	8.8830	8.8827	8.8824	8.8823	8.8820
	2	7.4863	7.4852	7.4848	7.4840	7.4837	7.4834	7.4833	7.4830
	2.5	5.8177	5.8179	5.8180	5.8181	5.8182	5.8182	5.8183	5.8184
	3	4.0572	4.0571	4.0581	4.0521	4.0571	4.0570	4.0563	4.0572

Table 6 - 6: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (m=50) for various positive shifts for the case of not using the skewness correction term when constructing the control limits of the chart

Additionally, comparing the ARL values for the EWMA in Tables 6-4 and 6-5 with the ARL values for the Shewhart-type control chart in Table 6-1, we can see that the EWMA control chart performs better than the Shewhart-type control chart for smaller shifts, since for the case of small shifts, the EWMA out-of-control ARL values are smaller than the corresponding ARL values for the Shewhart-type charts. When it comes to large shifts, however, EWMA ARL values are slightly larger and, therefore, make Shewhart-type control charts preferable for those cases.

6.7 Optimal Choice for the Parameters of the EWMA Control Charts for Individual Observations from the One-Parameter Lindley Distribution

When constructing an EWMA control chart, there are two parameters involved in the way the chart is going to perform, namely the constant λ which affects the weight we give to the past values of our observations and the value of L which affects the width of the chart's control limits. Therefore, we need to find the combination of the values of those two parameters which will lead us to the optimal performance of our control chart.

A lot of work has been done on optimal design of control charts in literature [e.g. Capizzi and Masarotto (2003), Castagliola et al. (2008), Khoo et al. (2013), Castagliola et al. (2019), Saha et al. (2019), Yeong et al. (2021), Chong et al. (2022), Tang et al. (2022), Xie et al. (2022), Yeong et al. (2023)] based on minimizing the out-of-control value of various performance criteria. Since all the study here has been based on ARL (which is the most commonly used performance criterion) the optimal design of the EWMA control chart will be done by minimizing the ARL. The algorithm applied here is as follows:

- Step 1: Set the desired in-control ARL value (e.g. ARL_0=370) and the size of the mean shift k to be detected (e.g. k = 0.5).
- Step 2: Set an initial value L = 1.

- Step 3: Vary the parameter λ (e.g. increasing by 0.01) so as $\lambda \in (0,1]$ and (using a nonlinear equation solver) find the value of λ for which the ARL_0 value in Step 1 is satisfied.
- Step 4: Calculate the ARL_1 value for the particular combination of λ and L resulting from Step 3. [The ARL_1 value is obtained as described in the previous section, using equation (6-9) for the computation of the transient probabilities along with equation (3-2) for the cumulative distribution function of the one-parameter Lindley distribution.]
- Step 5: Increase L by 0.01.
- Step 6: Repeat Steps 3-5 until the minimum ARL_1 value has been reached (i.e. until the ARL_1 value for L+0.01 is larger than the ARL_1 value for L).
- Step 7: Keep the combination of λ and L resulting from Step 6 for which the smallest ARL_1 value is obtained as the desired optimal one for the selected shift size in Step 1.
- Step 8: Repeat Steps 2-7 for all the desired values of shifts to be detected (e.g. k = {-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3}).

Application of this algorithm leads to Table 6-7 and Table 6-8 which present the optimal combination of values of the two parameters of concern (λ and L) of the EWMA chart with the corresponding ARL values for various values of the parameter θ of the one-parameter Lindley distribution and various positive and negative values, respectively, of k, which shows the shift of the process mean in terms of the process standard deviation which we want to be detected by the control chart we construct.

k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
0.2	(0.04, 2)	(0.04, 2)	(0.04, 2)	(0.04, 2)	(0.04, 2)	(0.04, 2)	(0.04, 2)	(0.04, 2)
	(375.4826, 53.6428)	(378.5962, 53.4848)	(371.4867, 53.7147)	(372.2494, 53.6397)	(375.6644, 53.5898)	(378.8584, 53.6146)	(370.1654, 53.6318)	(375.3154, 53.6157)
0.4	(0.03, 2)	(0.04, 2)	(0.69, 6.43)	(0.04, 2)	(0.04, 2)	(0.67, 6.08)	(0.04, 2)	(0.04, 2)
	(375.3626, 16.6965)	(378.4826, 16.9318)	(371.7679, 16.2948)	(372.2494, 16.7528)	(375.6644, 16.5926)	(370.1455, 16.5128)	(370.1454, 16.7543)	(375.3145, 16.6269)
0.6	(0.65, 6.48)	(0.68, 7.81)	(0.68, 6.73)	(0.67, 6.98)	(0.66, 6.76)	(0.67, 6.08)	(0.67, 7.7)	(0.67, 6.62)
	(370.0357, 10.1828)	(368.8257, 10.8197)	(371.7679, 10.2835)	(369.7372, 10.2842)	(370.2573, 10.2684)	(370.6545, 10.9919)	(368.6897, 10.6216)	(370.1487, 10.1488)
0.8	(0.67, 6.89)	(0.68, 7.81)	(0.68, 6.83)	(0.67, 6.98)	(0.67, 6.76)	(0.67, 6.08)	(0.67, 7.7)	(0.67, 6.62)
	(370.1887, 8.7544)	(368.8287, 8.2464)	(371.7518, 8.8948)	(369.7372, 8.9315)	(370.2403, 8.8245)	(370.2158, 8.6852)	(368.6459, 8.0485)	(370.1527, 8.7359)
1	(0.66, 6.89)	(0.05, 1.5)	(0.68, 6.68)	(0.67, 6.98)	(0.67, 6.76)	(0.66, 6.08)	(0.02, 1.8)	(0.66, 6.62)
	(370.1648, 7.5482)	(367.1808, 7.7548)	(371.5789, 7.7682)	(369.7522, 7.6488)	(370.4215, 7.5478)	(370.2484, 7.5154)	(359.9154, 7.7145)	(370.1597, 7.5157)
1.2	(0.04, 1.41)	(0.04, 1.4)	(0.04, 1.4)	(0.04, 1.4)	(0.04, 1.41)	(0.04, 1.4)	(0.03, 1.4)	(0.03, 1.41)
	(378.3796, 4.8822)	(367.5988, 4.7844)	(369.2518, 4.8428)	(360.8688, 4.8973)	(373.7948, 4.8548)	(362.8157, 4.7845)	(359.9145, 4.7978)	(378.3145, 4.8486)
1.4	(0.05, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)	(0.04, 1.4)	(0.04, 1.4)	(0.05, 1.41)
	(378.3537, 4.4648)	(367.1898, 4.5245)	(369.2918, 4.5715)	(360.8688, 4.5989)	(373.7948, 4.5788)	(362.8145, 4.2815)	(359.936, 4.2415)	(378.3486, 4.3214)
1.6	(0.04, 1.41)	(0.04, 1.4)	(0.04, 1.4)	(0.04, 1.4)	(0.04, 1.41)	(0.04, 1.4)	(0.04, 1.4)	(0.04, 1.41)
	(378.3646, 4.2055)	(367.1598, 3.9986)	(369.2789, 3.9818)	(360.8688, 3.9916)	(373.7948, 4.2045)	(362.8232, 3.9845)	(359.9684, 3.9848)	(378.3286, 4.0157)
1.8	(0.05, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)
	(378.3486, 3.9016)	(367.2718, 3.7924)	(369.2845, 3.7988)	(360.8688, 3.7972)	(373.7948, 3.8028)	(362.8598, 3.7918)	(359.948, 3.7928)	(378.3646, 3.8098)
2	(0.05, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)
	(378.3918, 3.6454)	(367.2835, 3.6461)	(369.2487, 3.6253)	(360.8697, 3.6253)	(373.7948, 3.6384)	(362.8484, 3.6848)	(359.9362, 3.6848)	(378.3166, 3.6798)
2.2	(0.06, 1.41)	(0.06, 1.4)	(0.06, 1.4)	(0.06, 1.4)	(0.06, 1.41)	(0.06, 1.4)	(0.06, 1.4)	(0.06, 1.41)
	(378.3458, 3.5928)	(367.1548, 3.5898)	(369.2487, 3.5878)	(360.8654, 3.5848)	(373.7848, 3.5899)	(362.8487, 3.5845)	(359.968, 3.5868)	(378.3148, 3.5915)
2.4	(0.06, 1.41)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.4)	(0.05, 1.41)	(0.05, 1.4)	(0.04, 1.4)	(0.05, 1.41)
	(378.3546, 3.5384)	(367.1878, 3.5439)	(369.2487, 3.5419)	(360.8165, 3.5418)	(373.7548, 3.5399)	(362.8984, 3.5428)	(359.9693, 3.5468)	(378.3148, 3.5388)
2.6	(0.97, 2.59)	(0.97, 2.57)	(0.97, 2.59)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.59)	(0.97, 2.59)
	(376.2486, 3.4468)	(377.6198, 3.3984)	(375.1257, 3.4578)	(379.2154, 3.3988)	(375.1463, 3.4128)	(375.2189, 3.3948)	(379.5454, 3.4368)	(376.2648, 3.4391)
2.8	(0.97, 2.59)	(0.97, 2.57)	(0.97, 2.59)	(0.97, 2.57)	(0.97, 2.57)	(0.98, 2.57)	(0.98, 2.59)	(0.98, 2.59)
	(376.2482, 3.2098)	(377.6468, 3.1887)	(375.1843, 3.2218)	(379.2571, 3.1764)	(375.2548, 3.1884)	(375.2085, 3.1742)	(379.5482, 3.2108)	(376.2486, 3.2098)
3	(0.97, 2.59)	(0.98, 2.57)	(0.97, 2.59)	(0.98, 2.57)	(0.97, 2.57)	(0.98, 2.57)	(0.98, 2.59)	(0.98, 2.59)
	(376.248, 3.0189)	(377.6425, 2.9964)	(375.1543, 3.0289)	(379.2684, 2.9893)	(375.2844, 2.9978)	(375.2146, 2.9845)	(379.5712, 3.0168)	(376.2489, 3.0098)

Table 6 - 7: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the one-parameter Lindley distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of positive shifts k (m=50)

k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
-0.2	(0.04, 1.6)	(0.04, 1.6)	(0.68, 6.64)	(0.04, 1.6)	(0.04, 1.6)	(0.04, 1.6)	(0.04, 1.6)	(0.04, 1.6)
	(375.3728, 52.3284)	(378.396, 53.6828)	(372.7889, 52.8936)	(372.2397, 52.7845)	(375.6444, 54.7573)	(378.8468, 52.8484)	(370.1648, 52.2678)	(375.3428, 52.3186)
-0.4	(0.08, 2.93)	(0.08, 2.96)	(0.08, 2.96)	(0.08, 2.93)	(0.1, 3.18)	(0.08, 2.93)	(0.1, 3.16)	(0.08, 2.93)
	(364.864, 15.3536)	(364.7846, 15.0784)	(368.7391, 15.0579)	(369.6378, 15.4884)	(368.435, 15.4124)	(372.4573, 15.2453)	(364.3862, 15.2935)	(364.864, 15.3536)
-0.6	(0.16, 3.98)	(0.16, 3.98)	(0.16, 3.97)	(0.16, 3.98)	(0.16, 3.98)	(0.16, 3.98)	(0.16, 3.98)	(0.16, 3.98)
	(377.9646, 10.7553)	(375.6868, 10.3932)	(372.2164, 10.2826)	(375.6445, 10.5782)	(378.1248, 10.8228)	(378.5038, 10.544)	(375.2757, 10.6204)	(377.9645, 10.7553)
-0.8	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 9.4254)	(372.369, 8.8457)	(372.888, 10.044)	(375.844, 9.2402)	(362.9997, 8.457)	(375.6884, 9.0393)	(375.6805, 9.2518)	(375.5408, 9.4254)
-1	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 6.4018)	(372.369, 6.228)	(372.888, 6.8786)	(375.844, 6.4837)	(362.9997, 5.9384)	(375.6884, 6.2642)	(375.6805, 6.2887)	(375.5408, 6.4018)
-1.2	(0.84, 2.55)	(0.79, 2.55)	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.55)	(0.79, 2.55)	(0.8, 2.55)	(0.84, 2.55)
	(399.6864, 4.9324)	(372.369, 5.2484)	(372.888, 5.6443)	(378.1879, 5.2816)	(372.3018, 5.1624)	(375.6884, 5.2612)	(397.4825, 5.2873)	(399.6864, 5.3524)
-1.4	(0.84, 2.55)	(0.88, 2.57)	(0.88, 2.57)	(0.84, 2.55)	(0.88, 2.55)	(0.82, 2.55)	(0.84, 2.55)	(0.84, 2.55)
	(372.5462, 4.4018)	(377.7973, 4.8642)	(377.057, 4.8268)	(378.5554, 4.9012)	(378.3272, 4.7848)	(379.7273, 4.9623)	(375.124, 4.8424)	(372.0724, 4.8022)
-1.6	(0.93, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.93, 2.57)
	(388.4888, 4.068)	(377.0362, 4.0284)	(375.2439, 4.141)	(364.4184, 4.1535)	(372.1288, 4.0454)	(377.9844, 4.1464)	(378.8097, 4.0841)	(388.4888, 4.089)
-1.8	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 3.9012)	(372.369, 3.9012)	(372.888, 3.9012)	(375.844, 3.9016)	(362.9997, 3.9014)	(375.6884, 3.9043)	(375.6805, 3.9014)	(375.5408, 3.9012)
-2	(0.93, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.93, 2.57)
	(388.4888, 3.6208)	(377.6469, 3.6284)	(375.2439, 3.6289)	(379.2528, 3.6264)	(393.2553, 3.6257)	(375.2069, 3.6252)	(369.3912, 3.6239)	(388.4888, 3.624)
-2.2	(0.93, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.93, 2.57)
	(388.4888, 3.6208)	(377.6469, 3.6284)	(375.2439, 3.6289)	(379.2528, 3.6264)	(393.2553, 3.6257)	(375.2069, 3.6252)	(369.3912, 3.6239)	(388.4888, 3.624)
-2.4	(0.93, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.97, 2.57)	(0.93, 2.57)
	(388.4888, 3.5308)	(377.6469, 1.0184)	(375.2439, 1.0189)	(379.2528, 1.0164)	(393.2553, 1.0157)	(375.2069, 1.0152)	(369.3912, 1.0139)	(388.4888, 1.014)
-2.6	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 3.3903)	(372.369, 3.3024)	(372.888, 3.3024)	(375.844, 3.3018)	(362.9997, 3.3105)	(375.6884, 3.3024)	(375.6805, 3.3024)	(375.5408, 3.3012)
-2.8	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 3.2012)	(372.369, 3.2012)	(372.888, 3.2012)	(375.844, 3.2015)	(362.9997, 3.2012)	(375.6884, 3.2014)	(375.6805, 3.2014)	(375.5408, 3.2015)
-3	(0.79, 2.57)	(0.79, 2.55)	(0.79, 2.57)	(0.75, 2.57)	(0.75, 2.55)	(0.79, 2.55)	(0.75, 2.57)	(0.79, 2.57)
	(375.5408, 2.9822)	(372.369, 2.9822)	(372.888, 2.9822)	(375.844, 2.9818)	(362.9997, 2.9814)	(375.6884, 2.9814)	(375.6805, 2.9818)	(375.5408, 2.9812)

Table 6 - 8: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the one-parameter Lindley distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of negative shifts k (m=50)

6.8 Examples on the Individual One-Parameter Lindley Probability-Type, Shewhart-Type and EWMA Control Charts

This section provides illustration of the proposed control charts by means of both simulated data generated from the distribution of concern and real data. The case of simulated data is presented in Subsection 6.8.1, while the real data case is covered in Subsection 6.8.2.

6.8.1 Examples with Simulated Data from the One-Parameter Lindley Distribution

For the simulation the R programming language version 4.0.2 (R Core Team (2020)) has been used along with the "LindleyR" package version 1.1.0 (Mazucheli et al. (2016)). The "lamW" package version 1.3.3 (Adler (2015)) has also been used for the quantile function of the distribution used in probability-type control charts.

Suppose we take a sample of n = 30 observations from a one-parameter Lindley distributed process as follows. First, we take a sample of 15 observations from a one-parameter Lindley process with in-control θ value equal to 55. Now suppose that a shift of one standard deviation unit occurs in the process mean, and after that shift, we draw another set of 15 observations from the process. The resulting data set can be seen in Table 6-9. For this data set, we construct the individual probability-type one-parameter Lindley control chart shown in Figure 6-1, using the most commonly used value for the significance level α = 0.27%, as mentioned in Section 6.2.

Data Set 1	0.014816	0.026409	0.002257	0.008270	0.067346
	0.032560	0.014201	0.024136	0.026196	0.004702
	0.005228	0.049403	0.008079	0.000664	0.023497
	0.085456	0.034413	0.029355	0.093822	0.067916
	0.032951	0.077530	0.035203	0.150783	0.053750
	0.098310	0.070499	0.214163	0.071007	0.093822

Table 6 - 9: Data from a one-parameter Lindley process with in control θ = 55 and a shift of one standard deviation unit in the process mean due to an increasing shift after the first 15 observations (gray shading)

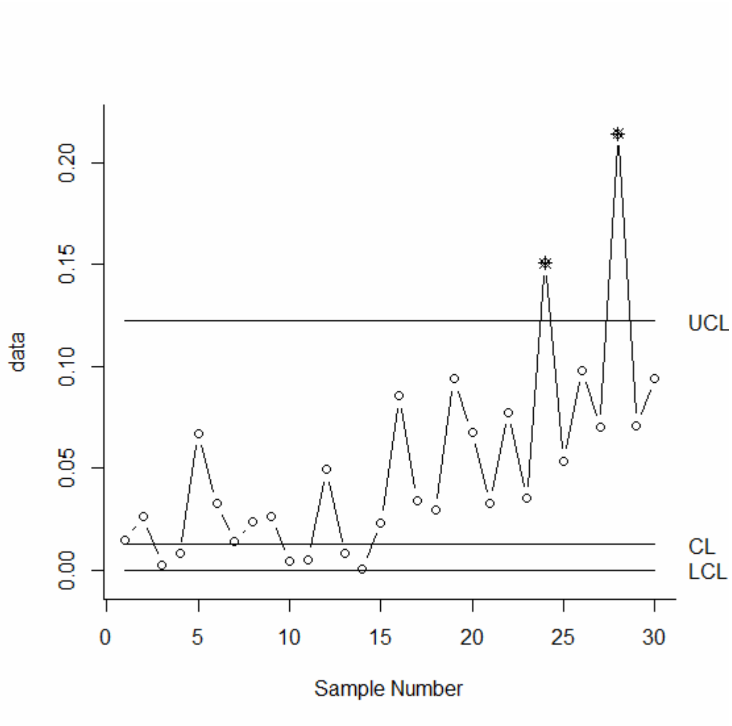

Figure 6 - 1: Individual probability-type one-parameter Lindley control chart for the data set in Table 6-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations and the control charts detect some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level.

For the same data with one standard deviation unit shift in Table 6-9, we now construct the Shewhart-type one-parameter Lindley control chart shown in Figure 6-2, using L = 3.431 standard deviations (which gives a desired value of in-control ARL close to 370).

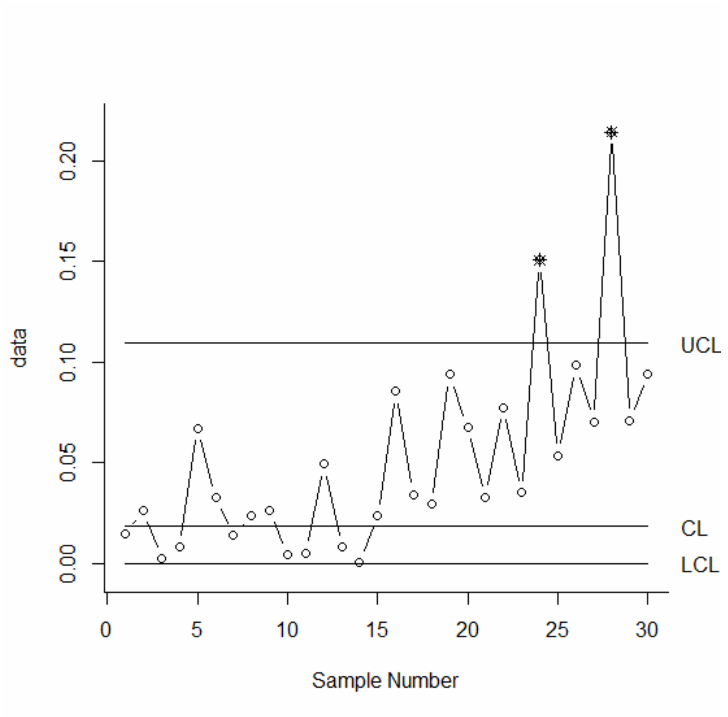

Figure 6 - 2: Individual Shewhart-type one-parameter Lindley control chart for the data set in Table 6-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations and the control charts detect some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level. Comparing this chart to the previous one (Figure 6-1), we observe similar behaviour of the probability-type chart to the Shewhart-type chart with skewness correction.

Using the data set in Table 6-9 for the case of a shift of one standard deviation unit, we now construct the individual EWMA one-parameter Lindley control chart shown in Figure 6-3, using $\lambda=0.05$ and $L=2.67445$ standard deviations (which gives a desired value of in-control ARL close to 370). As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 21^{st} observation.

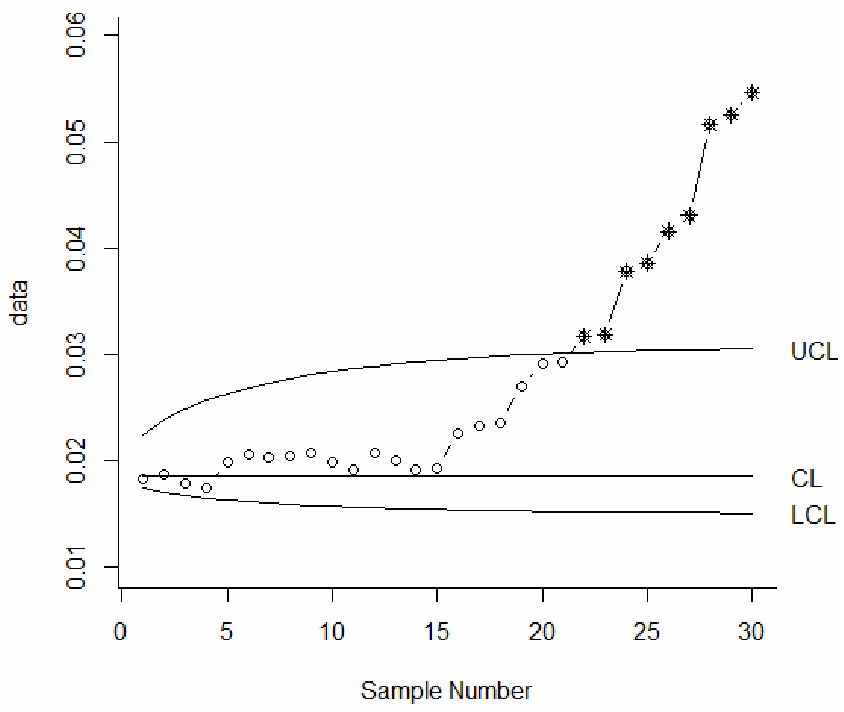

Figure 6 - 3: Individual EWMA one-parameter Lindley control chart for the data set in Table 6-9 with a shift of one standard deviation unit in the process mean

Comparing Figure 6-3 with Figure 6-2 we can see now that, as expected, the EWMA control chart detects the one-standard deviation-unit shift quicker than the corresponding Shewhart-type control chart.

6.8.2 Application of the Individual One-Parameter Lindley Probability-Type, Shewhart-Type and EWMA Control Charts to Real Data

Here we present the illustration of the proposed control charts through application to two real datasets. The first dataset was used Ghitany et al. (2008) representing waiting times before service of bank customers. This data set is presented here in Table 6-10.

13.9	21.9	8.8	3.1	14.1	8.6	8.0	12.9	6.2	4.9
13.7	1.9	4.3	27.0	6.3	9.5	11.9	9.6	2.6	17.3
1.8	4.0	11.0	3.3	13.6	5.7	5.3	21.3	21.4	4.2
4.4	12.5	6.9	4.1	18.1	8.9	7.7	11.2	7.1	2.1
6.2	18.9	2.7	4.6	38.5	10.7	6.1	2.9	13.1	4.9
3.2	11.5	9.8	11.1	19.0	4.3	15.4	1.5	0.8	13.3
6.2	4.7	18.2	4.4	3.6	31.6	7.1	6.7	11.2	1.9
5.0	15.4	7.1	23.0	8.9	8.2	18.4	4.2	5.7	33.1
7.4	8.6	10.9	7.6	4.7	11.0	4.8	3.5	19.9	9.7
8.6	13.0	7.1	17.3	5.5	8.8	12.4	1.3	0.8	20.6

Table 6 - 10: Waiting Times Data Set

First of all, when dealing with any dataset, the normality assumption should be checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the one-parameter Lindley distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.6994 with the presence of ties in our data and a p-value=0.8161 without them. In both cases p-value is very large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the one-parameter Lindley distribution fits our data well.

The value of the parameter of the assumed one-parameter Lindley distribution from our data as in Ghitany et al. (2008) being equal to 0.187 is going to be used for the construction of the individual probability-type control chart in Figure 6-4 for the dataset in hand. The Shewhart-type control chart for the particular dataset, using the above estimation along with the value of L=2.993 standard deviations (for which in-control ARL is close to 370), is presented in Figure 6-5. As we can see there, the data points are all inside the control limits in both charts and this means that the waiting times of bank customers are within the expected ranges.

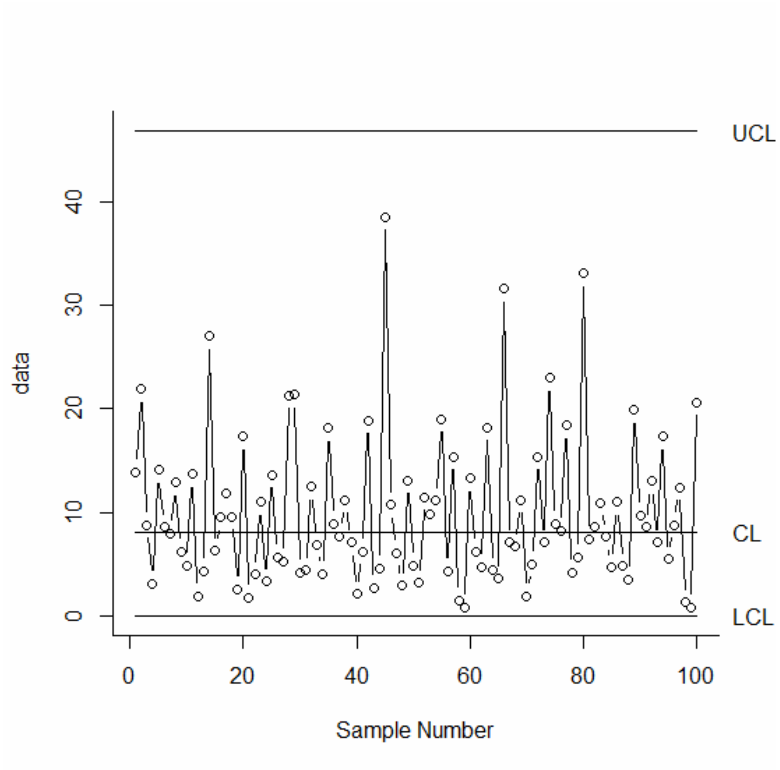

Figure 6 - 4: Individual probability-type control chart for the Waiting Times dataset assuming one-parameter Lindley distribution for the data

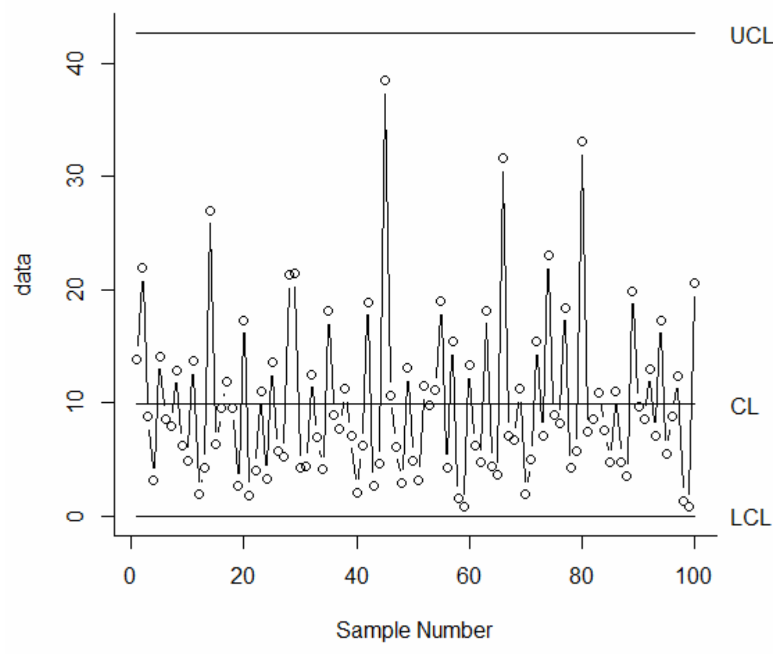

Figure 6 - 5: Individual Shewhart-type control chart for the Waiting Times dataset assuming one-parameter Lindley distribution for the data

For the construction of the individual EWMA control chart for the data set in hand, using the same parameter value of the assumed one-parameter Lindley distribution along with the values of λ=0.08 and L=2.623 standard deviations (for which in-control ARL is close to 370), we construct the control chart as presented in Figure 6-6. As we can see there, the data points are all inside the control limits and this means, once again, that the waiting times of bank customers are within the expected ranges.

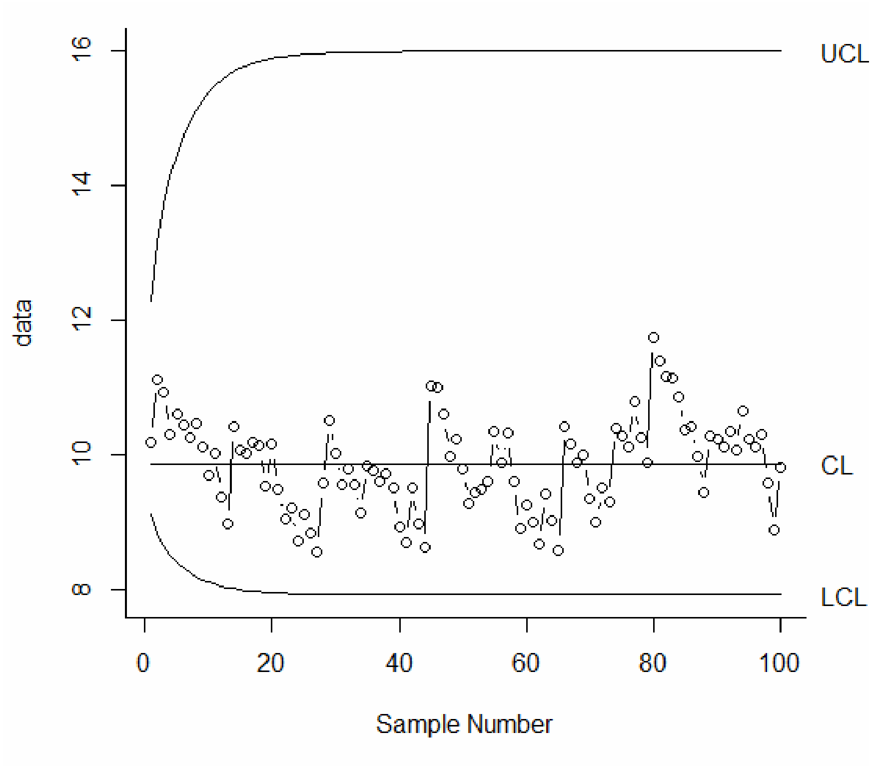

Figure 6 - 6: Individual EWMA control chart for the Waiting Times dataset assuming one-parameter Lindley distribution for the data

Now let's apply the proposed control charts on a second data set. The dataset comes from a paper by Proschan (1963) and can also be found in Cox and Snell (1981) and represents the time intervals between failures of the air-conditioning equipment of ten Boeing 720 aircrafts. Here we will use the data for the third aircraft, as presented, for convenience, in Table 6-11. First, as usual the normality assumption is checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the one-parameter Lindley distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.3752 with the presence of ties in our data and a p-value=0.3433 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the one-parameter Lindley distribution fits our data well. There are, however, some outliers in our data. Let's see if the control charts can detect them.

Times between failures	74	57	48	29	502
	12	70	21	29	386
	59	27	153	26	326

Table 6 - 11: Time (in hours) between failures of the air-conditioning equipment of the third Boeing 720 aircraft in Proschan (1963).

The value of the parameter θ of our assumed Lindley distribution being equal to 0.0164 is going to be used for the construction of the individual control charts. For the probability-type control chart the significance level value $\alpha = 0.27\%$ is used, while for the Shewhart-type control chart for our data the value of L=2.973 standard deviations (for which in-control ARL is close to 370) is used. The resulting control charts can be seen in Figure 6-7 and Figure 6-8 for the probability-type and Shewhart-type control chart, respectively. As we can see the probability control chart does not detect any out-of-control points, while the Shewhart-type control chart detects one.

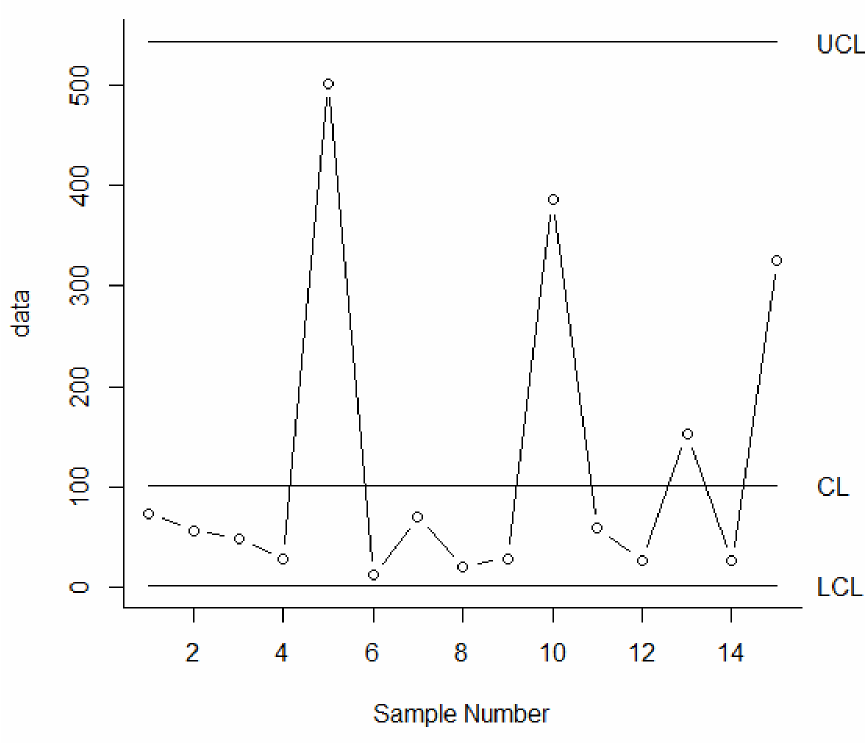

Figure 6 - 7: Individual probability-type control chart for the Failure Time Intervals of the third aircraft dataset assuming Lindley distribution for the data.

For the construction of the individual EWMA control chart, the same parameter θ value is going to be used along with the values of λ=0.05 and L=2.9734 standard deviations (for which in-control ARL is close to 370). The resulting control chart is shown in Figure 6-9, which presents no point outside the control limits, but shows one point almost on the lower control limit, which is an indication that the EWMA control chart (which is more sensitive to small shifts) was very close to give an out-of-control signal, because it detected that the previous values were decreasing and the process almost got out-of-control which the previous two charts did not detect.

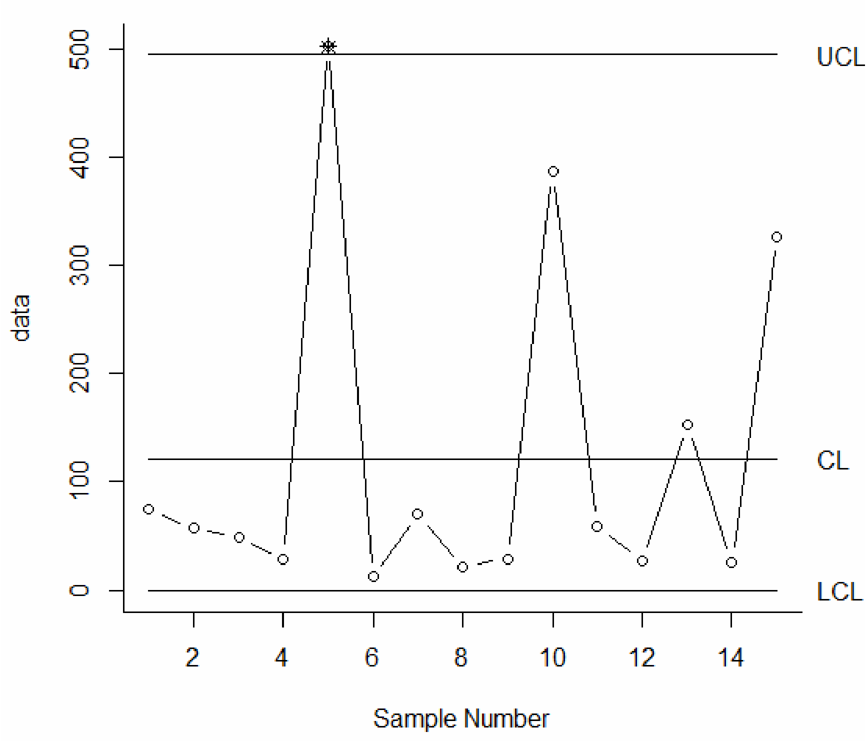

Figure 6 - 8: Individual Shewhart-type control chart for the Failure Time Intervals of the third aircraft dataset assuming Lindley distribution for the data.

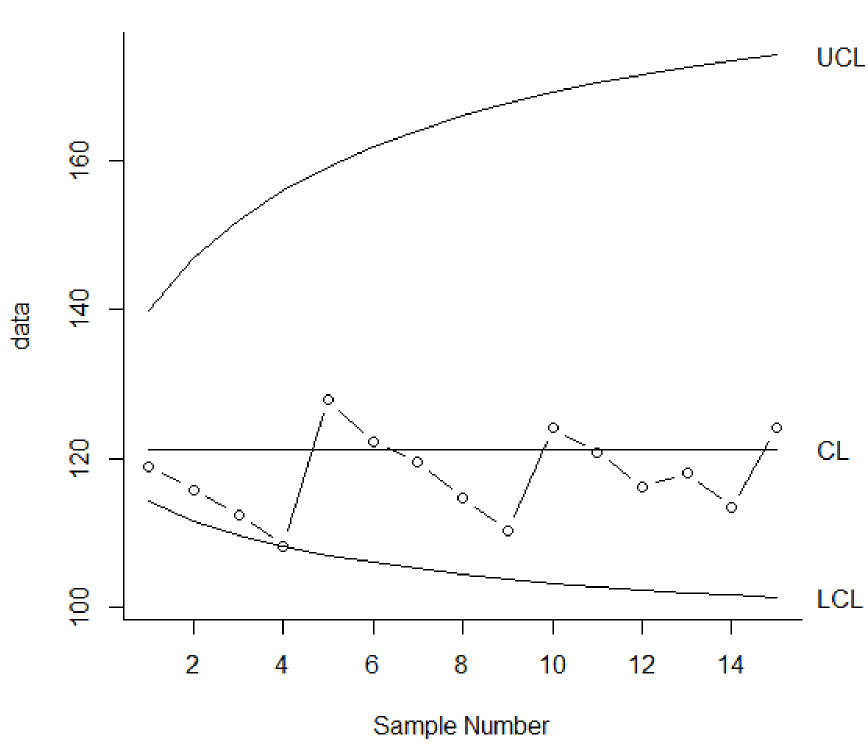

Figure 6 - 9: Individual EWMA control chart for the Failure Time Intervals of the third aircraft dataset assuming Lindley distribution for the data.

6.9 Control Charts for Individual Observations from the One-Parameter Lindley Distribution with the Scaled Weighted Variance Method

So far, we have presented and investigated Shewhart-type and EWMA control charts for individual observations from the one-parameter Lindley distribution using the skewness correction method proposed by Chan and Cui (2003). There are, however, other methods, too, for taking into consideration the distribution's skewness. One such method is the scaled weighted variance method proposed by Castagliola (2000). This method is going to be used in the following sections for constructing and investigating the performance of individual observations control charts and individual EWMA control charts for the one-parameter Lindley distribution and the resulting charts will be compared with those constructed so far.

6.9.1. Construction of Shewhart-type Control Charts for Individual Observations from a Process Following the One-Parameter Lindley Distribution Using the Scaled Weighted Variance Method

According to the method by Castagliola (2000), the construction procedure is the following: the central line is placed at the mean of the one-parameter Lindley distribution, which is computed using equation (3-3), while the control limits are placed around the mean at two different multiples of the standard deviation of the one-parameter Lindley distribution, which is computed using equation (3-4). These multiples are functions of appropriate values of the quantiles of the standardized Normal distribution, the probability of type I error or false alarm rate, α, and the cumulative distribution function of the one-parameter Lindley distribution, which is computed using equation (3-2). More specifically, the lower control limit is defined as

$LCL = \mu - \sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)\sigma$, while the upper control limit is defined as $UCL = \mu + \sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)\sigma$.

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the one-parameter Lindley control chart are as follows.

$$UCL = \frac{\theta+2}{\theta(\theta+1)} + \sqrt{\frac{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}$$

$$CL = \frac{\theta+2}{\theta(\theta+1)} \qquad (6\text{-}12)$$

$$LCL = \frac{\theta+2}{\theta(\theta+1)} - \sqrt{\frac{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}$$

6.9.2. Performance Investigation for the Individual One-Parameter Lindley Control Charts Constructed With the Scaled Weighted Variance Method

As performance measures of the chart we constructed above we will use the ARL_0 and ARL_1 values as in Section 6.4. So we will use again the equations (6-5) and (6-6) with $F_{in}(x)$ being the cumulative distribution function of the one-parameter Lindley distribution in equation (3-2) with in-control parameter, $F_{out}(x)$ being the cumulative distribution function for the distribution of concern with out-of-control parameter given by $\theta_{new} = \frac{1-(\mu_0+k\sigma)+\sqrt{(\mu_0+k\sigma)^2+6(\mu_0+k\sigma)+1}}{2(\mu_0+k\sigma)}$ (as earlier) and the control limits computed with equation (6-12) in both cases. Using the above formulas we obtain Table 6-12 which shows the in-control and out-of-control ARL values for the individual one-parameter Lindley control chart with scaled weighted variance for various values of the parameter θ of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. A significance level equal to the most commonly used value of 0.27% has been chosen, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
-3	2.4636	2.4543	2.2428	2.1991	2.1825	2.1714	2.0842	2.0361
-2.8	2.9644	2.7846	2.7352	2.7284	2.6910	2.6242	2.5703	2.5273
-2.6	3.9804	3.9732	3.7220	3.6848	3.6417	3.1289	3.1250	3.0848
-2.4	4.6482	4.5796	4.4693	4.3312	4.2893	4.1275	4.0089	3.8884
-2.2	4.8225	4.6420	4.6220	4.5404	4.5264	4.4634	4.1078	4.0932
-2	5.9371	5.8269	5.8197	5.7806	5.5935	5.2248	5.1288	5.1284
-1.8	6.3484	6.2319	6.1572	6.1028	6.0907	5.8268	5.5069	5.4448
-1.6	6.9012	6.8725	6.8037	6.7968	6.7704	6.4028	6.1719	6.0309
-1.4	9.8637	9.8408	9.6073	9.4893	9.4868	9.3757	9.3212	9.1206
-1.2	10.9075	10.5369	10.5125	10.5028	10.2546	10.1884	10.1648	10.1035
-1	12.9334	12.8486	12.7577	12.6208	12.4648	12.3709	12.1223	12.1093
-0.8	22.8419	22.1484	21.8480	21.2637	20.9687	20.7314	20.5778	20.2412
-0.6	39.3204	37.8645	37.2464	36.0399	35.4309	34.9336	34.6270	33.9315
-0.4	63.8640	61.6398	60.6960	57.9375	57.8448	57.7068	57.1881	55.6487
-0.2	139.5573	136.8450	135.6901	133.3715	132.2026	131.2532	130.6337	129.2644
0	370.3084	370.7530	371.4096	372.3759	373.5784	371.8484	370.9643	370.4880
0.2	130.9160	132.2463	132.8082	133.6217	134.5527	136.1202	136.8087	138.0898
0.4	57.6407	59.6378	60.0345	60.5784	61.1544	62.0128	62.3241	62.7812
0.6	37.5726	37.8257	37.9308	39.1287	39.4423	39.8070	39.8981	39.9346
0.8	25.0578	25.4648	25.6007	25.7518	25.8140	25.8935	25.9718	25.9880
1	16.2335	16.3122	16.4684	16.5375	16.5379	16.6026	16.6073	16.6451
1.2	14.2341	14.4808	14.5140	14.5442	14.6408	14.6486	14.6846	14.6873
1.4	12.4453	12.6933	12.7845	12.9125	12.9326	12.9637	12.9707	12.9757
1.6	8.8428	9.0937	9.1841	9.3373	9.3937	9.4143	9.4219	9.4312
1.8	7.3964	7.6277	7.7186	7.8751	7.9373	7.9759	7.9936	8.0046
2	7.0346	7.2593	7.3489	7.5046	7.5712	7.6155	7.6373	7.6428
2.2	5.5468	5.7522	5.8484	6.0031	6.0710	6.1284	6.1445	6.1842
2.4	4.8084	5.0184	5.1022	5.2543	5.3228	5.3717	5.3996	5.4457
2.6	3.5087	3.7151	3.7970	3.9364	4.0146	4.0642	4.0932	4.1442
2.8	2.9372	3.1431	3.2236	3.3704	3.4371	3.4879	3.5175	3.5718
3	2.0484	2.2557	2.3352	2.4899	2.5469	2.5968	2.6264	2.6840

Table 6 - 12: ARL values for individual one-parameter Lindley control charts with scaled weighted variance, with $\alpha = 0.0027$.

Comparison of Tables 6-12 and 6-2 reveals the improvement in the performance of the chart when the scaled weighted variance method is used instead of the skewness corrected limits, since the in-control ARL values when the scaled weighted variance method is used are greater by much more than 1% than the corresponding ones when the skewness correction is used and all the out-of-control ARL values for the scaled weighted variance method are smaller than the corresponding ones for the skewness correction method with almost all the differences being greater than 5%. Comparison of the ARL values for positive and negative shifts reveals that the ARL values for positive shifts are mostly larger than the ones for the negative shifts. The only cases for which ARL values for negative shifts are bigger than the corresponding ones for positive shifts are the cases of smaller θ values (equal to or less than 62) in conjunction with very small or very large shift sizes (equal to or smaller than 0.6 and equal to or larger than 2.6 standard deviation units).

6.9.3. Construction of the EWMA Control Charts For Individual Observations from the One-Parameter Lindley Distribution Using the Scaled Weighted Variance Method

The construction of the individual EWMA one-parameter Lindley control charts is going to be done here based on equation (2-3) for the traditional EWMA control charts using the scaled weighted variance method proposed by Castagliola (2000). More specifically, the procedure for the construction of the proposed control chart is as follows: in equation (2-3), L will be replaced by $\sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)$ for the lower control limit and $\sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)$ for the upper control limit, with μ being the mean of the one-parameter Lindley distribution, which is computed using equation (3-3), and $F_X(x)$ is its cumulative distribution function given by equation (3-2). For the construction of the EWMA control charts we will also need the standard deviation of the one-parameter Lindley distribution computed from equation (3-4).

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the one-parameter Lindley EWMA control chart are as follows.

$$UCL = \frac{\theta+2}{\theta(\theta+1)} + \sqrt{\frac{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = \frac{\theta+2}{\theta(\theta+1)}$$

$$LCL = \frac{\theta+2}{\theta(\theta+1)} - \sqrt{\frac{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

(6-13)

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our one-parameter Lindley distribution.

6.9.4. Performance Investigation for the Individual EWMA One-Parameter Lindley Control Charts Constructed With the Scaled Weighted Variance Method

In order to investigate the performance of the proposed individual EWMA chart with the scaled weighted variance method, we will use the ARL, computed with equation (6-10). For the transient probabilities in (6-9) the cumulative distribution function for the one-parameter Lindley distribution, i.e. equation (3-2), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equation (6-13) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = \frac{\theta+2}{\theta(\theta+1)} + \sqrt{\frac{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = \frac{\theta+2}{\theta(\theta+1)} - \sqrt{\frac{\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}{1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

(6-14)

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (3-3) and (3-4) in terms of the distribution's parameter, as earlier.

Using those formulae we get Tables 6-13, 6-14 and 6-15, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the one-parameter Lindley distribution for various values of its parameter θ and for various values of k which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 6-13 contains the ARL values for λ=0.3 for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=150, above which ARL increases very slightly. As a result, the suggested value of m for the computation of ARL in the formulae above is m=150. Therefore, Tables 6-14 and 6-15 show the ARL values for m=150 for various values of λ for positive and negative shifts, respectively.

m	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
50	0	370.0599	370.0892	370.1005	370.1203	370.1255	370.1291	370.1294	370.1470
	0.2	131.7207	131.7432	131.7518	131.7670	131.7737	131.7786	131.7816	131.7874
	0.5	42.4078	42.4170	42.4205	42.4267	42.4294	42.4314	42.4326	42.4350
	1	9.3044	9.3070	9.3080	9.3098	9.3106	9.3122	9.3123	9.3126
	1.5	6.1720	6.1739	6.1747	6.1760	6.1764	6.1770	6.1773	6.1778
	2	4.6498	4.6517	4.6524	4.6537	4.6543	4.6548	4.6550	4.6556
	2.5	3.7897	3.7916	3.7924	3.7938	3.7944	3.7949	3.7951	3.7957
	3	3.2465	3.2486	3.2494	3.2509	3.2516	3.2521	3.2524	3.2530
70	0	379.2972	379.3602	379.3844	379.4269	379.4458	379.4597	379.4680	379.4844
	0.2	138.7702	138.8126	138.8289	138.8575	138.8703	138.8795	138.8851	138.8962
	0.5	45.3247	45.3387	45.3441	45.3535	45.3577	45.3608	45.3626	45.3643
	1	9.9590	9.9624	9.9637	9.9640	9.9671	9.9678	9.9683	9.9692
	1.5	6.3990	6.4012	6.4020	6.4035	6.4042	6.4047	6.4050	6.4056
	2	4.7571	4.7591	4.7599	4.7614	4.7620	4.7625	4.7628	4.7634
	2.5	3.8509	3.8530	3.8538	3.8553	3.8560	3.8565	3.8568	3.8574
	3	3.2861	3.2882	3.2891	3.2906	3.2912	3.2919	3.2922	3.2929
90	0	382.4212	382.4979	382.5274	382.5792	382.6022	382.6190	382.6292	382.6492
	0.2	140.9614	141.0120	141.0301	141.0635	141.0784	141.0893	141.0958	141.1087
	0.5	46.1227	46.1482	46.1542	46.1646	46.1692	46.1726	46.1746	46.1786
	1	10.1257	10.1292	10.1406	10.1431	10.1442	10.1450	10.1454	10.1464
	1.5	6.4636	6.4648	6.4659	6.4684	6.4691	6.4696	6.4699	6.4706
	2	4.7896	4.7917	4.7926	4.7940	4.7947	4.7952	4.7955	4.7961
	2.5	3.8706	3.8727	3.8736	3.8751	3.8758	3.8763	3.8764	3.8772
	3	3.2994	3.3016	3.3024	3.3040	3.3047	3.3053	3.3056	3.3063
120	0	393.1243	393.1733	393.2019	393.2191	393.2531	400.9754	401.1601	401.2312
	0.2	147.9416	147.9639	147.9802	147.9900	148.0093	152.3802	152.4758	152.5125
	0.5	48.5277	48.5336	48.5380	48.5406	48.5457	49.7025	49.7251	49.7338
	1	10.6767	10.6780	10.6789	10.6795	10.6806	10.8568	10.8612	10.8630
	1.5	6.6786	6.6794	6.6800	6.6804	6.6812	6.7281	6.7307	6.7317
	2	4.9069	4.9076	4.9082	4.9085	4.9091	4.9250	4.9273	4.9282
	2.5	3.9468	3.9476	3.9481	3.9484	3.9491	3.9538	3.9561	3.9570
	3	3.3545	3.3553	3.3559	3.3562	3.3564	3.3569	3.3589	3.3598
150	0	409.8641	410.1207	410.2195	410.3935	410.4710	410.5277	410.5619	410.6294
	0.2	156.8980	157.0172	157.0631	157.1436	157.1794	157.2055	157.2212	157.2524
	0.5	50.7476	50.7727	50.7824	50.7993	50.8068	50.8122	50.8155	50.8221
	1	12.0018	12.0064	12.0082	12.0124	12.0128	12.0128	12.0144	12.0157
	1.5	6.7644	6.7693	6.7703	6.7722	6.7730	6.7736	6.7740	6.7747
	2	4.9396	4.9419	4.9429	4.9445	4.9453	4.9458	4.9462	4.9468
	2.5	3.9607	3.9630	3.9639	3.9643	3.9648	3.9655	3.9672	3.9679
	3	3.3602	3.3625	3.3634	3.3645	3.3648	3.3651	3.3659	3.3675
180	0	417.7751	418.1054	418.2328	418.4572	418.5572	418.6303	418.6745	418.7617
	0.2	160.5334	160.6736	160.7275	160.8222	160.8643	160.8951	160.9126	160.9502
	0.5	51.5063	51.5334	51.5438	51.5620	51.5701	51.5760	51.5796	51.5864
	1	12.1019	12.1067	12.1085	12.1222	12.1228	12.1243	12.1249	12.1262
	1.5	6.7929	6.7956	6.7967	6.7986	6.7994	6.8000	6.8004	6.8012
	2	4.9496	4.9520	4.9529	4.9546	4.9553	4.9559	4.9562	4.9569
	2.5	3.9653	3.9676	3.9685	3.9702	3.9709	3.9715	3.9719	3.9725
	3	3.3626	3.3650	3.3659	3.3676	3.3684	3.3689	3.3693	3.3700
210	0	424.8263	425.2306	425.3864	425.6414	425.7840	425.8737	425.9278	426.0348
	0.2	163.4993	163.6579	163.7188	163.8260	163.8736	163.9085	163.9295	163.9709
	0.5	52.0737	52.1023	52.1222	52.1225	52.1410	52.1472	52.1510	52.1584
	1	12.1729	12.1778	12.1797	12.1831	12.1845	12.1856	12.1863	12.1876
	1.5	6.8122	6.8129	6.8149	6.8168	6.8177	6.8183	6.8187	6.8195
	2	4.9564	4.9588	4.9597	4.9614	4.9621	4.9627	4.9630	4.9637
	2.5	3.9684	3.9707	3.9717	3.9733	3.9741	3.9746	3.9750	3.9757
	3	3.3642	3.3644	3.3675	3.3692	3.3700	3.3706	3.3709	3.3716
240	0	431.1934	431.6707	431.8550	432.1799	432.3249	432.4312	432.4951	432.6216
	0.2	165.9872	166.1620	166.2292	166.3474	166.4000	166.4384	166.4616	166.5073
	0.5	52.5212	52.5509	52.5623	52.5824	52.5912	52.5977	52.6017	52.6094
	1	12.2277	12.2327	12.2346	12.2380	12.2395	12.2406	12.2412	12.2426
	1.5	6.8251	6.8279	6.8290	6.8309	6.8318	6.8324	6.8328	6.8336
	2	4.9616	4.9640	4.9644	4.9650	4.9674	4.9680	4.9683	4.9690
	2.5	3.9708	3.9732	3.9741	3.9757	3.9765	3.9771	3.9774	3.9781
	3	3.3655	3.3678	3.3688	3.3705	3.3712	3.3718	3.3722	3.3729

Table 6 - 13: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (λ=0.3) with scaled weighted variance, with α = 0.0027.

λ	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
λ=0.05	0	372.5690	372.6485	372.6791	372.7327	372.7565	372.7739	372.7844	372.8051
	0.2	94.4349	94.5197	94.5523	94.6096	94.6351	94.6449	94.6537	94.6870
	0.4	42.6846	42.7312	42.7490	42.7803	42.7941	42.8043	42.8104	42.8224
	0.6	22.3993	22.4225	22.4314	22.4470	22.4539	22.4590	22.4620	22.4680
	0.8	15.7555	15.7680	15.7728	15.7812	15.7850	15.7878	15.7894	15.7927
	1	10.6034	10.6122	10.6141	10.6193	10.6216	10.6233	10.6243	10.6263
	1.5	5.2154	5.2190	5.2204	5.2229	5.2240	5.2248	5.2253	5.2263
	2	5.0339	5.0365	5.0376	5.0394	5.0402	5.0408	5.0412	5.0419
	2.5	4.1815	4.1839	4.1849	4.1865	4.1873	4.1879	4.1882	4.1889
	3	3.7123	3.7147	3.7156	3.7174	3.7181	3.7187	3.7191	3.7198
λ=0.08	0	377.3317	377.4331	377.4720	377.5404	377.5709	377.5931	377.6065	377.6329
	0.2	100.8241	100.9438	100.9899	101.0708	101.1068	101.1231	101.1489	101.1802
	0.4	44.4889	44.5412	44.5612	44.5962	44.6128	44.6232	44.6300	44.6436
	0.6	22.4888	22.5125	22.5216	22.5375	22.5445	22.5497	22.5528	22.5589
	0.8	15.1485	15.1606	15.1652	15.1733	15.1769	15.1796	15.1812	15.1843
	1	9.7519	9.7591	9.7619	9.7648	9.7690	9.7705	9.7715	9.7734
	1.5	4.3263	4.3297	4.3312	4.3334	4.3345	4.3353	4.3358	4.3367
	2	4.2595	4.2621	4.2631	4.2644	4.2648	4.2650	4.2658	4.2675
	2.5	3.5163	3.5187	3.5196	3.5212	3.5221	3.5227	3.5230	3.5238
	3	3.1250	3.1275	3.1284	3.1401	3.1409	3.1415	3.1419	3.1426
λ=0.10	0	384.0329	384.2007	384.2653	384.3789	384.4294	384.4644	384.4886	384.5326
	0.2	106.4964	106.6257	106.6754	106.7625	106.8012	106.8297	106.8467	106.8804
	0.4	46.6225	46.6820	46.7048	46.7447	46.7625	46.7755	46.7833	46.7987
	0.6	23.1077	23.1227	23.1423	23.1592	23.1644	23.1721	23.1754	23.1819
	0.8	15.1870	15.1993	15.2040	15.2123	15.2160	15.2187	15.2203	15.2235
	1	9.5700	9.5773	9.5801	9.5849	9.5871	9.5887	9.5897	9.5916
	1.5	4.0304	4.0339	4.0352	4.0376	4.0387	4.0394	4.0399	4.0409
	2	3.9820	3.9847	3.9857	3.9876	3.9884	3.9890	3.9894	3.9901
	2.5	3.2712	3.2736	3.2745	3.2763	3.2771	3.2776	3.2780	3.2787
	3	2.9193	2.9217	2.9227	2.9244	2.9253	2.9259	2.9262	2.9270
λ=0.12	0	386.1851	386.3626	386.4309	386.5510	386.6045	386.6436	386.6471	386.7126
	0.2	107.3786	107.5079	107.5577	107.6450	107.6839	107.7122	107.7294	107.7631
	0.4	46.7178	46.7756	46.7977	46.8364	46.8539	46.8645	46.8741	46.8890
	0.6	22.9612	22.9853	22.9946	23.0107	23.0179	23.0231	23.0263	23.0325
	0.8	14.9602	14.9720	14.9765	14.9844	14.9880	14.9905	14.9921	14.9951
	1	9.3188	9.3258	9.3284	9.3331	9.3352	9.3367	9.3377	9.3395
	1.5	3.7925	3.7959	3.7972	3.7995	3.8006	3.8012	3.8018	3.8027
	2	3.7765	3.7792	3.7802	3.7820	3.7829	3.7835	3.7839	3.7846
	2.5	3.0942	3.0967	3.0976	3.0994	3.1002	3.1008	3.1012	3.1019
	3	2.7652	2.7677	2.7686	2.7704	2.7712	2.7718	2.7722	2.7730
λ=0.15	0	389.2340	389.4334	389.5101	389.6451	389.7053	389.7492	389.7757	389.8281
	0.2	108.3412	108.4738	108.5249	108.6145	108.6543	108.6834	108.7010	108.7356
	0.4	46.4360	46.4915	46.5129	46.5502	46.5648	46.5789	46.5862	46.6006
	0.6	22.4363	22.4587	22.4673	22.4824	22.4890	22.4939	22.4969	22.5027
	0.8	14.4361	14.4469	14.4510	14.4583	14.4615	14.4639	14.4653	14.4681
	1	8.8374	8.8438	8.8462	8.8505	8.8524	8.8538	8.8547	8.8563
	1.5	3.4185	3.4217	3.4230	3.4252	3.4261	3.4269	3.4273	3.4282
	2	3.4788	3.4812	3.4823	3.4841	3.4849	3.4855	3.4858	3.4864
	2.5	2.8491	2.8515	2.8524	2.8541	2.8549	2.8555	2.8558	2.8565
	3	2.5578	2.5602	2.5612	2.5629	2.5636	2.5642	2.5646	2.5653
λ=0.20	0	400.2532	400.5307	400.6377	400.8259	400.9098	400.9712	401.0082	401.0812
	0.2	113.4129	113.5648	113.6236	113.7270	113.7729	113.8065	113.8268	113.8647
	0.4	47.8412	47.8976	47.9192	47.9571	47.9740	47.9863	47.9937	48.0083
	0.6	22.7226	22.7441	22.7524	22.7648	22.7733	22.7780	22.7808	22.7864
	0.8	14.3824	14.3926	14.3965	14.4034	14.4064	14.4086	14.4100	14.4126
	1	8.6439	8.6499	8.6722	8.6763	8.6781	8.6794	8.6802	8.6818
	1.5	3.1855	3.1886	3.1898	3.1919	3.1929	3.1936	3.1940	3.1949
	2	3.2606	3.2631	3.2641	3.2647	3.2659	3.2673	3.2677	3.2684
	2.5	2.6403	2.6409	2.6412	2.6420	2.6545	2.6569	2.6578	2.6596
	3	2.3849	2.3873	2.3883	2.3901	2.3909	2.3915	2.3918	2.3926

Table 6 - 14: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (m=150) with scaled weighted variance, with $\alpha = 0.0027$, for various positive shifts

λ	k	θ=48	θ=57	θ=62	θ=75	θ=84	θ=93	θ=100	θ=120
λ=0.05	0	372.5690	372.6485	372.6791	372.7327	372.7565	372.7739	372.7844	372.8051
	-0.2	95.2349	95.2185	95.2102	95.1964	95.1776	95.1251	95.1220	95.0481
	-0.4	42.8329	42.8239	42.8194	42.8128	42.8015	42.7782	42.7650	42.7304
	-0.6	22.8121	22.8082	22.8058	22.8017	22.7961	22.7835	22.7763	22.7576
	-0.8	16.0196	16.0168	16.0154	16.0121	16.0099	16.0026	15.9985	15.9878
	-1	10.9595	10.9461	10.9108	10.8369	10.6328	10.6061	10.5129	10.0737
	-1.5	5.9578	5.9271	5.8481	5.7918	5.6883	5.4623	5.1277	5.1258
	-2	4.3333	4.1996	4.1969	4.0184	3.9418	3.8787	3.7493	3.7235
	-2.5	2.9052	2.8199	2.7471	2.5700	2.5683	2.4958	2.4035	2.2109
	-3	3.0702	3.0085	2.9203	2.8252	2.7998	2.5610	2.2273	2.1728
λ=0.08	0	377.6329	377.6065	377.5931	377.5709	377.5404	377.4720	377.4331	377.3317
	-0.2	100.5359	100.5125	100.5005	100.4808	100.4537	100.3930	100.3583	100.2683
	-0.4	45.7314	45.7198	45.7129	45.7041	45.6907	45.6435	45.6406	45.5988
	-0.6	23.9723	23.9671	23.9644	23.9600	23.9540	23.9403	23.9326	23.9123
	-0.8	16.7794	16.7763	16.7747	16.7721	16.7685	16.7605	16.7559	16.7439
	-1	10.9395	10.9121	10.8719	10.5128	10.4064	10.3990	10.3707	10.1577
	-1.5	5.9648	5.9606	5.9322	5.7318	5.4648	5.1785	5.1619	5.0958
	-2	4.3165	4.3123	4.2791	4.1298	4.1019	4.0681	3.9885	3.9319
	-2.5	3.1697	2.9308	2.8287	2.6128	2.4077	2.2649	2.2497	2.2373
	-3	3.0953	2.8526	2.7915	2.7045	2.7005	2.5008	2.4564	2.2777
λ=0.10	0	384.5326	384.4886	384.4644	384.4294	384.3789	384.2653	384.2007	384.0329
	-0.2	106.7338	106.7000	106.6829	106.6545	106.6156	106.5283	106.4786	106.3494
	-0.4	49.3008	49.2849	49.2769	49.2635	49.2452	49.2041	49.1806	49.1296
	-0.6	25.4620	25.4558	25.4527	25.4475	25.4404	25.4243	25.4152	25.3914
	-0.8	17.8215	17.8177	17.8158	17.8126	17.8082	17.7984	17.7928	17.7783
	-1	10.9556	10.8452	10.6932	10.4175	10.2738	10.2695	10.2076	10.0722
	-1.5	5.9909	5.9488	5.8374	5.6083	5.3181	5.2926	5.2804	5.1860
	-2	4.5777	4.4573	4.4531	4.4362	4.1833	4.1016	3.6986	3.6520
	-2.5	3.0972	2.8721	2.8365	2.7641	2.6589	2.3438	2.2912	2.2456
	-3	3.0912	2.9712	2.7616	2.6348	2.6260	2.1931	2.1462	2.1297
λ=0.12	0	386.7126	386.6471	386.6436	386.6045	386.5510	386.4309	386.3626	386.1851
	-0.2	109.4062	109.3682	109.3490	109.3170	109.2734	109.1753	109.1295	108.9746
	-0.4	51.7269	51.7075	51.6977	51.6812	51.6590	51.6088	51.5802	51.5058
	-0.6	26.5270	26.5200	26.5165	26.5106	26.5026	26.4846	26.4743	26.4476
	-0.8	18.2234	18.2196	18.2176	18.2144	18.2100	18.2000	18.1943	18.1795
	-1	10.9879	10.9296	10.7679	10.5899	10.4469	10.3706	10.2780	10.1795
	-1.5	5.9246	5.8949	5.8498	5.5632	5.4945	5.3367	5.0928	5.0143
	-2	4.5676	4.3580	4.3426	4.2459	4.1459	4.1450	4.1259	4.0879
	-2.5	3.1612	3.0532	3.0122	2.8535	2.6982	2.6726	2.6371	2.4439
	-3	3.0779	2.9816	2.7262	2.6372	2.3126	2.2600	2.2245	2.1262
λ=0.15	0	389.8281	389.7757	389.7492	389.7053	389.6451	389.5101	389.4334	389.2340
	-0.2	113.0361	112.9902	112.9670	112.9284	112.8757	112.7573	112.6900	112.5152
	-0.4	55.0803	55.0536	54.9771	54.9325	54.8717	54.7352	54.6577	54.4564
	-0.6	27.6962	27.6875	27.6831	27.6758	27.6458	27.6433	27.6304	27.5970
	-0.8	17.8758	17.8723	17.8705	17.8675	17.8634	17.8543	17.8490	17.8354
	-1	10.9582	10.8536	10.8304	10.7251	10.6905	10.6848	10.4437	10.1917
	-1.5	5.9637	5.8391	5.8208	5.6019	5.5363	5.3407	5.0709	5.0144
	-2	4.5296	4.4926	4.4212	4.2855	4.1798	4.1554	3.9644	3.7201
	-2.5	3.1842	3.1446	3.0468	3.0359	2.6450	2.4533	2.3288	2.3248
	-3	2.7747	2.6597	2.5449	2.4077	2.3655	2.3312	2.2094	2.1221
λ=0.20	0	401.0812	401.0082	400.9712	400.9098	400.8259	400.6377	400.5307	400.2532
	-0.2	116.8223	116.7492	116.7121	116.6507	116.5648	116.3786	116.2717	115.9946
	-0.4	57.0569	57.0039	55.0401	55.0177	54.9871	54.9182	54.8791	54.7772
	-0.6	29.1870	29.1638	29.1521	29.1226	29.1060	29.0462	29.0121	28.3237
	-0.8	20.4474	20.4417	20.4388	20.4339	20.4274	20.4126	20.4041	20.3821
	-1	10.9129	10.8977	10.6551	10.6208	10.5089	10.4558	10.1263	10.0065
	-1.5	5.9268	5.8573	5.5176	5.3612	5.3455	5.3420	5.2126	5.0480
	-2	4.3912	4.3677	4.3042	3.9156	3.8090	3.7685	3.7584	3.6399
	-2.5	3.1738	3.1223	2.7898	2.7775	2.4141	2.3961	2.3771	2.2594
	-3	2.9729	2.7799	2.7600	2.7171	2.5503	2.4787	2.2084	2.1251

Table 6 - 15: ARL values for individual EWMA control charts for the one-parameter Lindley distribution (m=150) with scaled weighted variance, with $\alpha = 0.0027$, for various negative shifts

Comparing those two tables, we observe that the proposed control chart detects both positive and negative shifts well, but there are some differences in the ARL values between those two tables. Most of the ARL values for the negative shifts are bigger than the corresponding ones for the positive shifts. The out-of-control ARL values for the positive shifts are bigger than the corresponding ones for the negative shifts for cases of larger θ values in conjunction with large shift sizes. This makes sense because the larger the θ value the smaller the observation from the one-parameter Lindley distribution, which means that for a large negative shift the possibility of the shifted value getting out of control becomes larger and, therefore, the chart detects it more quickly.

Comparing Tables 6-14 and 6-15 with Tables 6-4 and 6-5 we see the improvement in the performance of the individual EWMA control chart when using the scaled weighted variance method instead of the skewness correction. The in-control ARL values are all larger when using the scaled weighted variance instead of the skewness correction method and all the out-of-control ARL values are smaller than the corresponding ones resulting from the skewness correction method and these are valid either the shift is positive or negative. Moreover, the differences are almost all higher than 5% for both positive and negative shifts, so the improvement is significant.

<u>6.9.5 Example on the one-parameter Lindley individual Shewhart-type and EWMA control charts with scaled weighted variance using simulated data</u>

This section contains the illustration of the proposed control charts by means of simulated data generated from the distribution of concern. The case of real data will be presented in section 6.9.6. For the same data set of Table 6-9, we construct the individual Shewhart-type and EWMA one-parameter Lindley control charts with scaled weighted variance presented in Figures 6-10 and 6-11, using the most commonly used value for the significance level $\alpha = 0.27\%$, as mentioned earlier. As we can see in those graphs, both charts detect the out-of-control state of the process sooner than the corresponding charts with the skewness correction method presented earlier in Figures 6-2 and 6-3, respectively.

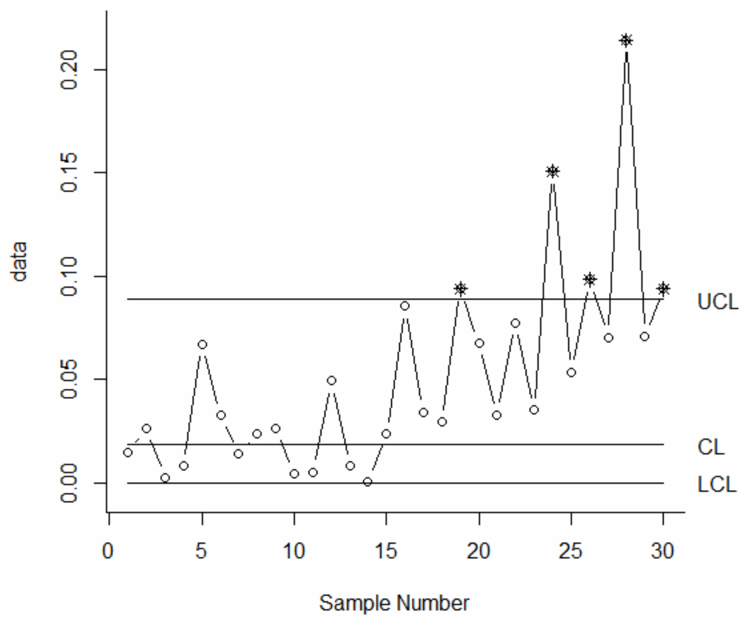

Figure 6 - 10: Individual one-parameter Lindley control chart with scaled weighted variance for the data set in Table 6-9 with a shift of one standard deviation unit in the process mean

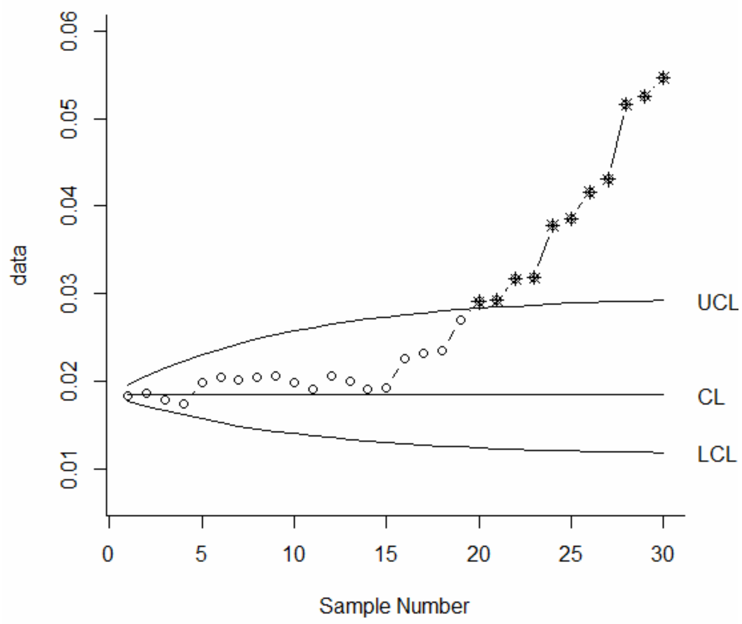

Figure 6 - 11: Individual EWMA one-parameter Lindley control chart with scaled weighted variance for the data set in Table 6-9 with a shift of one standard deviation unit in the process mean

6.9.6 Application of the one-parameter Lindley individual Shewhart-type and EWMA control charts with scaled weighted variance to real data

This section addresses the illustration of the proposed control charts through application to the same real data as in Tables 6-10 and 6-11. For the first case of the waiting times dataset, the individual one-parameter Lindley control chart with scaled weighted variance is presented in Figure 6-12 and it detects an out-of-control point which the other control charts seen so far had not detect. The individual EWMA one-parameter Lindley control chart with scaled weighted variance is shown in Figure 6-13. This chart does not present any out-of-control points, probably due to the inertia effect, we mentioned in Section 2.14.2. The value of λ=0.08 is quite small and does not give much weight to the present data and, therefore, the EWMA statistic is effected from the previous low values and does not react quickly to the shift in the opposite direction which the chart in Figure 6-12 detected.

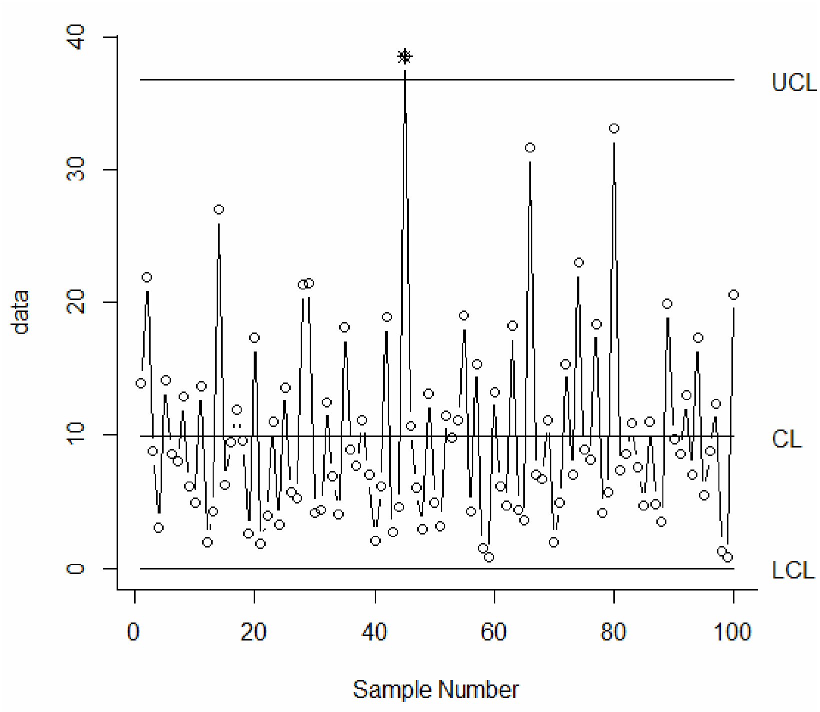

Figure 6 - 12: Individual one-parameter Lindley control chart with scaled weighted variance for the Waiting Times dataset

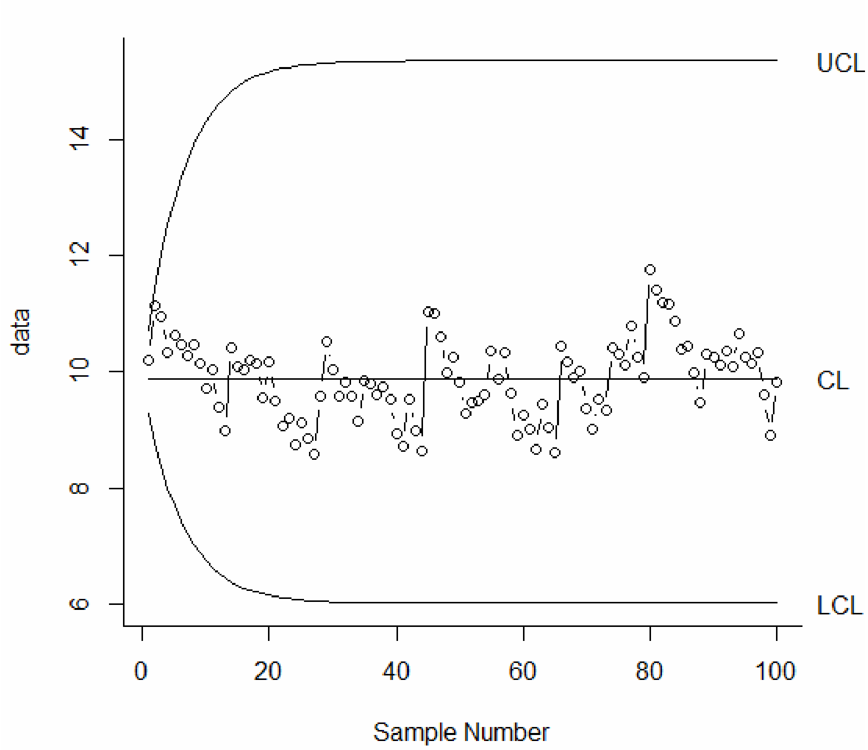

Figure 6 - 13: Individual EWMA one-parameter Lindley control chart with scaled weighted variance for the Waiting Times data set

For the case of the airplane air-conditioning failure times dataset, the corresponding individual one-parameter Lindley and EWMA one-parameter Lindley control charts with scaled weighted variance are presented in Figure 6-14 and Figure 6-15, respectively. The chart in Figure 6-14 detects the same out-of-control observation as the corresponding chart with the skewness correction, but the individual EWMA chart in Figure 6-15 does not detect that. This probably happened because of the inertia effect and the small value of λ=0.05 which gives small weight on the large present values to the opposite direction than the previous small ones. The EWMA chart, however, presents a point outside the lower control limit. This is an indication that a downwards shift occurred first and the EWMA chart which is sensitive to small shifts detected it, while all the other charts seen so far had not detected it.

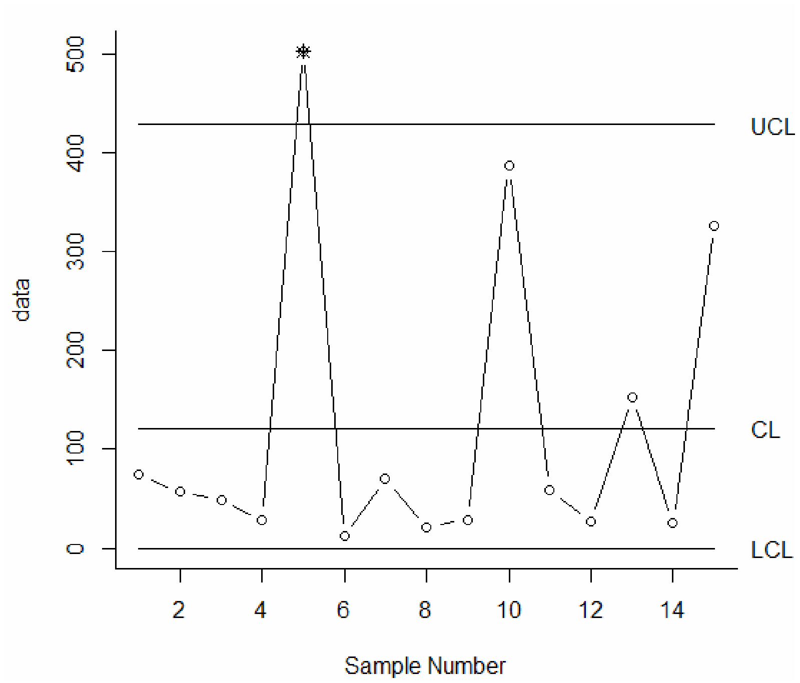

Figure 6 - 14: Individual one-parameter Lindley control chart with scaled weighted variance for the aircraft air-conditioning equipment failure dataset

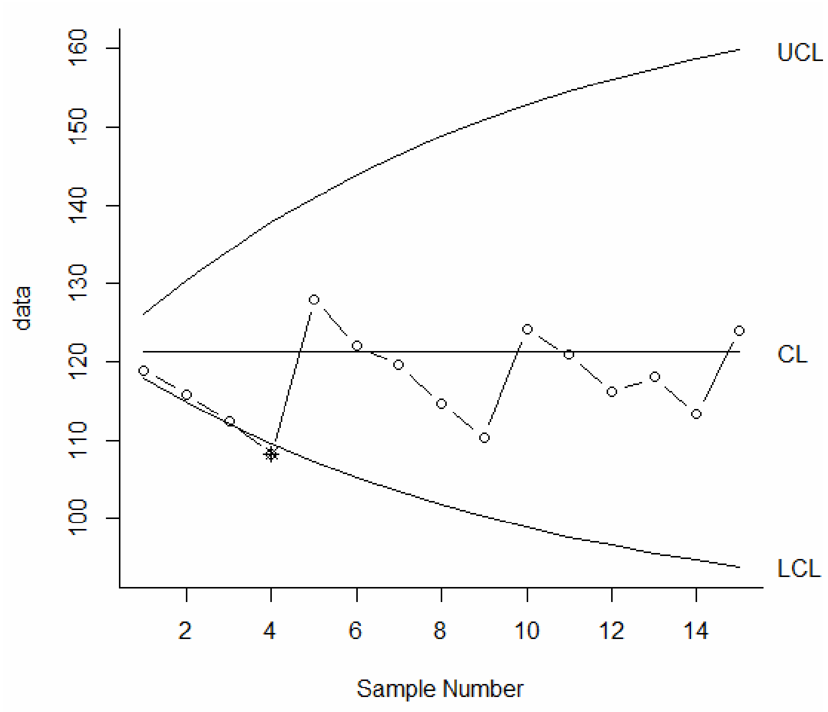

Figure 6 - 15: Individual EWMA one-parameter Lindley control chart with scaled weighted variance for the aircraft air-conditioning equipment failure dataset

6.10 Conclusions and Further Research

In this chapter probability-type, Shewhart-type and EWMA control charts have been constructed for monitoring individual observations from a process which is assumed to follow the one-parameter Lindley distribution for the theoretical scenario of known distributions' parameters. Two different methods for taking into account the distribution's skewness have been considered. The performance of the proposed control charts has been investigated for the cases of all the proposed control charts (probability-type, Shewhart-type and EWMA control charts with both skewness correction methods). Optimal design for the EWMA control chart has also been presented. The five types of proposed control charts have been illustrated with both simulated and real data.

The proposed control charts take into account the skewness of the distribution and this leads to a significant improvement of their performance as has been demonstrated along this chapter. The performance of the control charts seems to improve more when the scaled weighted variance method by Castagliola (2000) is used instead of the skewness correction method proposed by Chan and Cui (2003).

This study can also be applied to other Lindley-related distributions (generalizations, mixtures, transformations, etc.). Such an attempt is made in Chapter 7, where control charts are constructed for the two-parameter Lindley distribution by Shanker et al. (2013).

Moreover, for future research, the whole analysis can be extended to include supplementary runs rules for the detection of small shifts. For this purpose it would also be useful to construct CUSUM control charts for the one-parameter Lindley distribution, as well.

CHAPTER 7

CONTROL CHARTS FOR INDIVIDUAL OBSERVATIONS FROM THE TWO-PARAMETER LINDLEY DISTRIBUTION

7.1 Introduction

As pointed out in Chapter 3, Lindley-related distributions (extensions, modifications, mixtures) have various applications in our everyday lives, such as in medicine, genetics, epidemiology, biology, finance and actuarial sciences, ecology, meteorology, sociology, demography, agriculture, hydrology, geosciences, reliability and engineering, life testing and survival analysis, airborne systems and communications, environmental studies and modeling and describing of human mistakes, strikes, accidents, behavioural and emotional or IQ test scores and waiting times of customers in queues until service etc. As a result of the variety of its applications, it is important to develop control charts for detecting shifts in a process which follows a Lindley-related distribution.

In this chapter, the two-parameter Lindley distribution proposed by Shanker et al. (2013) is considered and the first part of it has already been published [Demertzi and Psarakis (2024)]. Probability-type, as well as Shewhart-type and EWMA control charts are constructed for individual observations from the chosen distribution using two different methods for taking into account its skewness when establishing the control limits of the Shewhart-type and EWMA charts. The performance of all the control charts proposed in this chapter is investigated and illustrated with both simulated and real datasets (same for each chart for the shake of comparisons). The whole analysis reveals the superiority of using skewness correction for the construction of the control charts against not using it, as well as the superiority of the scaled weighted variance method for taking into account the distribution’s skewness. The outline of this chapter is as follows: Section 7.2 deals with the construction of the probability control charts for individual observations from the two-parameter Lindley distribution, while section 7.3 describes the construction of Shewhart-type control charts for the case of using the skewness correction method proposed by Chan and Cui (2003). The performance of both charts is investigated in section 7.4, which reveals the superiority of the proposed Shewhart-type control charts. Section 7.5 presents the construction of EWMA control charts for individual observations form the two-parameter Lindley distribution using the same skewness correction method and the performance of these control charts is investigated in section 7.6, which reveals the superiority of the proposed control charts over EWMA charts without the skewness correction. Optimal design for the control charts of section 7.5 is discussed in section 7.7. Illustration of all the control charts proposed in the previous sections is provided in section 7.8 with both simulated and real data. Section 7.9 is dedicated to Shewhart-type and EWMA charts for individual observations from the two-parameter Lindley distribution using a different method for taking into consideration the distribution's skewness, namely the scaled weighted variance method proposed by Castagliola (2000). More specifically, subsections 7.9.1 and 7.9.2 discuss the construction and performance investigation, respectively, of the Shewhart-type charts with this method, while subsections 7.9.3 and 7.9.4 deal with the construction of the corresponding EWMA charts. Subsections 7.9.5 and 7.9.6 offer illustration of the proposed control charts with the scaled weighted variance method through application to the same simulated and real data, respectively, used in section 7.8 (for comparison reasons).

7.2 Probability-Type Control Charts for Individual Observations Following the Two-parameter Lindley Distribution

The control limits for the probability-type control chart for observations from the two-parameter Lindley distribution will be constructed in terms of the probability of type I error or false alarm rate, α, using our distribution of interest (see for example, Chang and Gan (1999) for the case of the modified geometric distribution). For this purpose we will need the quantile function of the two-parameter Lindley distribution, which is obtained in subsection 7.2.1.

7.2.1 The Quantile Function of the Two-parameter Lindley Distribution

For the case of using the probability of type I error to obtain the control charts for the two-parameter Lindley distribution we need the distribution's quantile function. Applying the methodology in Theorem 1 of Jodrá's (2010) paper, we can find a formula for the required quantile function in terms of the Lambert's W function [Corless et al. (1996)] as presented here.

The quantile function in general, is given by $Q_X(u)=F_X^{-1}(u)$, with u such as 0<u<1. For the case of the two-parameter Lindley distribution under study, we have:

$$F_X(x)=u\Rightarrow u=1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}\Rightarrow(\theta+r+r\theta x)e^{-\theta x}=(1-u)(\theta+r)\Rightarrow$$

$$\Rightarrow\left(\frac{\theta+r}{r}+\theta x\right)e^{-\theta x}=(1-u)\frac{\theta+r}{r}\Rightarrow-\left(\frac{\theta+r}{r}+\theta x\right)e^{-\frac{\theta+r}{r}-\theta x}=-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}\Rightarrow$$

$$\Rightarrow W\left(-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}\right)=-\left(\frac{\theta+r}{r}+\theta x\right)\Rightarrow$$

$$x=-\frac{\theta+r}{\theta r}-\frac{1}{\theta}W_{-1}\left(-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}\right) \tag{7-1}$$

It should be noted that we use the negative brunch of the Lambert's W function in the formula above. A detailed justification is provided below. By definition we have $\theta>0,\ r>-\theta,\ x>0$. In addition, $r>-\theta\Rightarrow\theta+r>0$. Now there are two possibilities r>0 and r<0.

For the first case $r>0\Rightarrow-\frac{\theta}{r}<0\Rightarrow-\frac{\theta}{r}-1<-1\Rightarrow e^{-\frac{\theta+r}{r}}<e^{-1}\Rightarrow-e^{-\frac{\theta+r}{r}}>-e^{-1}$ and $r>0\Rightarrow\frac{\theta+r}{r}>0$. Also $0<u<1\Rightarrow1-u>0$. Therefore, $-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}>-e^{-1}$.

For the second case $r<0\Rightarrow-\frac{\theta}{r}>0\Rightarrow-\frac{\theta}{r}-1>-1\Rightarrow e^{-\frac{\theta+r}{r}}>e^{-1}\Rightarrow-e^{-\frac{\theta+r}{r}}<-e^{-1}$ and $r<0\Rightarrow\frac{\theta+r}{r}<0$. Also $0<u<1\Rightarrow1-u>0$. Thus $-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}>-e^{-1}$. So in both cases $-(1-u)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}\in\left(-1/e,\infty\right)$.

Moreover, $\left.\begin{matrix}\theta>0\\x>0\end{matrix}\right\}\Rightarrow\theta x>0$. As a result, for the first case $r>0\Rightarrow-\frac{\theta}{r}<0\Rightarrow-\frac{\theta}{r}-1<-1\Rightarrow\frac{\theta+r}{r}>1\Rightarrow\frac{\theta+r}{r}+\theta x>1$. As for the second case, the inequality $\frac{\theta+r}{r}+\theta x>1$ holds only for r such that $x>-\frac{1}{r}$, since $\frac{\theta+r}{r}+\theta x>1\Rightarrow\frac{\theta}{r}+\theta x>0\Rightarrow\frac{\theta}{r}>-\theta x\overset{(\theta>0)}{\Rightarrow}\frac{1}{r}>-x\Rightarrow x>-\frac{1}{r}$. For every such r, all the above allow as to use the negative branch of the Lambert W function, considering its properties as presented in Section 2 of Jodrá's (2010) paper.

7.2.2 Control Limits of the Individual Probability-Type Two-Parameter Lindley Control Charts

In this subsection the computation of the control limits of the chart is presented in terms of the probability of type I error or false alarm rate, α. In order to do that we need to use the cumulative probability of the two-parameter Lindley distribution as presented in equation (3-7). The method is the following:
For a significance level α, we have

$$P\left(X<LCL\right)=\frac{\alpha}{2}$$

and

$$P\left(X<LCL\right)=1-\frac{\theta+r+r\theta\cdot LCL}{\theta+r}e^{-\theta\cdot LCL},\quad LCL>0,\quad\theta>0,\quad r>-\theta,$$

from which using equation (7-1) we obtain

$$1-\frac{\theta+r+r\theta\cdot LCL}{\theta+r}e^{-\theta\cdot LCL}=\frac{\alpha}{2}\Rightarrow LCL=-\frac{\theta+r}{\theta r}-\frac{1}{\theta}W_{-1}\left(-\left(1-\frac{\alpha}{2}\right)\frac{\theta+r}{r}e^{-\frac{\theta+r}{r}}\right),$$

where $W_{-1}\left(x\right)$ is the negative branch of the Lambert W function.
Similarly, for the upper control limit, we have

$$P\left(X>UCL\right)=\frac{\alpha}{2}$$

and

$$P\left(X>UCL\right)=1-P\left(X\leq UCL\right)=\frac{\theta+r+r\theta UCL}{\theta+r}e^{-\theta UCL},\quad\theta>0,\quad r>-\theta,$$

from which, using equation (7-1) once again, we get that

$$\frac{\theta + r + r\theta \cdot UCL}{\theta + r} e^{-\theta \cdot UCL} = \frac{\alpha}{2} \Rightarrow UCL = -\frac{\theta + r}{\theta r} - \frac{1}{\theta} W_{-1}\left(-\frac{\alpha}{2} \frac{\theta + r}{r} e^{-\frac{\theta + r}{r}} \right)$$

Similarly for the central line we obtain

$$CL = -\frac{\theta + r}{\theta r} - \frac{1}{\theta} W_{-1}\left(-0.5 \frac{\theta + r}{r} e^{-\frac{\theta + r}{r}} \right)$$

As a result from all the above, the control limits of the chart in terms of the probability of type I error, α, are as follows.

$$UCL_{\alpha} = -\frac{\theta + r}{\theta r} - \frac{1}{\theta} W_{-1}\left(-\frac{\alpha}{2} \frac{\theta + r}{r} e^{-\frac{\theta + r}{r}} \right)$$

$$CL_{\alpha} = -\frac{\theta + r}{\theta r} - \frac{1}{\theta} W_{-1}\left(-0.5 \frac{\theta + r}{r} e^{-\frac{\theta + r}{r}} \right) \quad , \quad \theta > 0, \quad r > -\theta \qquad (7\text{-}2)$$

$$LCL_{\alpha} = -\frac{\theta + r}{\theta r} - \frac{1}{\theta} W_{-1}\left(-\left(1 - \frac{\alpha}{2} \right) \frac{\theta + r}{r} e^{-\frac{\theta + r}{r}} \right)$$

7.3 Shewhart-Type Control Charts for Individual Two-Parameter Lindley-Distributed Observations

Here the individual two-parameter Lindley control charts are constructed based on the Shewhart-type individual control charts using the skewness correction as in Chan and Cui (2003). More specifically, the central line is placed at the mean of the two-parameter Lindley distribution, which is computed using equation (3-8), while the control limits are placed around the mean at L times its standard deviation (the square root of the quantity computed by equation (3-9)) plus c_4^* times its standard deviation, where $c_4^*(x) = \frac{\frac{4}{3}\left[sk(x)\right]}{1+0.2\left[sk(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient computed from equation (3-10). This means that the skewness correction for the two-parameter Lindley distribution will be

$$c_4^*(x) = \frac{8\left[2(\theta+r)^3 - \theta^3\right]\left(\theta^2+4\theta r+2r^2\right)^{3/2}}{3\left(\theta+4\theta r+2r^2\right)^3+0.24\left[2(\theta+r)^3-\theta^3\right]^2} \tag{7-3}$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the two-parameter Lindley control chart are as follows.

$$UCL = \frac{\theta+2r}{\theta(\theta+r)} + \left[L + c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}$$

$$CL = \frac{\theta+2r}{\theta(\theta+r)} \tag{7-4}$$

$$LCL = \frac{\theta+2r}{\theta(\theta+r)} + \left[-L + c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}$$

7.4 Performance Investigation for the Individual Two-parameter Lindley Control Charts

As a performance measure of the charts we just constructed, we can use the ARL_0 and ARL_1 values as in the previous chapter. The formulae for their computation will be

$$ARL_0 = \frac{1}{1 - F_{in}(UCL) + F_{in}(LCL)} \tag{7-5}$$

where $F_{in}(x)$ is the cumulative distribution function of the two-parameter Lindley distribution in equation (3-7) with in-control parameters and control limits as computed with equation (7-2) for the probability-type control charts or equations (7-4) and (7-3) for the Shewhart-type control charts and

$$ARL_1 = \frac{1}{1 - F_{out}(UCL) + F_{out}(LCL)} \tag{7-6}$$

where $F_{out}(x)$ is the cumulative distribution function for the distribution of concern with out-of-control parameters and same control limits as before. For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (3-8) and (3-9) in terms of the distribution's two parameters. The resulting values for them are given by $\theta_{new} = \frac{\sqrt{2}}{\sqrt{2}(\mu_0 + k\sigma) + \sqrt{(\mu_0 + k\sigma)^2 - \sigma_{new}^2}}$ and $r_{new} = \frac{\sqrt{2}(\mu_0 + k\sigma) + \sqrt{(\mu_0 + k\sigma)^2 - \sigma_{new}^2} - \sqrt{2}\sqrt{(\mu_0 + k\sigma)^2 - \sigma_{new}^2}}{\sqrt{2}(\mu_0 + k\sigma) + 2\sqrt{(\mu_0 + k\sigma)^2 - \sigma_{new}^2}}$.

Using the above formulas we obtain Table 7-1 and Table 7-2, which show the in-control and out-of-control ARL values for the individual probability-type and individual Shewhart-type control chart, respectively, for the two-parameter Lindley distribution for various values of the two parameters θ and r of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the probability-type control charts we have chosen a significance level equal to the most commonly used value of 0.27%, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

k	θ=48, r=54	θ=57, r=68	θ=62, r=75	θ=75, r=86	θ=84, r=92	θ=93, r=108	θ=100, r=114	θ=120, r=135
-3	3.7097	3.8882	3.9239	3.4675	3.9414	3.9370	3.9570	3.6987
-2.8	4.6828	4.2818	4.2410	4.4103	4.2842	4.7850	4.6041	4.6228
-2.6	4.8812	4.8232	4.8712	5.2018	5.3578	5.2253	5.4305	5.2035
-2.4	5.3481	5.3693	5.3424	5.4870	5.4668	5.3235	5.4455	5.3750
-2.2	6.5266	6.4505	6.3775	6.8272	6.9610	6.5316	6.8257	6.4175
-2	7.5189	7.5452	7.7870	7.8532	7.8491	7.5750	7.7510	7.5189
-1.8	8.8427	8.6828	8.4817	8.3486	8.9937	8.4873	8.8468	8.7954
-1.6	9.6484	9.4228	9.8628	9.2648	9.8680	9.7557	9.7593	9.5734
-1.4	10.8893	10.6875	10.6890	10.7562	10.8122	10.7890	10.8082	10.7824
-1.2	14.8728	14.5261	14.2086	14.2846	14.8276	14.4061	14.6086	14.7898
-1	18.8483	18.5712	18.7056	19.0856	18.5012	18.2386	19.0039	18.8483
-0.8	21.0459	21.0863	21.0205	21.0686	21.0957	21.0690	21.0326	21.0278
-0.6	35.3144	35.7536	35.1487	35.6824	35.5912	35.4184	35.2171	35.6207
-0.4	68.8793	66.5877	66.9504	66.3822	68.7273	66.0041	68.4025	66.0435
-0.2	202.3536	202.1264	200.7909	203.3778	204.5724	200.3095	200.1893	201.8153
0	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704
0.2	200.7918	199.9578	198.8480	200.9370	202.1457	198.6588	199.7332	200.5969
0.4	68.2787	64.3418	64.1493	64.5996	68.1235	64.4184	68.3683	64.7052
0.6	34.2857	34.2109	34.3700	34.6199	34.5482	34.2035	34.7353	34.9702
0.8	20.6998	20.4610	20.9084	20.0728	20.1273	20.8203	20.4066	20.3990
1	18.5961	18.3085	18.2419	18.5015	18.7310	18.4392	18.5303	18.5961
1.2	14.8719	14.1577	14.3707	14.9395	14.5128	14.9302	14.9322	14.5785
1.4	10.7957	10.8619	10.4648	10.7318	10.9668	10.9606	10.4681	10.5019
1.6	8.6198	8.3319	8.3784	8.7123	8.6891	8.7164	8.2077	8.0648
1.8	7.5128	7.7308	7.0373	7.9697	7.0497	7.6287	7.6045	7.6004
2	6.1915	6.1533	6.1445	6.1790	6.2094	6.1707	6.1828	6.1915
2.2	5.9975	5.5888	5.8753	5.7003	5.6882	5.8457	5.6037	5.6348
2.4	5.2088	5.3466	5.0361	5.1269	5.4226	5.4878	5.1463	5.0930
2.6	4.6121	4.9964	4.8433	4.9350	4.8272	4.9795	4.9428	4.8007
2.8	4.4266	4.4888	4.0260	4.2817	4.0482	4.4181	4.0719	4.0457
3	3.8068	3.7933	3.7901	3.8023	3.8031	3.7994	3.8037	3.8068

Table 7 - 1: ARL values for individual probability-type control charts for the two-parameter Lindley distribution, with $\alpha = 0.0027$.

k	θ=48, r=54 L=3.082	θ=57, r=68 L=3.075	θ=62, r=75 L=3.073	θ=75, r=86 L=3.079	θ=84, r=92 L=3.084	θ=93, r=108 L=3.078	θ=100, r=114 L=3.08	θ=120, r=135 L=3.081
-3	2.5230	2.6206	2.6277	2.8280	2.6402	2.6877	2.6270	2.6846
-2.8	3.5726	3.6350	3.6459	3.7580	3.6409	3.7505	3.6096	3.6448
-2.6	3.8671	3.9350	3.8468	3.9071	3.9526	3.9884	3.9732	3.9520
-2.4	4.0022	4.2645	4.2459	4.5072	4.3702	4.1683	4.0591	4.0268
-2.2	4.6439	5.1206	5.1639	5.1416	5.1579	5.1889	5.1278	5.1620
-2	5.1698	5.4480	5.5142	5.4071	5.5979	5.5789	5.5832	5.5759
-1.8	6.5989	6.3172	6.1957	6.2098	6.8957	6.5387	6.7280	6.8285
-1.6	7.2541	7.9373	7.2572	7.2098	7.6120	7.1543	7.7280	7.1534
-1.4	8.3825	8.6430	8.4260	8.1954	8.4878	8.0891	8.2536	8.3657
-1.2	10.6359	10.6469	10.6275	10.6162	10.6059	10.6324	10.6826	10.6904
-1	14.1732	14.3703	14.1546	14.1287	14.7355	14.8436	14.8455	14.1286
-0.8	18.6855	18.9797	18.9214	18.9782	18.7222	19.0228	19.4816	19.4955
-0.6	32.8425	32.9696	32.9897	32.8398	32.7384	32.8935	32.8444	32.7897
-0.4	62.7501	62.5289	62.2237	62.2930	64.2380	62.0662	62.4381	62.6757
-0.2	198.0554	190.4812	190.2530	196.2882	202.3122	196.7594	196.3208	198.4925
0	370.5489	370.4182	370.4248	370.4019	370.4248	370.4018	370.4462	370.4075
0.2	197.5759	186.8994	184.3355	193.6925	200.1273	191.6104	194.9069	197.1084
0.4	62.7680	60.7371	60.2432	62.0405	63.6273	61.6413	62.2682	62.6875
0.6	30.8697	30.2056	30.0421	30.6319	31.1459	30.5022	30.7066	30.8421
0.8	19.0063	18.7095	18.6357	18.8996	19.1279	18.8421	18.9332	18.9930
1	14.3086	14.1488	14.1087	14.2508	14.3731	14.2202	14.2690	14.3009
1.2	10.1099	10.0128	9.9883	10.0746	10.1486	10.0562	10.0858	10.1049
1.4	8.1274	8.0532	8.0370	8.0939	8.1427	8.0819	8.1014	8.1239
1.6	7.7812	7.7360	7.7245	7.7645	7.7987	7.7571	7.8698	7.7785
1.8	6.8343	6.8010	6.7925	6.8220	6.8471	6.8158	6.8259	6.8322
2	5.1243	5.1089	5.1023	5.1248	5.1440	5.1202	5.1279	5.1227
2.2	4.8992	4.5792	4.5740	4.5917	4.6068	4.5881	4.5941	4.8978
2.4	4.1788	4.1626	4.1584	4.1727	4.1848	4.1698	4.1747	4.1777
2.6	3.8410	3.8277	3.8242	3.8360	3.8459	3.8336	3.8376	3.8400
2.8	3.5545	3.5433	3.5404	3.5502	3.5585	3.5483	3.5516	3.5536
3	2.3344	2.3248	2.3224	2.3307	2.3378	2.3291	2.3319	2.3436

Table 7 - 2: ARL values for individual Shewhart-type control charts for the two-parameter Lindley distribution

Comparison of Tables 7-1 and 7-2 reveals the improvement in the performance of the chart when the skewness corrected limits are used instead of the probability-based ones. The difference in ARL values between Shewhart-type and probability-type control charts is greater than 5% for all shift sizes except k=±0.2 where it is slightly smaller than 5%.

Comparison of the ARL values for positive and negative shifts shows that, although the control charts can detect both positive and negative shifts well, there are some slight differences with most values being a little higher for the negative shifts than for the corresponding positive ones. This holds for either the probability-type or the Shewhart-type control chart. The only differences (in either direction) that are above 5% concern the shifts corresponding to values of k between 1.6 and 2.2 for the probability-type control charts and values of k=±{0.6, 2.2, 3} for the Shewhart-type control charts.

7.5 Construction of the EWMA Control Charts for Individual Observations from the Two-Parameter Lindley Distribution

When dealing with individual observations besides the Shewhart-type control charts we need EWMA charts which are a better alternative, as mentioned in Section 2.14.2. So it is useful to construct EWMA control charts for individual observations from the two-parameter Lindley distribution. In order to do so, we will need to remember the general guidelines for the construction of EWMA charts as presented in equation (2-3) and the statistic to be plotted on those charts presented in equation (2-2), with the constant λ representing the weight assigned to each of the past observations (usually chosen to be smaller for detecting smaller shifts) and the statistic's starting value being the distribution's mean.

So here, the construction of the individual two-parameter Lindley control charts is going to be done based on equation (2-3) for the general construction of EWMA charts, using the skewness correction as in Chan and Cui (2003), which is chosen since the distribution of concern is asymmetric and, as also mentioned in Weiß and Atzmüller (2011), this is an easily applied method for taking the distribution's skewness into consideration and leads to a better ARL performance of the resulting control chart. In the next section, where we deal with the performance investigation of the constructed control chart, we will further demonstrate the need for this adjustment considering the asymmetry of the distribution and the improvement in the performance of the chart when using the skewness correction contrary to not using it but using the traditionally used symmetric EWMA control limits instead.

More specifically, the procedure for the construction of the proposed control chart is as follows: in equation (2-3) we will replace L by L plus c_4^*, where $c_4^*(x)=\dfrac{\frac{4}{3}\left[\mathrm{sk}(x)\right]}{1+0.2\left[\mathrm{sk}(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient. EWMA control charts for individual observations from the two-parameter Lindley distribution are constructed using the mean of the two-parameter Lindley distribution, which is computed using equation (3-8), its standard deviation (the square root of the quantity computed by equation (3-9)) and the distribution's skewness coefficient computed from equation (3-10). This means that the skewness correction for the mean of the two-parameter Lindley distribution will be

$$c_4^*(x)=\frac{8\left[2(\theta+r)^3-\theta^3\right]\left(\theta^2+4\theta r+2r^2\right)^{3/2}}{3\left(\theta+4\theta r+2r^2\right)^3+0.24\left[2(\theta+r)^3-\theta^3\right]^2} \tag{7-7}$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the two-parameter Lindley EWMA control chart are as follows.

$$UCL=\frac{\theta+2r}{\theta(\theta+r)}+\left[L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL=\frac{\theta+2r}{\theta(\theta+r)} \tag{7-8}$$

$$LCL=\frac{\theta+2r}{\theta(\theta+r)}+\left[-L+c_4^*(x)\right]\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our two-parameter Lindley distribution.

7.6 Performance Investigation for the Individual Two-Parameter Lindley EWMA Control Charts

In order to investigate the performance of the proposed EWMA control charts, the ARL will be used. According to Lucas and Saccucchi (1990) the ARL of the EWMA control chart is computed by means of the Markov chain method and discretization of the control statistic. More specifically, the region between the upper and lower control limits is divided into $2m$+1 subintervals. Each subinterval S_j (j=1,2,…,2m+1) is taken to be represented by its midpoint s_j and then if δ is the half size of each subinterval, which means that $\delta = \frac{UCL - LCL}{2(2m+1)}$, then whenever $s_j - \delta < Z_i < s_j + \delta$ the process is in a transient state. Otherwise, the process is in the absorbing state. Therefore, the in-control transition probability from one transient state S_j to another transient state S_k is given by

$$\begin{aligned} p_{kj} &= P\left(Z_i \in S_k \middle| Z_{i-1} \in S_j\right) \\ &= P\left(s_k - \delta < Z_i < s_k + \delta \middle| Z_{i-1} = s_j\right) \\ &= P\left(s_k - \delta < \lambda X_i + (1-\lambda) Z_{i-1} < s_k + \delta \middle| Z_{i-1} = s_j\right) \\ &= P\left(\frac{s_k - \delta - (1-\lambda)s_j}{\lambda} < X_i < \frac{s_k + \delta - (1-\lambda)s_j}{\lambda}\right), \quad j,k = 1,2,\ldots,2m+1 \end{aligned} \tag{7-9}$$

The ith-stage transition probability matrix $\mathbf{P}^i$ is, then, defined as $\mathbf{P}^i = \begin{pmatrix} \mathbf{R}^i & \left(\mathbf{I} - \mathbf{R}^i\right)\mathbf{1} \\ \mathbf{0}^T & 1 \end{pmatrix}$, where $\mathbf{R}$ is the (2m+1, 2m+1) matrix of the transient probabilities p_{kj} mentioned in (7-9) above and $\mathbf{0}^T$=(0,0,…,0), i.e. $\mathbf{0}^T$ is the transpose of $\mathbf{0}$ which is a vector of 2m+1 zeros. The ith-stage transition probability matrix $\mathbf{P}^i$ contains the probabilities that the control statistic goes from one transient state to another in i steps and is used for the computation of the ARL of the EWMA control chart, which is given by

$$ARL = \mathbf{p}^T \left(\mathbf{I} - \mathbf{R}\right)^{-1} \mathbf{1} \tag{7-10}$$

where $\mathbf{p} = \left(p_{-m}, p_{-m+1}, \cdots p_{m-1}, p_m\right)^T$ is the vector of the initial probabilities related to the 2m+1 transient states.

For the transient probabilities in (7-9) the cumulative distribution function for the two-parameter Lindley distribution, i.e. equation (3-7), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equations (7-8) and (7-7) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = \frac{\theta + 2r}{\theta(\theta + r)} + \left[L + c_4^*(x)\right]\sqrt{\frac{\theta^2 + 4\theta r + 2r^2}{\theta^2(\theta + r)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = \frac{\theta + 2r}{\theta(\theta + r)} + \left[-L + c_4^*(x)\right]\sqrt{\frac{\theta^2 + 4\theta r + 2r^2}{\theta^2(\theta + r)^2}}\sqrt{\frac{\lambda}{2-\lambda}} \tag{7-11}$$

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (3-8) and (3-9) in terms of its two parameters, as for the Shewhart-type control chart.

Using those formulae we get Tables 7-3, 7-4, 7-5, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the two-parameter Lindley distribution for various values of the two parameters θ and r of the distribution of concern and for various values of k which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 7-3 contains the ARL values for λ=0.3 and L=6.876 (combination which gives in-control ARL value close to 370) for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ and L the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=50, above which ARL increases very slightly. Consequently, the suggested value of m for the computation of ARL in the formulae above is m=50. Therefore, Tables 7-4 and 7-5 show the ARL values for m=50 for various values of L and λ for positive and negative shifts, respectively.

m	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
5	0	370.0897	375.2128	376.2969	376.8948	372.6326	372.4493	374.8081	370.0897
	0.2	68.9238	63.0851	64.7330	64.4774	64.0125	67.0963	66.5433	68.9238
	0.5	42.2371	41.8757	41.7961	42.1251	42.4157	42.0364	42.1519	42.2371
	1	9.3380	9.2600	9.2417	9.3126	9.3740	9.2957	9.3204	9.3380
	1.5	5.1603	5.1464	5.1433	5.1558	5.1666	5.1529	5.1572	5.1603
	2	4.0558	4.0515	4.0505	4.0544	4.0578	4.0535	4.0549	4.0558
	2.5	3.6130	3.6114	3.6110	3.6125	3.6137	3.6122	3.6127	3.6130
	3	3.4178	3.4173	3.4171	3.4176	3.4181	3.4175	3.4177	3.4178
10	0	475.1814	499.2480	505.0432	482.9328	464.4122	488.1217	480.5558	475.1814
	0.2	64.3020	66.4160	64.8572	64.2298	68.1758	64.8248	64.0999	64.3020
	0.5	42.0840	41.6971	41.6113	41.9538	42.2741	41.8697	41.9932	42.0840
	1	9.7091	9.6270	9.6077	9.6823	9.7469	9.6645	9.6904	9.7091
	1.5	5.3633	5.3490	5.3456	5.3586	5.3698	5.3555	5.3601	5.3633
	2	4.2322	4.2278	4.2268	4.2308	4.2342	4.2298	4.2312	4.2322
	2.5	3.7858	3.7842	3.7838	3.7853	3.7864	3.7849	3.7854	3.7858
	3	3.5943	3.5938	3.5937	3.5942	3.5946	3.5940	3.5942	3.5943
20	0	507.5054	535.7341	542.5361	516.5938	494.8840	522.6798	513.8064	507.5054
	0.2	64.4678	63.9295	68.0826	69.2118	68.7455	66.7315	68.8804	64.4678
	0.5	42.0836	41.6901	41.6027	41.9513	42.2766	41.8648	41.9913	42.0836
	1	9.8509	9.7677	9.7482	9.8237	9.8893	9.8057	9.8320	9.8509
	1.5	5.4570	5.4426	5.4392	5.4523	5.4636	5.4492	5.4538	5.4570
	2	4.3199	4.3155	4.3145	4.3185	4.3219	4.3175	4.3189	4.3199
	2.5	3.8742	3.8726	3.8723	3.8737	3.8749	3.8734	3.8739	3.8742
	3	3.6859	3.6854	3.6853	3.6858	3.6862	3.6857	3.6858	3.6859
30	0	518.9206	547.1774	553.9859	528.0183	506.2861	534.1103	525.2281	518.9206
	0.2	64.6197	68.4417	64.8396	69.9101	67.3761	68.4758	66.7892	66.4197
	0.5	42.0968	41.7016	41.6140	41.9636	42.2913	41.8778	42.0037	42.0968
	1	9.8903	9.8069	9.7873	9.8631	9.9287	9.8451	9.8714	9.8903
	1.5	5.4868	5.4724	5.4690	5.4821	5.4934	5.4790	5.4836	5.4868
	2	4.3489	4.3445	4.3435	4.3475	4.3510	4.3466	4.3480	4.3489
	2.5	3.9039	3.9024	3.9020	3.9034	3.9046	3.9031	3.9036	3.9039
	3	3.7170	3.7164	3.7163	3.7168	3.7172	3.7167	3.7168	3.7170
40	0	521.2206	549.2798	556.0405	530.2548	508.6739	536.3042	527.4841	521.2206
	0.2	68.7942	64.6075	66.7345	64.5559	67.6033	64.4532	64.9102	68.7942
	0.5	42.1098	41.7131	41.6250	41.9764	42.3043	41.8902	42.0168	42.1098
	1	9.9082	9.8247	9.8050	9.8809	9.9467	9.8629	9.8893	9.9082
	1.5	5.5014	5.4870	5.4836	5.4967	5.5080	5.4936	5.4982	5.5014
	2	4.3634	4.3590	4.3580	4.3620	4.3644	4.3610	4.3624	4.3634
	2.5	3.9188	3.9172	3.9169	3.9183	3.9195	3.9180	3.9185	3.9188
	3	3.7326	3.7321	3.7319	3.7324	3.7328	3.7323	3.7324	3.7326
50	0	521.2830	549.1432	555.8558	530.2532	508.8251	536.2597	527.5021	521.2830
	0.2	64.4376	64.9864	66.4328	63.1674	68.7014	64.3018	66.8054	64.4376
	0.5	42.1170	41.7205	41.6324	41.9838	42.3114	41.8976	42.0241	42.1170
	1	9.9183	9.8348	9.8152	9.8910	9.9568	9.8730	9.8994	9.9183
	1.5	5.5101	5.4957	5.4923	5.5054	5.5167	5.5023	5.5068	5.5101
	2	4.3721	4.3677	4.3666	4.3706	4.3741	4.3697	4.3711	4.3721
	2.5	3.9278	3.9262	3.9258	3.9273	3.9285	3.9269	3.9274	3.9278
	3	3.7419	3.7414	3.7413	3.7418	3.7422	3.7417	3.7418	3.7419
80	0	521.8035	550.2889	557.1528	530.9745	509.0674	537.1158	528.1618	521.8035
	0.2	64.7159	64.1924	67.1846	67.6262	64.9244	68.5577	64.0190	64.7159
	0.5	42.1217	41.7253	41.6372	41.9884	42.3160	41.9023	42.0287	42.1217
	1	9.9249	9.8413	9.8217	9.8976	9.9634	9.8795	9.9059	9.9249
	1.5	5.5158	5.5014	5.4980	5.5111	5.5224	5.5080	5.5125	5.5158
	2	4.3778	4.3734	4.3724	4.3764	4.3798	4.3754	4.3768	4.3778
	2.5	3.9337	3.9322	3.9318	3.9332	3.9344	3.9329	3.9334	3.9337
	3	3.7482	3.7477	3.7476	3.7480	3.7484	3.7479	3.7481	3.7482
100	0	522.7688	551.1099	557.9388	531.8935	510.0969	538.0037	529.0950	522.7688
	0.2	68.5078	66.8682	68.5044	68.3883	64.0667	64.3351	66.4057	68.5078
	0.5	42.1259	41.7287	41.6405	41.9925	42.3206	41.9061	42.0328	42.1259
	1	9.9294	9.8458	9.8262	9.9021	9.9679	9.8841	9.9105	9.9294
	1.5	5.5199	5.5055	5.5021	5.5152	5.5264	5.5121	5.5166	5.5199
	2	4.3819	4.3775	4.3764	4.3805	4.3839	4.3795	4.3809	4.3819
	2.5	3.9380	3.9364	3.9361	3.9375	3.9387	3.9371	3.9376	3.9380
	3	3.7527	3.7522	3.7521	3.7525	3.7529	3.7524	3.7526	3.7527

Table 7 - 3: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (λ=0.3 and L=6.876)

λ, L	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
λ=0.05 L=2.123	0	371.9161	376.2120	376.3999	373.9491	372.6393	375.9587	373.0230	371.9161
	0.2	48.3202	46.2353	45.7872	47.6057	49.3912	47.1512	47.8201	48.3202
	0.4	17.7546	18.8316	19.1076	18.0900	17.3057	18.3204	17.9860	17.7546
	0.6	9.2662	9.1094	9.0732	9.2145	9.3398	9.1806	9.2302	9.2662
	0.8	7.9257	8.0669	8.1017	7.9705	7.8647	8.0009	7.9577	7.9257
	1	6.2649	6.2173	6.2062	6.2493	6.2869	6.2390	6.2541	6.2649
	1.5	4.2710	4.2635	4.2617	4.2685	4.2744	4.2669	4.2693	4.2710
	2	3.7329	3.7322	3.7320	3.7327	3.7333	3.7325	3.7327	3.7329
	2.5	3.5530	3.5548	3.5552	3.5536	3.5522	3.5539	3.5534	3.5530
	3	3.5268	3.5300	3.5308	3.5278	3.5253	3.5285	3.5275	3.5268
λ=0.08 L=2.752	0	371.8978	376.7121	376.7821	373.7746	372.8384	375.6799	372.8963	371.8978
	0.2	58.6105	50.2399	48.5997	55.5958	63.4499	53.7602	57.4836	58.6105
	0.4	15.0478	15.7129	15.8794	15.2575	14.7654	15.3989	15.1920	15.0478
	0.6	10.9881	10.7548	10.7012	10.9121	10.0983	10.8605	10.9345	10.9881
	0.8	8.3200	8.4245	8.4504	8.3529	8.2753	8.3754	8.3427	8.3200
	1	6.8745	6.8126	6.7969	6.8539	6.9036	6.8403	6.8602	6.8745
	1.5	4.4284	4.4187	4.4165	4.4252	4.4328	4.4231	4.4262	4.4284
	2	3.7950	3.7935	3.7931	3.7945	3.7958	3.7942	3.7947	3.7950
	2.5	3.5725	3.5736	3.5739	3.5728	3.5719	3.5731	3.5727	3.5725
	3	3.5175	3.5202	3.5208	3.5184	3.5163	3.5190	3.5181	3.5175
λ=0.10 L=3.158	0	371.8455	376.1009	376.0379	373.5424	372.0365	375.3274	372.7193	371.8455
	0.2	55.0755	57.8024	54.8160	55.8589	55.0655	57.4943	57.9733	55.0755
	0.4	18.1554	18.6125	18.7253	18.2995	18.9592	18.3975	18.2550	18.1554
	0.6	12.0973	12.7495	12.6700	12.9820	12.2629	12.9066	12.0170	12.0973
	0.8	8.3890	8.4645	8.4835	8.4126	8.3571	8.4287	8.4053	8.3890
	1	7.5037	7.4230	7.4041	7.4772	7.5412	7.4598	7.4853	7.5037
	1.5	4.5700	4.5583	4.5557	4.5762	4.5753	4.5737	4.5773	4.5700
	2	3.8475	3.8451	3.8446	3.8467	3.8485	3.8462	3.8469	3.8475
	2.5	3.5871	3.5878	3.5880	3.5873	3.5868	3.5875	3.5873	3.5871
	3	3.5077	3.5088	3.5015	3.5084	3.5067	3.5089	3.5082	3.5077
λ=0.12 L=3.586	0	371.7126	376.8984	376.5785	373.0641	372.3812	374.6186	372.3466	371.7126
	0.2	57.6981	53.6179	55.5820	57.7353	54.1285	53.1944	58.0931	57.6981
	0.4	18.0629	17.3589	17.2003	17.8278	18.4042	17.6749	17.8990	18.0629
	0.6	12.0957	12.3804	12.4508	12.1861	10.9720	12.2473	12.1582	12.0957
	0.8	10.6319	10.5142	10.4868	10.5933	10.6869	10.5778	10.6050	10.6319
	1	8.2399	8.2820	8.2930	8.2528	8.2230	8.2618	8.2488	8.2399
	1.5	4.7872	4.7723	4.7688	4.7823	4.7940	4.7791	4.7838	4.7872
	2	3.9257	3.9223	3.9215	3.9246	3.9273	3.9238	3.9249	3.9257
	2.5	3.6108	3.6108	3.6108	3.6108	3.6108	3.6108	3.6108	3.6108
	3	3.4981	3.4998	3.5002	3.4987	3.4974	3.4990	3.4985	3.4981
λ=0.15 L=4.602	0	371.8097	376.1836	376.8434	376.3150	372.4182	376.3514	376.0122	371.8097
	0.2	66.7398	63.8259	63.8881	61.3677	69.8217	68.3693	63.0312	66.7398
	0.4	20.2328	20.5058	20.5733	20.3195	20.1241	20.3782	20.2928	20.2328
	0.6	14.0077	14.6454	14.5714	14.8886	14.1772	14.8102	14.9249	14.0077
	0.8	10.9662	10.7994	10.7617	10.9106	10.0468	10.8744	10.9275	10.9662
	1	8.7247	8.6463	8.6279	8.6991	8.7608	8.6822	8.7069	8.7247
	1.5	5.1949	5.1844	5.1820	5.1915	5.1997	5.1892	5.1925	5.1949
	2	4.1402	4.1274	4.1268	4.1293	4.1414	4.1287	4.1295	4.1402
	2.5	3.7395	3.7387	3.7386	3.7393	3.7399	3.7391	3.7394	3.7395
	3	3.5763	3.5761	3.5761	3.5762	3.5763	3.5762	3.5762	3.5763
λ=0.20 L=5.935	0	371.5025	376.1243	376.1212	376.1412	372.2892	376.5886	375.1047	371.5025
	0.2	61.1739	63.4543	63.9326	61.8879	60.3223	62.3766	61.7870	61.2739
	0.4	25.7320	23.7788	23.1262	24.8784	26.4897	24.4812	25.2758	25.5320
	0.6	16.9948	16.8725	16.6897	16.8120	16.1234	16.8785	16.9486	16.9948
	0.8	10.6950	10.5377	10.5593	10.6471	10.6822	10.6423	10.5788	10.6950
	1	8.9159	8.8424	8.8260	8.8857	8.9375	8.8724	8.8942	8.9359
	1.5	5.1842	5.1809	5.1882	5.1875	5.2875	5.1862	5.1986	5.1842
	2	4.2091	4.1986	4.1953	4.1986	4.2026	4.1982	4.1993	4.2091
	2.5	3.8264	3.8254	3.8240	3.8260	3.8269	3.8268	3.8273	3.8264
	3	3.6845	3.6843	3.6842	3.6844	3.6846	3.6843	3.6844	3.6845

Table 7 - 4: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (m=50) for various positive shifts

λ, L	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
λ=0.05 L=2.123	0	371.9161	376.2120	376.3999	373.9491	372.6393	375.9587	373.0230	371.9161
	-0.2	50.2773	48.2881	46.8287	48.6218	46.2128	48.1910	48.8213	50.2773
	-0.4	18.4579	18.4610	18.2293	18.1305	18.9231	18.9148	18.2302	18.4579
	-0.6	10.2462	10.0712	10.9366	10.4579	10.9129	10.3335	10.5153	10.2462
	-0.8	8.2750	8.0724	8.6816	8.6555	8.9435	8.5764	8.6920	8.2750
	-1	6.5206	6.4087	6.3224	7.5426	8.6501	7.0005	6.4563	6.5206
	-1.5	4.8990	4.2343	4.1660	4.4280	4.0838	4.3662	4.3393	4.8990
	-2	4.3610	4.0523	3.9776	4.2618	4.4986	4.1953	4.2922	4.3610
	-2.5	4.2841	3.8955	3.7999	4.1603	4.4540	4.0768	4.1984	4.2841
	-3	3.9807	3.9282	3.9416	3.9452	3.9512	3.9379	3.9562	3.9807
λ=0.08 L=2.752	0	371.8978	376.7121	376.7821	373.7746	372.8384	375.6799	372.8963	371.8978
	-0.2	47.9015	46.1402	45.7330	47.3214	48.7290	46.9400	47.4980	47.9015
	-0.4	16.6293	16.7219	16.5108	16.3315	16.0524	16.1351	16.4222	16.6293
	-0.6	10.9126	10.3740	10.3407	10.7364	10.1620	10.6199	10.7901	10.9126
	-0.8	9.7179	9.5561	9.2480	8.8640	8.7108	9.1660	8.4653	9.7179
	-1	7.5464	7.1881	7.1036	7.4296	7.6335	7.3522	7.3989	7.5464
	-1.5	5.3496	5.0438	5.9705	5.2508	5.4873	5.1849	5.2812	5.3496
	-2	4.0062	4.9516	4.0726	4.8943	4.1610	4.8192	4.9287	4.0062
	-2.5	3.5906	3.6573	3.5726	3.6518	3.7579	3.7453	3.6095	3.5206
	-3	3.5277	3.5651	3.5621	3.5737	3.6334	3.5709	3.5798	3.5977
λ=0.10 L=3.158	0	371.8455	376.1009	376.0379	373.5424	372.0365	375.3274	372.7193	371.8455
	-0.2	48.7721	47.2150	46.8545	48.2596	49.5023	47.9225	48.4156	48.7721
	-0.4	18.7332	18.9005	18.7066	18.4600	19.1209	18.2799	19.5433	18.7332
	-0.6	12.3247	14.6365	14.5477	12.4253	12.0500	14.0174	12.0033	12.3247
	-0.8	9.8412	9.4121	9.2815	9.7544	9.1572	9.6441	9.8053	9.8412
	-1	8.7399	8.5967	8.5122	8.8349	8.6017	8.7586	8.8701	8.7399
	-1.5	4.7361	5.3573	4.5230	4.9288	5.4845	5.0606	4.8691	4.7361
	-2	4.2844	4.4975	4.4378	4.6804	4.9292	4.6124	4.7122	4.2844
	-2.5	3.5380	3.7128	3.7601	3.5933	3.4713	3.6308	3.5762	3.5380
	-3	3.3668	3.3952	3.4840	3.3437	3.3990	3.3983	3.3508	3.3668
λ=0.12 L=3.586	0	371.7126	376.8984	376.5785	373.0641	376.8985	374.6186	372.3466	371.7126
	-0.2	48.1201	46.8644	46.5726	47.7077	48.7064	47.4359	47.8333	48.1201
	-0.4	19.0620	22.9330	23.9928	20.2235	17.5701	21.0421	19.8597	19.0620
	-0.6	12.2820	12.5400	12.3891	12.0522	12.6320	12.3937	12.1253	12.2820
	-0.8	10.6847	10.2087	10.0969	10.5292	10.9043	10.4263	10.5786	10.6847
	-1	8.8688	8.6640	8.8731	8.1253	8.5369	8.2840	8.0390	8.8688
	-1.5	5.1976	5.8476	5.7647	5.0838	5.3574	5.0082	5.1286	5.1976
	-2	4.6860	4.9995	4.0786	4.7851	4.5512	4.8524	4.7545	4.6860
	-2.5	3.7051	3.6935	3.7168	3.6794	3.7412	3.6823	3.6873	3.7051
	-3	3.6000	3.6260	3.6072	3.6296	3.5596	3.6497	3.6204	3.6000
λ=0.15 L=4.602	0	371.8097	376.1836	376.8434	376.3150	372.4182	376.3514	376.0122	371.8097
	-0.2	54.0675	59.5365	50.7719	57.4012	53.3518	57.3490	55.9828	54.0675
	-0.4	19.0532	18.4455	18.3045	18.8534	18.3374	18.7219	18.9143	19.0532
	-0.6	12.5346	12.4761	12.7161	12.8282	12.1479	12.0300	12.7371	12.5346
	-0.8	10.6882	10.3447	10.2645	10.5757	10.8476	10.5014	10.6100	10.6882
	-1	8.4790	8.9231	9.0343	8.6182	8.5970	8.7139	8.5748	8.4790
	-1.5	5.9751	5.9837	5.9388	5.9126	5.9635	5.9712	5.9317	5.9751
	-2	4.5721	4.8036	4.8614	4.6440	4.4807	4.6943	4.6210	4.5721
	-2.5	3.9354	3.8670	3.8510	3.9120	3.9671	3.8982	3.9198	3.9354
	-3	3.5939	3.6884	3.7107	3.6240	3.5527	3.6443	3.6147	3.5939
λ=0.20 L=5.935	0	371.5025	376.1243	376.1212	376.1412	372.2892	376.5886	375.1047	371.5025
	-0.2	54.1730	51.5571	53.0632	57.7702	54.1381	57.9072	57.2680	54.1730
	-0.4	22.8535	22.8078	22.1788	22.1801	22.4986	22.4050	22.0789	22.8535
	-0.6	12.2853	12.2328	12.1285	12.1260	12.4154	12.0380	12.1814	12.2853
	-0.8	10.2064	10.7120	10.8405	10.3642	10.9921	10.4716	10.3155	10.2064
	-1	8.6463	8.6771	8.689	8.687	8.6448	8.6414	8.6081	8.6463
	-1.5	6.0967	6.0455	6.0966	6.0846	6.0766	6.0341	6.0575	6.0967
	-2	4.8720	4.8776	4.8757	4.8410	4.9161	4.8206	4.8505	4.8720
	-2.5	4.4868	4.6364	4.6745	4.5312	4.4260	4.5642	4.5175	4.4868
	-3	3.8266	3.8234	3.8257	3.8253	3.8289	3.8245	3.8256	3.8266

Table 7 - 5: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (m=50) for various negative shifts

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some slight differences in ARL values between those two tables, with most of the differences being in favour of the ARL values for negative shifts. The only differences (in either direction) that are above 5% concern values of k=0.2 for values of λ greater than 0.08 and values of k between 0.4 and 0.6 for values of λ greater than 0.15. Moreover, comparing Table 7-4 and Table 7-5 we observe that as the value of λ increases ARL values for negative shifts are smaller than the corresponding ones for the positive shifts for small values of k and the reverse holds for larger values of k. Large negative shifts present smaller ARL values than the large positive ones for small values of λ. Furthermore, for k=0.2 negative shifts give smaller ARL values than the corresponding positive ones, with the exception of very small λ values.

The need for using the skewness correction for the construction of the individual EWMA control charts for the two-parameter Lindley distribution is justified by the fact that if we had used the traditional symmetric EWMA control limits without the skewness correction term $c_4^*(x)$ in equation (7-11) above, the ARL performance of the chart would have been worse, as can be seen when comparing the results in Table 7-6 for the case of not using the skewness correction term against the results in Table 7-4 for the case of using it. It should be noted that the ARL values in Table 7-6 have resulted from using the same values for λ and L as the ones in Table 7-4 for the shake of making comparisons between the two tables easier. The differences between the ARL values in Tables 7-4 and 7-6 are almost all higher than 5%. The only values for which the difference is less than 5% concern the values of k=±1 for very small values of λ, absolute values of k greater than 2.5 for values of λ between 0.08 and 0.12 and values of k=±3 for λ greater than 0.15. Comparison is similar for the case of negative shifts so the corresponding table is omitted for space reasons.

λ, L	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
	0	351.5735	360.6012	362.7269	354.5120	347.4059	355.4657	353.6127	351.5735
	0.2	58.6186	55.8586	55.2794	57.6604	60.0795	57.0577	57.9466	58.6186
	0.4	21.7749	22.5735	22.7642	22.0214	21.4437	22.1902	21.9450	21.7749
λ=0.05	0.6	10.9302	10.7722	10.7358	10.8781	10.9545	10.8439	10.8939	10.9302
	0.8	7.1891	7.3183	7.3502	7.2302	7.1226	7.2579	7.2175	7.1891
L=2.123	1	6.5171	6.4709	6.4601	6.5020	6.5385	6.4920	6.5066	6.5171
	1.5	5.1290	5.1218	5.1202	5.1267	5.1223	5.1251	5.1274	5.1290
	2	4.1200	4.1228	4.1212	4.1277	4.1233	4.1261	4.1284	4.1200
	2.5	3.9530	3.9547	3.9550	3.9535	3.9523	3.9539	3.9534	3.9530
	3	3.9267	3.9298	3.9305	3.9277	3.9254	3.9284	3.9274	3.9267
	0	351.3259	359.9847	362.0194	354.1522	347.3380	355.0241	353.2894	351.3259
	0.2	58.8162	55.7549	55.9937	57.3527	60.5780	57.2600	57.7815	58.0162
	0.4	21.9528	22.2348	22.3449	22.9333	21.0781	22.0279	21.8920	21.9528
λ=0.08	0.6	9.7942	9.6891	9.6288	9.8655	10.6054	9.8083	9.8904	9.7942
	0.8	7.6464	7.5822	7.5573	7.6254	7.6763	7.6105	7.6318	7.6464
L=2.752	1	6.0570	6.1512	6.1745	6.0871	6.0157	6.1074	6.0779	6.0570
	1.5	5.2579	5.2486	5.2463	5.2549	5.2622	5.2528	5.2558	5.2579
	2	4.2680	4.2587	4.2564	4.2650	4.2723	4.2629	4.2659	4.2680
	2.5	3.4522	3.4532	3.4535	3.4525	3.4517	3.4527	3.4524	3.4522
	3	3.3960	3.3984	3.3990	3.3968	3.3949	3.3973	3.3965	3.3960
	0	351.1786	359.3048	361.2125	353.8333	347.4292	355.5903	353.0231	351.1786
	0.2	58.1460	55.5463	55.7499	57.8796	60.6463	57.6582	57.6848	58.1460
	0.4	21.0120	22.5408	22.4343	22.8543	21.2376	22.7523	21.9018	21.0120
λ=0.10	0.6	10.0505	10.3378	10.4089	10.1417	10.8261	10.2034	10.1236	10.0505
	0.8	7.3985	7.3099	7.2893	7.3694	7.4398	7.3502	7.3783	7.3985
L=3.158	1	6.8686	6.9376	6.9547	6.8906	6.8385	6.9054	6.8838	6.8686
	1.5	5.4153	5.4037	5.4009	5.3915	5.4207	5.4090	5.4127	5.4153
	2	4.3952	4.3836	4.3808	4.3914	4.4006	4.3889	4.3926	4.3952
	2.5	3.4606	3.4621	3.4612	3.4607	3.4603	3.4608	3.4607	3.4606
	3	3.3783	3.3802	3.3807	3.3789	3.3774	3.3793	3.3787	3.3783
	0	351.7797	360.9182	362.3524	354.7896	347.9331	355.1275	353.1767	351.7797
	0.2	58.5548	55.4321	55.2044	57.4335	60.4339	57.0428	57.1619	58.5548
	0.4	21.4457	22.5850	22.3935	22.1571	21.8689	22.9691	21.2436	21.4457
λ=0.12	0.6	9.2482	9.4653	9.5187	9.3173	10.2796	9.3640	9.2961	9.2482
	0.8	7.2241	7.1054	7.0778	7.1851	7.1540	7.1594	7.1970	7.2241
L=3.586	1	6.9350	6.9857	6.9982	6.9510	6.9141	6.9619	6.9461	6.9350
	1.5	5.5376	5.5236	5.5203	5.5330	5.5440	5.5300	5.5344	5.5376
	2	4.5586	4.5446	4.5412	4.5540	4.5750	4.5510	4.5554	4.5586
	2.5	3.4615	3.4606	3.4595	3.4612	3.4603	3.4586	3.4614	3.4615
	3	3.3667	3.3683	3.3687	3.3672	3.3660	3.3675	3.3670	3.3667
	0	351.0993	360.1788	362.5507	354.1071	347.9486	355.8930	353.9338	351.0993
	0.2	78.0079	75.5720	75.9615	77.9582	70.1806	77.1285	77.4206	78.0079
	0.4	21.8606	22.1426	22.2123	22.9502	21.7381	22.0108	21.9226	21.8606
λ=0.15	0.6	16.3231	16.1523	16.1236	16.2662	16.4052	16.2292	16.2835	16.3231
	0.8	12.4266	12.1667	12.1060	12.3415	12.5473	12.2852	12.3674	12.4266
L=4.602	1	10.3216	10.2624	10.2484	10.3023	10.3488	10.2895	10.3082	10.3216
	1.5	6.1037	6.0963	6.0945	6.1014	6.1070	6.0997	6.1020	6.1037
	2	5.0936	5.0862	5.0844	5.0912	5.0969	5.0896	5.0919	5.0936
	2.5	4.7236	4.7233	4.7233	4.7235	4.7237	4.7234	4.7235	4.7236
	3	3.5796	3.5707	3.5794	3.5798	3.5786	3.5797	3.5794	3.5796
	0	351.7070	360.8029	362.6865	354.8069	347.4858	355.9944	353.6062	351.7070
	0.2	68.4857	65.0015	65.7876	67.1800	68.3412	67.6640	67.0432	68.4857
	0.4	46.5038	48.4673	48.4669	48.4858	46.5389	48.4771	46.4907	46.5038
λ=0.20	0.6	35.9434	35.5459	35.4534	35.8129	36.1286	35.7269	35.8527	35.9434
	0.8	15.4853	15.3954	15.3743	15.4570	15.5266	15.4366	15.4649	15.4853
L=5.935	1	10.2695	10.2357	10.2277	10.2585	10.2851	10.2512	10.2619	10.2695
	1.5	9.7285	9.7223	9.7208	9.7265	9.7314	9.7252	9.7271	9.7285
	2	5.7495	5.7433	5.7418	5.7475	5.7524	5.7462	5.7481	5.7495
	2.5	4.9305	4.9302	4.9301	4.9304	4.9307	4.9303	4.9304	4.9305
	3	3.6971	3.6971	3.6981	3.6921	3.6971	3.6970	3.6962	3.6971

Table 7 - 6: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (m=50) for various positive shifts for the case of not using the skewness correction term when constructing the control limits of the chart

Additionally, comparing the ARL values for the EWMA in Tables 7-4 and 7-5 with the ARL values for the Shewhart-type control chart in Table 7-1, we can see that the EWMA control chart performs better than the Shewhart-type control chart for smaller shifts, since for the case of small shifts, the EWMA out-of-control ARL values are smaller than the corresponding ARL values for the Shewhart-type charts. When it comes to large shifts, however, EWMA ARL values are slightly larger and, therefore, make Shewhart-type control charts preferable for those cases.

7.7 Optimal Choice for the Parameters of the EWMA Control Charts for Individual Observations from the Two-Parameter Lindley Distribution

When constructing an EWMA control chart, there are two parameters involved in the way the chart is going to perform, namely the constant λ which affects the weight we give to the past values of our observations and the value of L which affects the width of the chart's control limits. Therefore, we need to find the combination of the values of those two parameters which will lead us to the optimal performance of our control chart.

As already mentioned in Section 6.7 various methods have been proposed in literature for optimizing the design of control charts based on minimizing the out-of-control value of various performance criteria. Since all the study here has been based on ARL (which is the most commonly used performance criterion) the optimal design of the EWMA control chart will be done by minimizing the ARL. The algorithm applied here is as follows:

- Step 1: Set the desired in-control ARL value (e.g. ARL_0=370) and the size of the mean shift k to be detected (e.g. k = 0.5).
- Step 2: Set an initial value L = 1.
- Step 3: Vary the parameter λ (e.g. increasing by 0.01) so as $\lambda \in (0,1]$ and (using a nonlinear equation solver) find the value of λ for which the ARL_0 value in Step 1 is satisfied.
- Step 4: Calculate the ARL_1 value for the particular combination of λ and L resulting from Step 3. [The ARL_1 value is obtained as described in the previous section, using equation (7-9) for the computation of the transient

probabilities along with equation (3-7) for the cumulative distribution function of the two-parameter Lindley distribution.]

- Step 5: Increase L by 0.01.
- Step 6: Repeat Steps 3-5 until the minimum ARL_1 value has been reached (i.e. until the ARL_1 value for L+0.01 is larger than the ARL_1 value for L).
- Step 7: Keep the combination of λ and L resulting from Step 6 for which the smallest ARL_1 value is obtained as the desired optimal one for the selected shift size in Step 1.
- Step 8: Repeat Steps 2-7 for all the desired values of shifts to be detected (e.g. k = {-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3}).

Application of this algorithm leads to Table 7-7 and Table 7-8 which present the optimal combination of values of the two parameters of concern (λ and L) of the EWMA chart with the corresponding ARL values for various values of the parameters θ and r of the two-parameter Lindley distribution and various positive and negative values, respectively, of k, which shows the shift of the process mean in terms of the process standard deviation which we want to be detected by the control chart we construct.

k	θ=48, r=54	θ=57, r=68	θ=62, r=75	θ=75, r=86	θ=84, r=92	θ=93, r=108	θ=100, r=114	θ=120, r=135
0.2	(0.01, 1) (375.3626, 53.6138)	(0.01, 1) (378.496, 53.6948)	(0.01, 1) (371.477, 53.7147)	(0.01, 1) (372.2494, 53.6397)	(0.01, 1) (375.6644, 53.5777)	(0.01, 1) (378.8167, 53.6571)	(0.01, 1) (370.1457, 53.6318)	(0.01, 1) (375.3626, 53.6138)
0.4	(0.01, 1) (375.3626, 16.6965)	(0.01, 1) (378.496, 16.9317)	(0.67, 6.63) (371.7679, 16.948)	(0.01, 1) (372.2494, 16.7713)	(0.01, 1) (375.6644, 16.5938)	(0.66, 6.03) (370.2775, 16.5571)	(0.01, 1) (370.1457, 16.7483)	(0.01, 1) (375.3626, 16.6965)
0.6	(0.65, 6.59) (370.0687, 10.1728)	(0.67, 7.71) (368.8467, 10.8036)	(0.67, 6.63) (371.7679, 10.2787)	(0.66, 6.96) (369.7372, 10.3861)	(0.65, 6.76) (370.2403, 10.2582)	(0.66, 6.03) (370.2775, 10.9919)	(0.66, 7.4) (368.6207, 10.6096)	(0.65, 6.59) (370.0687, 10.1728)
0.8	(0.65, 6.59) (370.0687, 8.7584)	(0.67, 7.71) (368.8467, 8.2384)	(0.67, 6.63) (371.7679, 8.8955)	(0.66, 6.96) (369.7372, 8.9282)	(0.65, 6.76) (370.2403, 8.8124)	(0.66, 6.03) (370.2775, 8.6852)	(0.66, 7.4) (368.6207, 8.0755)	(0.65, 6.59) (370.0687, 8.7584)
1	(0.65, 6.59) (370.0687, 7.5322)	(0.02, 1.3) (367.0708, 7.7208)	(0.67, 6.63) (371.7679, 7.6934)	(0.66, 6.96) (369.7372, 7.6684)	(0.65, 6.76) (370.2403, 7.5626)	(0.66, 6.03) (370.2775, 7.5444)	(0.02, 1.3) (359.952, 7.7492)	(0.65, 6.59) (370.0687, 7.5322)
1.2	(0.02, 1.31) (378.3186, 4.8538)	(0.02, 1.3) (367.0708, 4.7862)	(0.02, 1.3) (369.2367, 4.7823)	(0.02, 1.3) (360.8687, 4.7972)	(0.02, 1.31) (373.7946, 4.8618)	(0.02, 1.3) (362.8582, 4.7937)	(0.02, 1.3) (359.952, 4.7989)	(0.02, 1.31) (378.3186, 4.8538)
1.4	(0.02, 1.31) (378.3186, 4.3218)	(0.02, 1.3) (367.0708, 4.2834)	(0.02, 1.3) (369.2367, 4.2815)	(0.02, 1.3) (360.8687, 4.2889)	(0.02, 1.31) (373.7946, 4.3258)	(0.02, 1.3) (362.8582, 4.2871)	(0.02, 1.3) (359.952, 4.2897)	(0.02, 1.31) (378.3186, 4.3218)
1.6	(0.02, 1.31) (378.3186, 4.0045)	(0.02, 1.3) (367.0708, 3.9816)	(0.02, 1.3) (369.2367, 3.9806)	(0.02, 1.3) (360.8687, 3.9843)	(0.02, 1.31) (373.7946, 4.0065)	(0.02, 1.3) (362.8582, 3.9834)	(0.02, 1.3) (359.952, 3.9847)	(0.02, 1.31) (378.3186, 4.0045)
1.8	(0.02, 1.31) (378.3186, 3.8036)	(0.02, 1.3) (367.0708, 3.7902)	(0.02, 1.3) (369.2367, 3.7898)	(0.02, 1.3) (360.8687, 3.7912)	(0.02, 1.31) (373.7946, 3.8044)	(0.02, 1.3) (362.8582, 3.7909)	(0.02, 1.3) (359.952, 3.7914)	(0.02, 1.31) (378.3186, 3.8036)
2	(0.02, 1.31) (378.3186, 3.6734)	(0.02, 1.3) (367.0708, 3.6868)	(0.02, 1.3) (369.2367, 3.6868)	(0.02, 1.3) (360.8687, 3.6868)	(0.02, 1.31) (373.7946, 3.6735)	(0.02, 1.3) (362.8582, 3.6868)	(0.02, 1.3) (359.952, 3.6868)	(0.02, 1.31) (378.3186, 3.6734)
2.2	(0.02, 1.31) (378.3186, 3.5901)	(0.02, 1.3) (367.0708, 3.5881)	(0.02, 1.3) (369.2367, 3.5884)	(0.02, 1.3) (360.8687, 3.5874)	(0.02, 1.31) (373.7946, 3.5897)	(0.02, 1.3) (362.8582, 3.5876)	(0.02, 1.3) (359.952, 3.5873)	(0.02, 1.31) (378.3186, 3.5901)
2.4	(0.02, 1.31) (378.3186, 3.5399)	(0.02, 1.3) (367.0708, 3.5418)	(0.02, 1.3) (369.2367, 3.5423)	(0.02, 1.3) (360.8687, 3.5405)	(0.02, 1.31) (373.7946, 3.5391)	(0.02, 1.3) (362.8582, 3.5409)	(0.02, 1.3) (359.952, 3.5403)	(0.02, 1.31) (378.3186, 3.5399)
2.6	(0.98, 2.57) (376.2326, 3.4388)	(0.98, 2.56) (377.6369, 3.3894)	(0.98, 2.58) (375.1443, 3.456)	(0.98, 2.56) (379.2431, 3.3991)	(0.98, 2.56) (375.2533, 3.4102)	(0.98, 2.56) (375.2069, 3.396)	(0.98, 2.57) (379.5451, 3.4355)	(0.98, 2.57) (376.2326, 3.4388)
2.8	(0.98, 2.57) (376.2326, 3.2089)	(0.98, 2.56) (377.6369, 3.1677)	(0.98, 2.58) (375.1443, 3.2237)	(0.98, 2.56) (379.2431, 3.1756)	(0.98, 2.56) (375.2533, 3.1848)	(0.98, 2.56) (375.2069, 3.1731)	(0.98, 2.57) (379.5451, 3.2062)	(0.98, 2.57) (376.2326, 3.2089)
3	(0.98, 2.57) (376.2326, 3.0168)	(0.98, 2.56) (377.6369, 2.982)	(0.98, 2.58) (375.1443, 3.0298)	(0.98, 2.56) (379.2431, 2.9886)	(0.98, 2.56) (375.2533, 2.9961)	(0.98, 2.56) (375.2069, 2.9865)	(0.98, 2.57) (379.5451, 3.0146)	(0.98, 2.57) (376.2326, 3.0168)

Table 7 - 7: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the two-parameter Lindley distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of positive shifts k (m=50)

k	θ=48, r=54	θ=57, r=68	θ=62, r=75	θ=75, r=86	θ=84, r=92	θ=93, r=108	θ=100, r=114	θ=120, r=135
-0.2	(0.01, 1)	(0.01, 1)	(0.67, 6.63)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)
	(375.3626,	(376.496, 53.7728)	(371.7679, 52.9726)	(372.2494, 52.7525)	(375.6644,	(376.8167,	(370.1457, 52.2827)	(375.3626,
-0.4	(0.09, 2.95)	(0.09, 2.94)	(0.09, 2.94)	(0.09, 2.95)	(0.1, 3.16)	(0.09, 2.95)	(0.1, 3.15)	(0.09, 2.95)
	(366.865,	(366.7856,	(368.7191, 15.0466)	(369.6278, 15.2879)	(368.435,	(371.4571,	(366.3862, 15.2955)	(366.865,
-0.6	(0.14, 3.97)	(0.14, 3.95)	(0.14, 3.94)	(0.14, 3.96)	(0.14, 3.97)	(0.14, 3.96)	(0.14, 3.96)	(0.14, 3.97)
	(377.9655,	(374.6868,	(372.0164, 10.2535)	(375.6645, 10.5961)	(376.1348,	(376.5038, 10.565)	(375.2758, 10.6104)	(377.9655,
-0.8	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 8.8559)	(371.887, 10.014)	(374.834, 9.2302)	(361.9997, 8.459)	(373.7881, 9.0492)	(373.6705, 9.2403)	(373.3306,
-1	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 6.128)	(371.887, 6.6889)	(374.834, 6.3037)	(361.9997,	(373.7881, 6.2245)	(373.6705, 6.3087)	(373.3306,
-1.2	(0.81, 2.55)	(0.77, 2.55)	(0.78, 2.57)	(0.79, 2.55)	(0.78, 2.54)	(0.78, 2.55)	(0.8, 2.55)	(0.81, 2.55)
	(399.6865,	(371.367, 5.2181)	(371.887, 5.543)	(376.1879, 5.2805)	(372.3009,	(373.7881, 5.2539)	(394.4725, 5.3073)	(399.6865,
-1.4	(0.85, 2.55)	(0.87, 2.56)	(0.87, 2.56)	(0.83, 2.55)	(0.87, 2.55)	(0.82, 2.55)	(0.84, 2.55)	(0.85, 2.55)
	(372.0721,	(377.7971, 4.8662)	(377.057, 4.8257)	(378.5556, 4.9001)	(376.3272,	(379.7271, 4.9603)	(373.123, 4.8324)	(371.0721,
-1.6	(0.92, 2.56)	(0.95, 2.56)	(0.93, 2.56)	(0.95, 2.56)	(0.95, 2.56)	(0.95, 2.56)	(0.95, 2.56)	(0.92, 2.56)
	(389.4788, 4.067)	(377.0362, 4.0082)	(374.2349, 4.042)	(364.4171, 4.0325)	(372.1188,	(377.9854, 4.0262)	(378.8094, 4.0831)	(389.4788, 4.077)
-1.8	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 3.9001)	(371.887, 3.9001)	(374.834, 3.9005)	(361.9997,	(373.7881, 3.9003)	(373.6705, 3.9002)	(373.3306,
-2	(0.92, 2.56)	(0.98, 2.56)	(0.93, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.96, 2.56)	(0.92, 2.56)
	(389.4788,	(377.6369, 3.7085)	(374.2349, 3.7079)	(379.2431, 3.7065)	(395.2533,	(375.2069, 3.7052)	(369.4912, 3.7049)	(389.4788, 3.704)
-2.2	(0.92, 2.56)	(0.98, 2.56)	(0.93, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.96, 2.56)	(0.92, 2.56)
	(389.4788,	(377.6369, 3.6085)	(374.2349, 3.6079)	(379.2431, 3.6065)	(395.2533,	(375.2069, 3.6052)	(369.4912, 3.6049)	(389.4788, 3.604)
-2.4	(0.92, 2.56)	(0.98, 2.56)	(0.93, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.98, 2.56)	(0.96, 2.56)	(0.92, 2.56)
	(389.4788,	(377.6369, 1.0085)	(374.2349, 1.0079)	(379.2431, 1.0065)	(395.2533,	(375.2069, 1.0052)	(369.4912, 1.0049)	(389.4788, 1.004)
-2.6	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 3.3001)	(371.887, 3.3001)	(374.834, 3.3009)	(361.9997,	(373.7881, 3.3004)	(373.6705, 3.3004)	(373.3306,
-2.8	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 3.2001)	(371.887, 3.2001)	(374.834, 3.2006)	(361.9997,	(373.7881, 3.2004)	(373.6705, 3.2004)	(373.3306,
-3	(0.77, 2.56)	(0.77, 2.55)	(0.78, 2.57)	(0.76, 2.56)	(0.76, 2.54)	(0.78, 2.55)	(0.76, 2.56)	(0.77, 2.56)
	(373.3306,	(371.367, 2.9801)	(371.887, 2.9801)	(374.834, 2.98008)	(361.9997,	(373.7881,	(373.6705, 2.98001)	(373.3306,

Table 7 - 8: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the two-parameter Lindley distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of negative shifts k (m=50)

7.8 Examples on the Individual Two-Parameter Lindley Probability-Type, Shewhart-Type and EWMA Control Charts

This section offers illustration of the proposed control charts by means of both simulated data generated from the distribution of concern and real data. The case of simulated data is discussed in Subsection 7.8.1, while the real data case is presented in Subsection 7.8.2.

7.8.1 Examples with Simulated Data from the Two-Parameter Lindley Distribution

For the simulation the R programming language version 4.0.2 (R Core Team (2020)) has been used along with the “LindleyR” package version 1.1.0 (Mazucheli et al. (2016)). The “lamW” package version 1.3.3 (Adler (2015)) has also been used for the quantile function of the distribution used in probability-type control charts.

Suppose we take a sample of n = 30 observations from a two-parameter Lindley process as follows. First, we take a sample of 15 observations from a two-parameter Lindley process with in-control θ value equal to 55 and in-control r value equal to 68. Now suppose that a shift of one standard deviation unit occurs in the process mean, and after that shift, we draw another set of 15 observations from the process. The resulting data set can be seen in Table 7-9. For this data set, we construct the individual probability-type two-parameter Lindley control chart shown in Figure 7-1, using the most commonly used value for the significance level α = 0.27%, as mentioned in Section 7.4.

Data Set 1	0.02223885	0.08479636	0.03570984	0.00241743	0.03109676
	0.06124349	0.00533464	0.00170250	0.05933743	0.00482841
	0.08727944	0.01340051	0.01963175	0.05211700	0.02316009
	0.06953373	0.09714948	0.06269988	0.10391495	0.07246218
	0.06395473	0.16562432	0.07364729	0.17930232	0.05985985
	0.05223898	0.18346928	0.09460042	0.06044129	0.09525128

Table 7 - 9: Data from a two-parameter Lindley process with in control θ=55, in-control r=68 and a shift of one standard deviation unit in the process mean due to an increasing shift after the first 15 observations (gray shading)

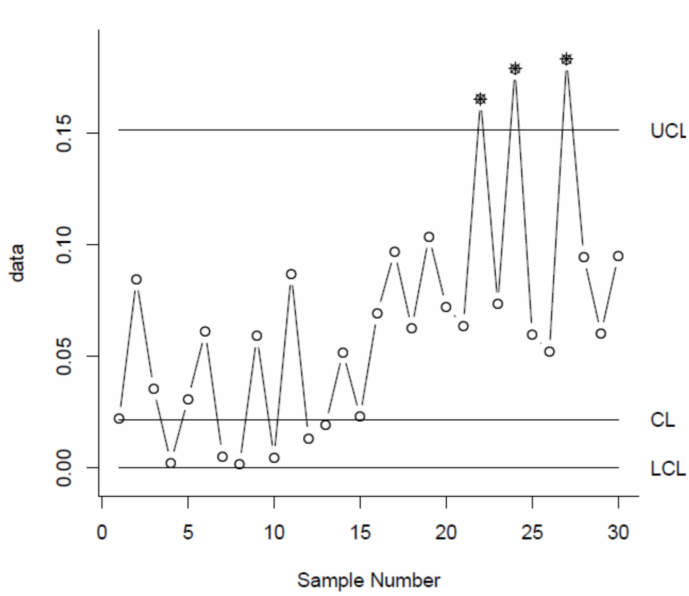

Figure 7 - 1: Individual probability-type two-parameter Lindley control chart for the data set in Table 7-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations and the control charts detect some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level.

For the same data with one standard deviation unit shift in Table 7-9, we now construct the Shewhart-type two-parameter Lindley control chart shown in Figure 7-2, using L = 3.071 standard deviations (which gives a desired value of in-control ARL close to 370).

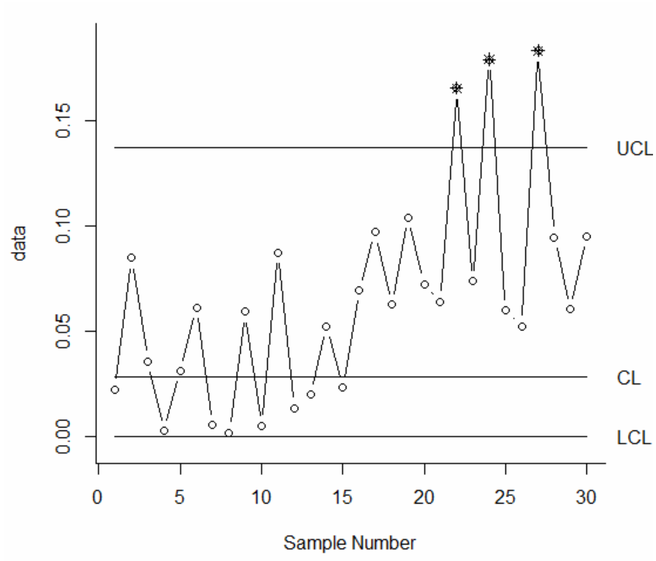

Figure 7 - 2: Individual Shewhart type two-parameter Lindley control chart for the data set in Table 7-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations and the control charts detect some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level. Comparing this chart to the previous one (Figure 7-1), we observe similar behaviour of the probability-type chart to the Shewhart-type chart with skewness correction.

Using the data set in Table 7-9 for the case of a shift of one standard deviation unit, we now construct the individual EWMA two-parameter Lindley control chart shown in Figure 7-3, using $\lambda=0.05$ and L = 2.10045 standard deviations (which gives a desired value of in-control ARL close to 370). As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 19th observation. Comparing Figure 7-3 with Figure 7-2 we can see now that, as expected, the EWMA control chart detects the one-standard deviation-unit shift quicker than the corresponding Shewhart-type control chart.

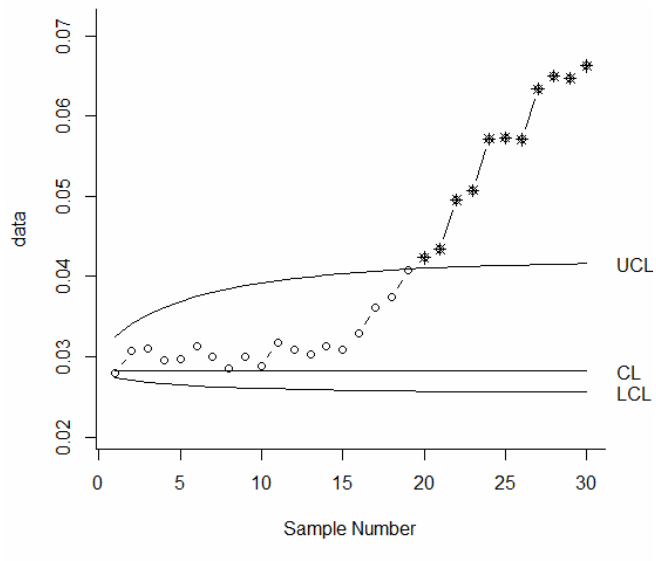

Figure 7 - 3: Individual EWMA two-parameter Lindley control chart for the data set in Table 7-9 with a shift of one standard deviation unit in the process mean

<u>7.8.2 Application of the Individual Two-Parameter Lindley Probability-Type, Shewhart-Type and EWMA Control Charts to Real Data</u>

This section deals with the illustration of the proposed control charts through application to real data used by Ghitany et al. (2008) representing waiting times before service of bank customers. This data set, which is presented in Table 7-10, was also used by Shanker et al. (2013) for illustration of the applicability of the two-parameter Lindley distribution they introduced.

13.9	21.9	8.8	3.1	14.1	8.6	8.0	12.9	6.2	4.9
13.7	1.9	4.3	27.0	6.3	9.5	11.9	9.6	2.6	17.3
1.8	4.0	11.0	3.3	13.6	5.7	5.3	21.3	21.4	4.2
4.4	12.5	6.9	4.1	18.1	8.9	7.7	11.2	7.1	2.1
6.2	18.9	2.7	4.6	38.5	10.7	6.1	2.9	13.1	4.9
3.2	11.5	9.8	11.1	19.0	4.3	15.4	1.5	0.8	13.3
6.2	4.7	18.2	4.4	3.6	31.6	7.1	6.7	11.2	1.9
5.0	15.4	7.1	23.0	8.9	8.2	18.4	4.2	5.7	33.1
7.4	8.6	10.9	7.6	4.7	11.0	4.8	3.5	19.9	9.7
8.6	13.0	7.1	17.3	5.5	8.8	12.4	1.3	0.8	20.6

Table 7 - 10: Waiting Times data set

First of all, when dealing with any dataset, the normality assumption should be checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the two-parameter Lindley distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.3667 with the presence of ties in our data and a p-value=0.9184 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the two-parameter Lindley distribution fits our data well.

The values of the parameters of our assumed two-parameter Lindley distribution as in Shanker et al. (2013) and being equal to 0.196 and 2.967 for θ and r, respectively, are going to be used for the construction of the individual probability-type control chart (along with the significance level value $\alpha = 0.27\%$) and for the Shewhart-type control chart for our data, in conjunction with the value of L=2.986 standard deviations (for which in-control ARL is close to 370). The resulting control charts can be seen in Figure 7-4 and Figure 7-5 for the probability-type and Shewhart-type control chart, respectively, which show all the observations being inside the control limits, which is an indication that the waiting times of bank customers are within the expected ranges.

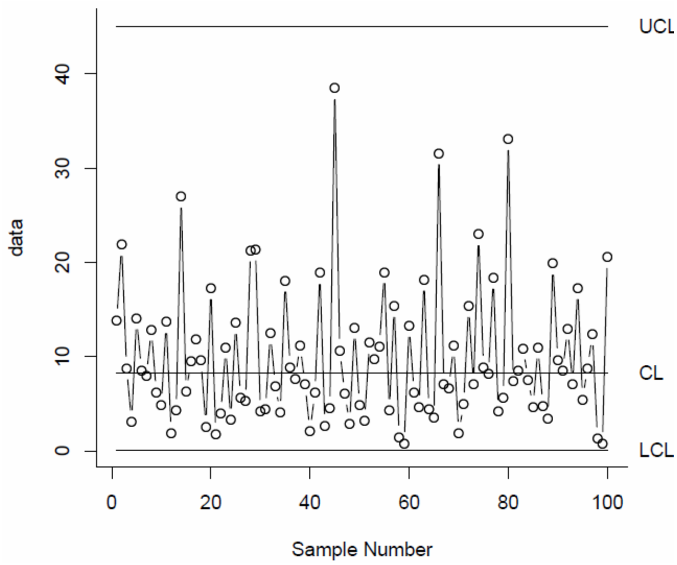

Figure 7 - 4: Individual probability-type control chart for the Waiting Times dataset assuming two-parameter Lindley distribution for the data

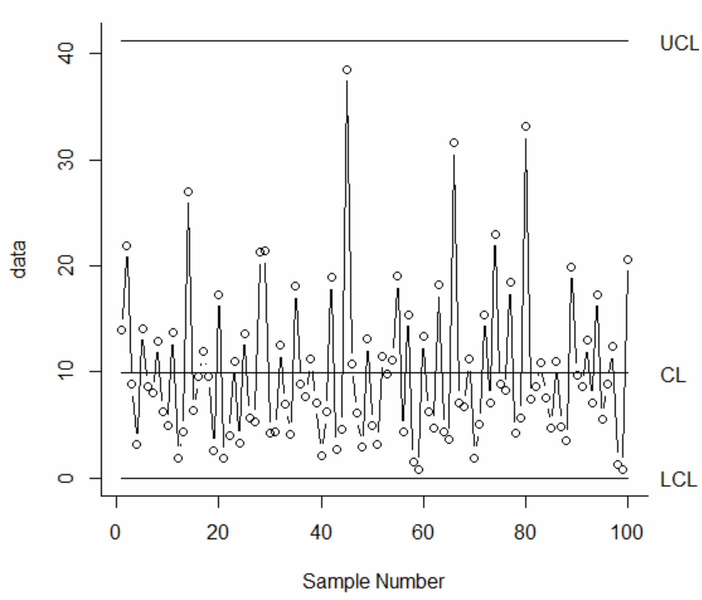

Figure 7 - 5: Individual Shewhart-type control chart for the Waiting Times dataset assuming two-parameter Lindley distribution for the data

For the construction of the individual EWMA control chart for our data, using the same parameter values of the assumed two-parameter Lindley distribution from the data in conjunction with the values of λ=0.08 and L=2.61 standard deviations (for which in-control ARL is close to 370), the resulting control chart can be seen in Figure 7-6, which shows all the observations being inside the control limits, which, once again, is an indication that the waiting times of bank customers are within the expected ranges.

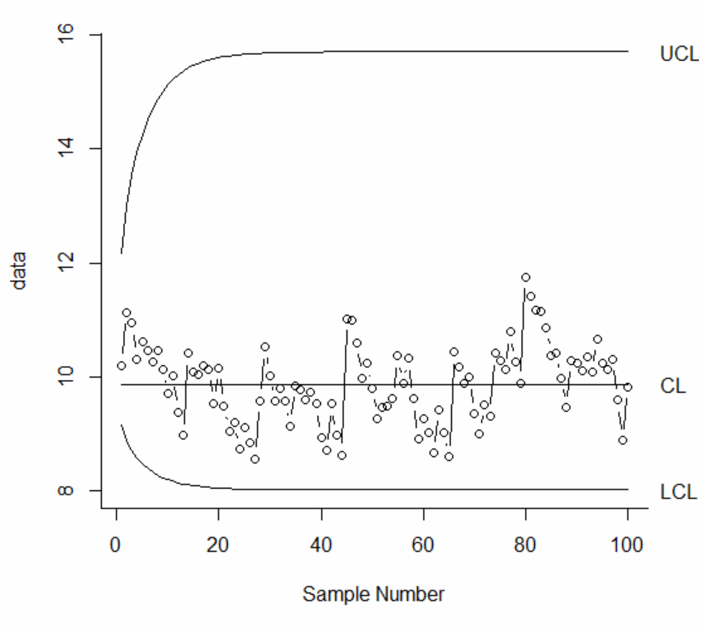

Figure 7 - 6: Individual EWMA control chart for the Waiting Times dataset assuming two-parameter Lindley distribution for the data

Now let's see the application on a second data set. This particular data set comes from Proschan (1963), also dealt with in Cox and Snell (1981), and represents the time intervals between failures of the air-conditioning equipment of ten Boeing 720 aircrafts. Here we use the data for the fifth aircraft. The data we use are presented in Table 7-11. First, as usual the normality assumption is checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the two-parameter Lindley distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.2364 with the presence of ties in our data and a p-value=0.888 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the two-parameter Lindley distribution fits our data well. We can see that there are some outliers in our data. Let's see if the control charts can detect them.

Times between failures	32	261	87	7	120	14	62	47	225	71
	246	21	42	20	5	12	120	11	3	14
	71	11	14	11	16	90	1	16	52	95

Table 7 - 11: Time (in hours) between failures of the air-conditioning equipment of the fifth Boeing 720 aircraft in Proschan (1963).

The values of the parameters of our assumed two-parameter Lindley distribution being equal to 0.0298 and 0.1088 for θ and r, respectively, are going to be used for the construction of the individual control charts. For the probability-type control chart the significance level value α = 0.27% is used, while for the Shewhart-type control chart for our data the value of L=3.426 standard deviations (for which in-control ARL is close to 370) is used. The resulting control charts can be seen in Figure 7-7 and Figure 7-8 for the probability-type and Shewhart-type control chart, respectively, which do not show any out-of-control points, but they present a clear downwards shift.

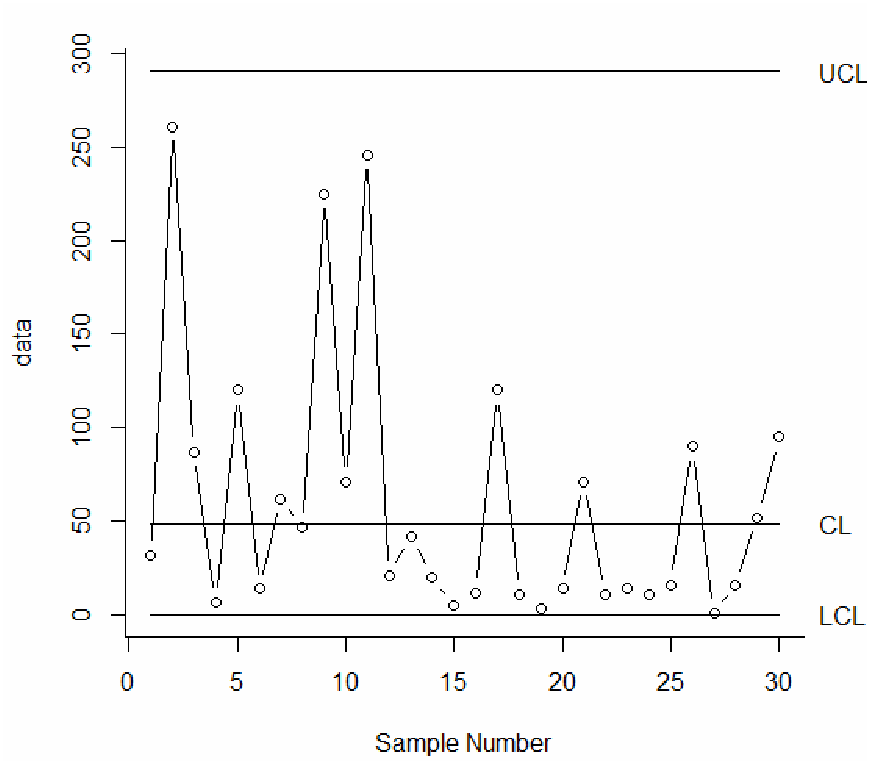

Figure 7 - 7: Individual probability-type control chart for the Failure Time Intervals of the fifth aircraft dataset assuming a two-parameter Lindley distribution for the data.

For the construction of the individual EWMA control chart, the same distribution's parameter values are going to be used in conjunction with the values of λ=0.05 and L=2.9318 standard deviations (for which in-control ARL is close to 370). The resulting control chart is presented in Figure 7-9 where we can also see that there are no out-of-control points, but there is a downwards movement indicating that the observed values have decreased from some point forward.

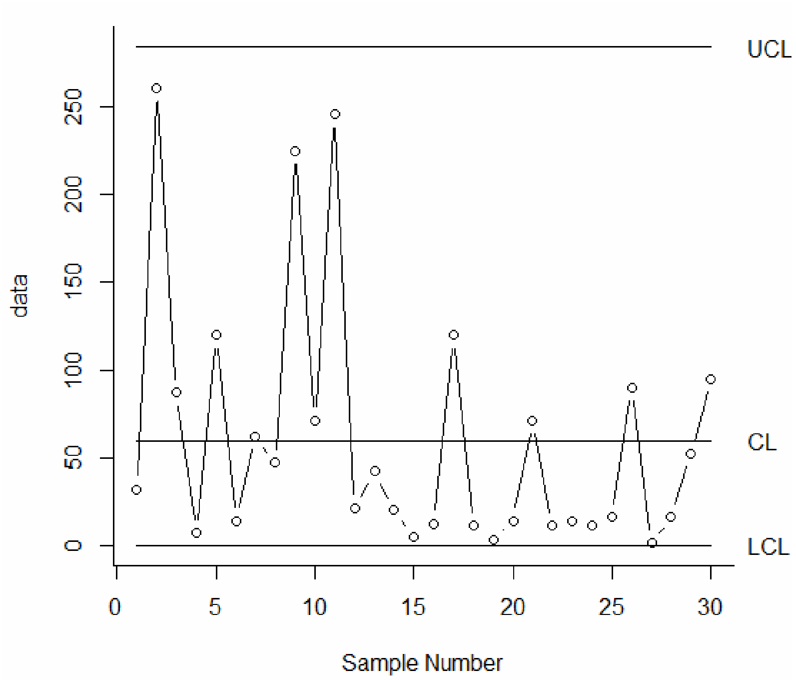

Figure 7 - 8: Individual Shewhart-type control chart for the Failure Time Intervals of the fifth aircraft dataset assuming a two-parameter Lindley distribution for the data.

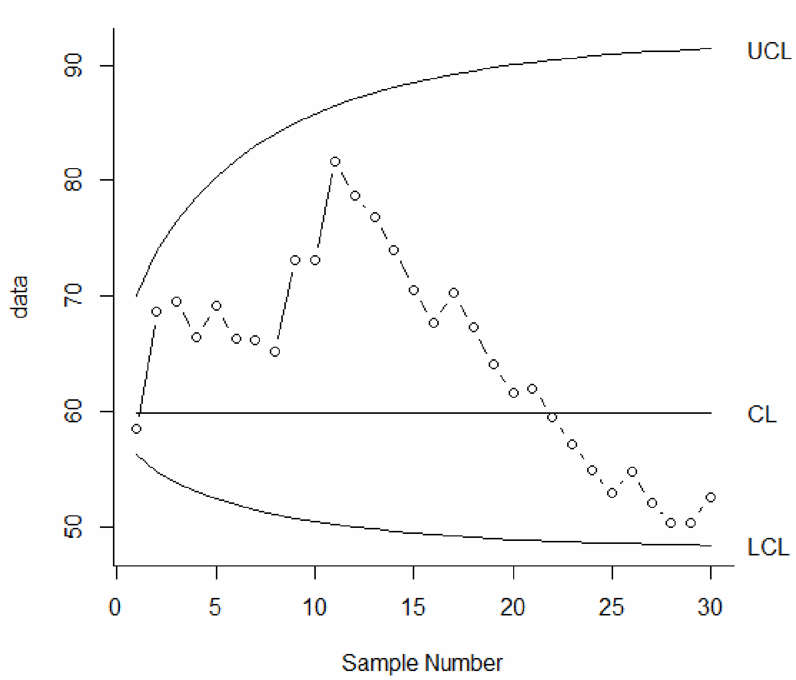

Figure 7 - 9: Individual EWMA control chart for the Failure Time Intervals of the fifth aircraft dataset assuming a two-parameter Lindley distribution for the data.

7.9 Control Charts for Individual Observations from the Two-Parameter Lindley Distribution with the Scaled Weighted Variance Method

The control charts constructed for the two-parameter Lindley distribution in previous sections used the skewness correction method by Chan and Cui (2003). This, however, is not the only method considering the skewness of a distribution. One more method doing that is the one proposed by Castagliola (2000). This is the method that is going to be used in this section for constructing control charts for individual two-parameter Lindley-distributed observations and comparison will be conducted with the corresponding previously presented control charts for this distribution.

7.9.1. Construction of Shewhart-type Control Charts for Individual Observations from a Process Following the Two-Parameter Lindley Distribution Using the Scaled Weighted Variance Method

The construction procedure according to the scaled weighted variance method by Castagliola (2000) is the following: the central line is placed at the mean of the two-parameter Lindley distribution, which is computed with equation (3-8), and the control limits are placed around the mean at two different multiples of the standard deviation of the two-parameter Lindley distribution, which is computed with equation (3-9). These multiples are functions of appropriate values of the quantiles of the standardized Normal distribution, the probability of type I error or false alarm rate, α, and the cumulative distribution function of the two-parameter Lindley distribution, which is computed with equation (3-7). More specifically, the lower control limit is defined as $LCL = \mu - \sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)\sigma$, while the upper control limit is defined as $UCL = \mu + \sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)\sigma$.

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the two-parameter Lindley control chart are as follows.

$$UCL = \frac{\theta+2r}{\theta(\theta+r)} + \sqrt{\frac{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}$$

$$CL = \frac{\theta+2r}{\theta(\theta+r)} \qquad (7\text{-}12)$$

$$LCL = \frac{\theta+2r}{\theta(\theta+r)} - \sqrt{\frac{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}$$

7.9.2. Performance Investigation for the Individual Two-Parameter Lindley Control Charts Constructed With the Scaled Weighted Variance Method

Once again, the performance of the proposed control chart will be investigated using the ARL (ARL_0 and ARL_1) as computed by equations (7-5) and (7-6) where $F_{in}\left(x\right)$ is the cumulative distribution function of the two-parameter Lindley distribution in equation (3-7) with in-control parameters, $F_{out}\left(x\right)$ is the cumulative distribution function for the distribution of concern with out-of-control parameters given by $\theta_{new} = \frac{\sqrt{2}}{\sqrt{2}\left(\mu_0 + k\sigma\right) + \sqrt{\left(\mu_0 + k\sigma\right)^2 - \sigma_{new}^2}}$ and $r_{new} = \frac{\frac{-\sqrt{2}\sqrt{\left(\mu_0 + k\sigma\right)^2 - \sigma_{new}^2}}{\sqrt{2}\left(\mu_0 + k\sigma\right) + \sqrt{\left(\mu_0 + k\sigma\right)^2 - \sigma_{new}^2}}}{\sqrt{2}\left(\mu_0 + k\sigma\right) + 2\sqrt{\left(\mu_0 + k\sigma\right)^2 - \sigma_{new}^2}}$ (as earlier) and the control limits obtained by equation (7-12) in both cases. Using the above formulas we obtain Table 7-11 which shows the in-control and out-of-control ARL values for the individual two-parameter Lindley control chart with scaled weighted variance for various values of the two parameters θ and r of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. A significance level equal to the most commonly used value of 0.27% has been chosen, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

Comparison of Tables 7-11 and 7-12 reveals the improvement in the performance of the chart when the skewness corrected limits are used instead of the probability-based ones. The difference in ARL values between Shewhart-type and probability-type control charts is greater than 5% for all shift sizes except k=±0.2 where it is slightly smaller than 5%.

Comparison of the ARL values for positive and negative shifts shows that, although the control chart can detect both positive and negative shifts well, there are some slight differences with the ARL values for positive shifts being mostly larger than the ones for the negative shifts. The only cases for which ARL values for negative shifts are bigger than the corresponding ones for positive shifts are the cases of larger values of the distribution's parameters (θ and r) in conjunction with shifts smaller than or equal to 1.6 standard deviation units.

k	θ=48, r=54	θ=57, r=68	θ=62, r=75	θ=75, r=86	θ=84, r=92	θ=93, r=108	θ=100, r=114	θ=120, r=135
-3	2.1697	2.1280	2.2010	2.1418	2.0737	2.1232	2.8264	2.1464
-2.8	2.3484	2.8848	2.3980	2.8428	2.5173	2.5901	2.9599	2.4680
-2.6	3.3933	3.3086	3.0364	3.0686	3.6006	3.3180	3.7070	3.6097
-2.4	3.9754	4.2281	4.4843	4.4848	4.3284	4.1524	4.0341	4.0257
-2.2	4.6030	5.1014	5.1519	5.1212	5.0977	5.1697	5.0451	5.1224
-2	5.1643	5.3437	5.0243	5.0799	5.4804	5.4225	5.4075	5.4636
-1.8	6.4157	6.2840	6.1773	6.2041	6.1557	6.4527	6.6841	6.7357
-1.6	7.2345	7.1869	7.2519	7.1848	7.5782	7.1489	7.5200	7.0975
-1.4	8.0246	8.4880	8.3714	8.1509	8.3548	8.0146	8.1534	8.3212
-1.2	10.4393	10.5359	10.0379	10.1612	10.5488	10.6284	10.4840	10.1580
-1	14.1517	14.2845	14.1227	14.1234	14.2418	14.2212	14.8973	14.0773
-0.8	18.3415	18.3122	18.1240	18.4808	18.4122	19.1975	19.4451	19.2842
-0.6	31.7228	31.3918	31.0303	31.6226	31.7121	31.8459	31.0891	31.2642
-0.4	60.8487	60.4124	60.0875	60.7316	60.1210	60.9098	60.2641	60.2371
-0.2	170.2523	165.9812	168.8684	176.5433	174.9737	167.9812	180.3179	171.8848
0	376.0171	370.5981	374.7361	384.8698	383.1093	373.4046	388.3091	378.5510
0.2	171.3464	167.7535	170.2528	175.6488	174.6424	169.5193	177.8442	172.5440
0.4	61.4284	61.1934	61.7331	61.6330	61.1287	61.3030	61.7312	62.0699
0.6	30.5793	30.2012	30.0223	30.6293	31.0190	30.4354	30.2393	30.9037
0.8	18.4488	18.6827	19.2054	19.0400	18.8973	18.0702	19.3469	18.6241
1	14.1226	14.1214	14.0879	14.2484	14.2030	14.1952	14.1821	14.0444
1.2	9.8684	9.5484	9.7548	10.0575	10.0325	9.7108	9.2062	9.9326
1.4	8.0848	8.0400	8.0108	8.0321	8.1234	8.0737	8.0927	8.1221
1.6	7.0414	7.6173	7.0007	7.1204	7.1000	7.4805	7.1644	7.0643
1.8	6.4840	6.3314	6.4414	6.6009	6.5737	6.4145	6.6486	6.5257
2	5.0848	5.0759	5.0421	5.0348	5.0281	5.0363	5.0790	5.0016
2.2	4.3963	4.3184	4.3708	4.4448	4.4322	4.3572	4.4819	4.4104
2.4	4.1608	4.0971	4.1289	4.1604	4.1702	4.1286	4.1624	4.1723
2.6	3.8240	3.8128	3.8177	3.8180	3.8195	3.8390	3.8162	3.8236
2.8	3.4057	3.4454	2.8604	3.2336	3.3645	3.1628	3.3759	3.1281
3	2.2800	2.2550	2.2844	2.2880	2.2208	2.2781	2.2433	2.2882

Table 7 - 12: ARL values for individual control charts for the two-parameter Lindley distribution with scaled weighted variance, with α = 0.0027.

7.9.3. Construction of the EWMA Control Charts for Individual Observations from the Two-Parameter Lindley Distribution Using the Scaled Weighted Variance Method

The procedure for the construction of the individual EWMA two-parameter Lindley control charts with the scaled weighted variance method proposed by Castagliola (2000) will be the following: in equation (2-3) for the traditional EWMA control charts, we will replace L by $\sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)$ for the lower control limit and $\sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)$ for the upper control limit, where μ is the mean of the two-parameter Lindley distribution, which is computed using equation (3-8), and $F_X(x)$ is its cumulative distribution function given by equation (3-7). For the construction of the EWMA control charts we will also need the standard deviation of the two-parameter Lindley distribution computed from equation (3-9).

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the two-parameter Lindley EWMA control chart are as follows.

$$UCL=\frac{\theta+2r}{\theta(\theta+r)}+\sqrt{\frac{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL=\frac{\theta+2r}{\theta(\theta+r)}$$

$$LCL=\frac{\theta+2r}{\theta(\theta+r)}-\sqrt{\frac{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

(7-12)

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our two-parameter Lindley distribution.

<u>7.9.4. Performance Investigation for the Individual EWMA Two-Parameter Lindley Control Charts Constructed With the Scaled Weighted Variance Method</u>

The performance of the proposed individual EWMA chart with the scaled weighted variance method will be investigated once again with the ARL computed by equation (7-10). For the transient probabilities in (7-9) the cumulative distribution function for the two-parameter Lindley distribution, i.e. equation (3-7), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equation (7-12) for $i \to \infty$. This means that the control limits to be used for the computation of ARL will be of the form

$$UCL = \frac{\theta+2r}{\theta(\theta+r)} + \sqrt{\frac{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = \frac{\theta+2r}{\theta(\theta+r)} - \sqrt{\frac{\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}{1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\frac{\theta+r+r\theta x}{\theta+r}e^{-\theta x}\right)}\right)\sqrt{\frac{\theta^2+4\theta r+2r^2}{\theta^2(\theta+r)^2}}\sqrt{\frac{\lambda}{2-\lambda}}$$

(7-13)

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (3-8) and (3-9) in terms of its two parameters, as for the Shewhart-type control chart.

Using those formulae we get Tables 7-12, 7-13, 7-14, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the two-parameter Lindley distribution for various values of the two parameters θ and r of the distribution of concern and for various values of k

which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 7-12 contains the ARL values for λ=0.3 for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=50, above which ARL increases very slightly. For that reason, the suggested value of m for the computation of ARL in the formulae above is m=50. Therefore, Tables 7-13 and 7-14 show the ARL values for m=50 for various values λ for positive and negative shifts, respectively.

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some slight differences in ARL values between those two tables, with most of the ARL values being bigger for the negative shifts than for the positive ones. The only differences (in either direction) that are above 5% concern values of k greater than 0.6 for values of λ greater than 0.12 and values of k greater than 0.8 for values of λ greater than 0.08.

Additionally, comparing the ARL values for the EWMA in Tables 7-13 and 7-14 with the corresponding tables with the ARL values for the EWMA control chart with the skewness correction method, we can see that the EWMA control chart with the weighted scaled variance performs better than the previous one since its in-control ARL values are higher and its out-of-control ARL values are smaller than the corresponding ARL values for the EWMA control chart with skewness correction with most differences between the ARL values for the two different methods being greater than 5% for either positive or negative shifts, which means that when using the scaled weighted variance instead of the skewness correction for the construction of the control chart the improvement of the performance is significant.

m	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
5	0	370.1261	370.2206	370.2201	370.6881	370.1242	370.1264	370.2950	370.3423
	0.2	63.5935	63.2194	63.5391	63.5230	63.6054	63.9014	63.0642	63.4247
	0.5	41.1008	41.0339	41.5496	41.0984	41.3985	41.3283	41.7648	41.0939
	1	10.3883	10.2701	10.1416	10.0774	10.2691	10.4127	10.0561	10.1270
	1.5	5.0212	5.2294	5.1246	5.0821	5.0764	5.3297	5.0451	5.2472
	2	4.1509	4.0504	4.2222	4.4021	4.2553	4.0712	4.0649	4.0267
	2.5	3.1255	3.1295	3.0785	3.1220	3.0347	3.0853	3.0360	3.1487
	3	3.1023	3.0267	2.9687	2.9937	2.9181	3.0379	3.0214	3.0673
10	0	480.8892	480.5558	480.3169	480.1228	480.6304	480.7833	480.1237	480.0920
	0.2	64.5217	64.0790	64.5641	64.3633	64.2741	64.3485	64.4687	64.2053
	0.5	42.1887	42.1283	42.3356	42.4935	42.0575	42.0803	42.0894	42.3280
	1	10.4106	10.2743	10.1886	10.1091	10.2826	10.4635	10.1223	10.2877
	1.5	5.2399	5.2741	5.3068	5.2070	5.1764	5.3454	5.2567	5.3036
	2	4.2350	4.2517	4.3443	4.4534	4.4488	4.3456	4.2918	4.3178
	2.5	3.1414	3.2677	3.1230	3.3201	3.3001	3.3889	3.0955	3.1712
	3	3.1097	3.0629	3.0689	3.2812	3.1250	3.1042	3.0676	3.1537
20	0	510.8680	510.5152	510.9225	510.5932	510.9300	510.5599	510.2650	510.7842
	0.2	64.8085	64.6412	64.5954	64.5158	64.2790	64.6871	64.5178	64.3558
	0.5	42.4197	42.3348	42.8034	42.5987	42.3884	42.5205	42.8642	42.4837
	1	10.4295	10.3019	10.2640	10.3386	10.3821	10.4919	10.1628	10.4636
	1.5	5.3359	5.3742	5.4158	5.3078	5.2432	5.4564	5.3163	5.4558
	2	4.4103	4.3979	4.4550	4.4709	4.4838	4.4142	4.4196	4.5000
	2.5	3.2450	3.5264	3.2604	3.5052	3.3163	3.5379	3.3871	3.2886
	3	3.1562	3.1537	3.1280	3.3355	3.2369	3.1972	3.1230	3.2416
30	0	520.0977	520.1433	520.5877	520.2374	520.0783	520.8647	520.8074	520.8575
	0.2	64.8978	64.9121	64.6078	64.9595	64.2930	64.6915	64.8906	64.4399
	0.5	42.8029	42.5734	42.9619	42.6489	42.5015	42.5659	42.9988	42.6227
	1	10.4592	10.4047	10.3752	10.3612	10.5344	10.5364	10.5281	10.5353
	1.5	5.3759	5.5170	5.4390	5.3705	5.4858	5.5203	5.5762	5.4700
	2	4.4125	4.4532	4.4841	4.5317	4.5771	4.4748	4.4329	4.6757
	2.5	3.4234	3.5337	3.5260	3.6847	3.5284	3.7383	3.5082	3.2956
	3	3.1751	3.1563	3.3034	3.3604	3.4382	3.3988	3.2547	3.2892
40	0	530.1298	530.0737	530.4859	530.4095	530.1640	530.8708	530.3243	530.0616
	0.2	64.9444	64.9474	64.6822	64.9748	64.6297	64.7593	64.9579	64.6174
	0.5	43.2006	43.8767	43.1784	43.2642	43.2923	43.3904	43.1930	43.3488
	1	10.5483	10.6488	10.4563	10.4129	10.7732	10.5955	10.5376	10.5540
	1.5	5.7636	5.6882	5.5031	5.3812	5.5400	5.6125	5.6859	5.6081
	2	4.5443	4.6247	4.5218	4.7367	4.6541	4.6432	4.6441	4.8242
	2.5	3.5464	3.7095	3.5397	3.6961	3.6406	3.7471	3.6392	3.3575
	3	3.4791	3.5570	3.3533	3.4571	3.5393	3.5225	3.2787	3.3310
50	0	530.1625	530.1957	530.8907	530.8125	530.4015	530.9269	530.7792	530.6061
	0.2	64.9719	65.0064	64.7912	64.9936	64.7327	64.9771	65.0367	65.2542
	0.5	43.4812	43.9289	43.7322	43.4583	43.4772	43.4363	43.2426	43.3677
	1	10.5649	10.8492	10.5287	10.5691	10.7847	10.5976	10.5776	10.7204
	1.5	5.8308	5.7145	5.5822	5.5526	5.5531	5.8091	5.7417	5.6223
	2	4.5878	4.8469	4.5293	4.7408	4.7141	4.7375	4.7156	4.8647
	2.5	3.7643	3.7644	3.8757	3.7185	3.7683	3.8794	3.8226	3.6492
	3	3.5486	3.6073	3.3770	3.5517	3.6823	3.5250	3.5035	3.3627
80	0	540.2935	540.3157	540.8370	540.4691	540.7555	540.5697	540.4752	540.4785
	0.2	65.1581	65.2059	65.3778	65.3386	65.8857	65.6016	65.5716	65.3087
	0.5	43.5804	43.9681	43.8637	43.4584	43.6528	43.9934	43.2470	43.6446
	1	10.7790	10.8899	10.5787	10.6860	10.9198	10.6171	10.6377	10.7504
	1.5	5.9087	5.8508	5.6775	5.7154	5.7725	5.9351	5.7426	5.8002
	2	4.7126	4.8559	4.7400	4.7632	4.8512	4.8612	4.8043	4.9301
	2.5	3.8074	3.9051	3.9120	3.7284	3.8109	3.8930	3.9387	3.7035
	3	3.6120	3.8275	3.3844	3.7234	3.7815	3.5316	3.5089	3.4453
100	0	541.0758	541.0477	541.0598	540.8075	540.9473	540.6294	541.0012	541.0336
	0.2	65.9128	65.9956	65.6435	65.3907	65.8897	65.7372	67.0544	67.0724
	0.5	43.9901	44.0674	44.0212	44.0109	44.0560	44.0149	43.7054	44.0409
	1	10.9639	10.9512	10.6502	10.6946	10.9421	10.9232	10.7810	10.8002
	1.5	5.9377	5.9356	5.7023	5.8561	5.7897	5.9907	5.9545	5.9122
	2	4.7373	5.0992	4.8150	5.0257	4.9264	4.8844	4.8482	4.9512
	2.5	4.0006	4.0048	4.0337	3.7549	3.8408	3.9624	3.9873	3.8967
	3	3.8148	3.9300	3.5542	3.7285	3.8257	3.9021	3.6489	3.6249

Table 7 - 13: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (λ=0.3) with scaled weighted variance, with α=0.0027, for various values of m.

λ	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
λ=0.05	0	378.0307	378.8861	378.7348	377.8468	378.3023	377.7287	377.9022	378.0307
	0.2	44.5863	45.0964	44.8985	44.3384	44.9496	44.1785	44.4123	44.5863
	0.4	17.1888	17.5505	17.3898	16.9731	17.4879	16.8496	17.0454	17.1888
	0.6	8.6153	8.8282	8.7229	8.4779	8.8121	8.3882	8.5196	8.6153
	0.8	6.8365	6.9612	6.8936	6.7473	6.9641	6.6887	6.7744	6.8365
	1	5.3755	5.4518	5.4067	5.3156	5.4607	5.2761	5.3338	5.3755
	1.5	3.9515	3.9785	3.9587	3.9249	3.9890	3.9073	3.9330	3.9515
	2	3.5569	3.5686	3.5579	3.5426	3.5772	3.5331	3.5470	3.5569
	2.5	3.3141	3.3197	3.3121	3.3052	3.3265	3.2994	3.3079	3.3141
	3	2.8906	2.8935	2.8891	2.8847	2.8990	2.8807	2.8865	2.8906
λ=0.08	0	383.2804	383.6928	383.4320	382.9353	382.2780	382.7150	383.0390	383.2804
	0.2	48.7699	48.7650	48.4635	48.3642	48.0776	48.1042	48.4864	48.7699
	0.4	17.9912	17.9251	17.7149	17.7057	17.5624	17.5212	17.7919	17.9912
	0.6	9.5208	9.4561	9.3356	9.3565	9.3008	9.2495	9.4064	9.5208
	0.8	6.9498	6.9002	6.8298	6.8538	6.8337	6.7909	6.8830	6.9498
	1	5.1220	5.0750	5.0310	5.0520	5.0461	5.0125	5.0703	5.1220
	1.5	3.4228	3.4032	3.3855	3.3987	3.4017	3.3827	3.4060	3.4228
	2	3.0289	3.0169	3.0078	3.0164	3.0201	3.0082	3.0202	3.0289
	2.5	2.8327	2.8246	2.8191	2.8252	2.8284	2.8202	2.8275	2.8327
	3	2.4592	2.4534	2.4498	2.4543	2.4569	2.4510	2.4558	2.4592
λ=0.10	0	384.6322	387.2985	386.9364	384.2402	385.2182	383.9904	384.3579	384.6322
	0.2	49.6493	51.8348	51.4199	49.2226	50.3326	48.9368	49.3570	49.6493
	0.4	18.0936	18.3212	18.0831	17.7948	18.1284	17.6018	17.8850	18.0936
	0.6	9.3826	9.9738	9.8393	9.2176	9.6209	9.1201	9.2676	9.3826
	0.8	6.6589	6.9641	6.8901	6.5652	6.7930	6.5038	6.5937	6.6589
	1	4.7757	4.9487	4.9043	4.7183	4.8574	4.6805	4.7358	4.7757
	1.5	3.1057	3.1633	3.1464	3.0832	3.1274	3.0684	3.0901	3.1057
	2	2.7597	2.7853	2.7768	2.7483	2.7758	2.7407	2.7518	2.7597
	2.5	2.6042	2.6177	2.6126	2.5974	2.6128	2.5929	2.5995	2.6042
	3	2.2625	2.2703	2.2670	2.2580	2.2688	2.2550	2.2593	2.2625
λ=0.12	0	386.9931	387.8095	387.4190	386.5037	385.8831	388.0439	386.6505	386.9931
	0.2	51.2573	51.0828	50.7008	50.7335	50.5743	51.8643	50.8909	51.2573
	0.4	18.4820	19.3124	19.0548	18.1544	18.5314	18.8014	18.2533	18.4820
	0.6	9.5345	9.4337	9.3084	9.3625	9.3767	9.6829	9.4146	9.5345
	0.8	6.5872	6.5256	6.4564	6.4926	6.5123	6.6418	6.5214	6.5872
	1	4.6180	4.5781	4.5364	4.5612	4.5789	4.6595	4.5786	4.6180
	1.5	2.9126	2.8951	2.8793	2.8910	2.9020	2.9258	2.8976	2.9126
	2	2.5855	2.5757	2.5677	2.5747	2.5818	2.5912	2.5780	2.5855
	2.5	2.4524	2.4461	2.4414	2.4460	2.4510	2.4552	2.4480	2.4524
	3	2.1298	2.1253	2.1222	2.1256	2.1292	2.1212	2.1269	2.1298
λ=0.15	0	388.6258	391.1241	390.6549	388.0637	389.4737	392.0942	388.2321	388.6258
	0.2	52.1559	53.8257	53.3609	51.5795	53.0186	54.7796	51.7526	52.1559
	0.4	18.9225	20.0176	19.7482	18.5818	19.4232	20.2814	18.6846	18.9225
	0.6	9.3172	10.0361	9.9012	9.1454	9.5657	10.3043	9.1975	9.3172
	0.8	6.2612	6.7630	6.6912	6.1693	6.3930	6.9047	6.1973	6.2612
	1	4.2749	4.6454	4.6032	4.2207	4.3520	4.7283	4.2372	4.2749
	1.5	2.6159	2.8231	2.8075	2.5958	2.6442	2.8534	2.6019	2.6159
	2	2.3421	2.4778	2.4701	2.3322	2.3561	2.4927	2.3352	2.3421
	2.5	2.2497	2.3472	2.3426	2.2438	2.2579	2.3559	2.2456	2.2497
	3	1.9572	2.0317	2.0288	1.9534	1.9626	2.0374	1.9545	1.9572
λ=0.20	0	394.5009	392.0554	391.5438	393.6509	395.7984	393.1267	393.9047	394.5009
	0.2	55.7256	53.4980	53.0243	54.9578	56.8851	54.4718	55.1877	55.7256
	0.4	22.2093	21.0376	20.7798	21.8129	22.7938	21.5580	21.9324	22.2093
	0.6	9.6916	9.1294	9.0146	9.5075	9.9581	9.3876	9.5633	9.6916
	0.8	6.2630	5.9786	5.9123	6.1690	6.3976	6.1074	6.1976	6.2630
	1	4.1274	3.9739	3.9360	4.0837	4.2129	4.0483	4.1001	4.1274
	1.5	2.4082	2.3498	2.3360	2.3891	2.4352	2.3765	2.3950	2.4082
	2	2.1481	2.1296	2.1229	2.1288	2.1612	2.1227	2.1417	2.1481
	2.5	2.0780	2.0614	2.0575	2.0726	2.0856	2.0690	2.0743	2.0780
	3	1.8059	1.7951	1.7925	1.8024	1.8108	1.8000	1.8034	1.8059

Table 7 - 14: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (m=50) with scaled weighted variance, with α=0.0027, for various positive shifts

λ	k	θ=48 r=54	θ=57 r=68	θ=62 r=75	θ=75 r=86	θ=84 r=92	θ=93 r=108	θ=100 r=114	θ=120 r=135
λ=0.05	0	378.0307	378.8861	378.7348	377.8468	378.3023	377.7287	377.9022	378.0307
	-0.2	45.6140	45.8729	45.7360	45.4412	45.8678	45.3296	45.4933	45.6140
	-0.4	16.3352	16.2008	16.0077	16.0691	15.9059	15.8986	16.1490	16.3352
	-0.6	8.4978	8.3183	8.1808	8.3035	8.1653	8.1784	8.3620	8.4978
	-0.8	7.2554	7.2419	7.0187	7.1759	7.3701	7.1239	7.2000	7.2554
	-1	5.7775	5.7494	5.7129	5.7278	5.8491	5.6954	5.7429	5.7775
	-1.5	3.9419	3.6570	3.5520	3.8603	3.9957	3.8212	3.8899	3.9419
	-2	3.5007	3.5014	3.5129	3.5024	3.5160	3.5087	3.5012	3.5047
	-2.5	3.3325	3.3021	3.3024	3.3125	3.3042	3.3003	3.3009	3.3125
	-3	2.8025	2.8075	2.8039	2.8127	2.8212	2.8422	2.8391	2.8052
λ=0.08	0	383.2804	383.6928	383.4320	382.9353	382.2780	382.7150	383.0390	383.2804
	-0.2	47.2241	47.1235	46.9084	46.9190	46.7314	46.7237	47.0108	47.2241
	-0.4	16.5372	16.7365	16.6256	16.2842	16.7625	16.0936	16.3737	16.5372
	-0.6	9.0355	9.8330	9.6104	8.8223	9.3463	8.6843	8.8868	9.0355
	-0.8	8.2706	7.2908	7.2308	7.9196	8.7983	7.6956	8.0249	8.2706
	-1	7.3265	7.1873	7.1239	7.2512	7.2269	7.2017	7.2740	7.3265
	-1.5	4.0496	3.6739	3.5565	3.8677	4.3412	3.8753	3.9164	4.0496
	-2	3.5034	3.5055	3.5196	3.5045	3.5254	3.5123	3.5299	3.5068
	-2.5	3.3420	3.3264	3.3051	3.3327	3.3075	3.3048	3.3292	3.3271
	-3	2.8031	2.8080	2.8371	2.8239	2.8228	2.8425	2.8404	2.8144
λ=0.10	0	384.6322	387.2985	386.9364	384.2402	385.2182	383.9904	384.3579	384.6322
	-0.2	47.5952	47.5334	47.2231	47.2517	47.1262	47.0321	47.3550	47.5952
	-0.4	17.0812	18.4607	18.1899	16.8831	17.5202	16.7834	16.9297	17.0812
	-0.6	9.7087	9.9579	9.7424	9.5912	9.8384	9.5150	9.6269	9.7087
	-0.8	9.3691	9.1946	9.1092	9.2483	9.1644	9.1693	9.2849	9.3691
	-1	7.4232	7.7453	7.6884	7.3496	7.5285	7.3012	7.3720	7.4232
	-1.5	4.1049	3.7654	3.7123	3.9161	4.4269	3.9010	3.9724	4.1049
	-2	3.5089	3.5079	3.5273	3.5251	3.5390	3.5276	3.5424	3.5351
	-2.5	3.3459	3.3384	3.3073	3.3365	3.3550	3.3075	3.3322	3.3487
	-3	2.8401	2.8123	2.8456	2.8361	2.8645	2.8514	2.8582	2.8424
λ=0.12	0	386.9931	387.8095	387.4190	386.5037	385.8831	388.0439	386.6505	386.9931
	-0.2	47.6939	48.3856	47.9328	47.2688	48.1061	47.8261	47.3964	47.6939
	-0.4	18.1684	18.0886	17.8177	17.8004	17.6825	18.6124	17.9120	18.1684
	-0.6	10.2050	10.2688	10.1659	9.9433	9.9791	10.5541	10.0223	10.2050
	-0.8	9.6514	10.0255	9.8417	9.5261	9.8317	9.4442	9.5642	9.6514
	-1	7.7618	7.7869	7.7897	7.6838	7.6093	7.8571	7.7075	7.7618
	-1.5	4.1601	4.0649	3.8142	3.9852	4.5579	3.9228	4.0374	4.1601
	-2	3.5124	3.5580	3.5641	3.5418	3.5496	3.5510	3.5488	3.5433
	-2.5	3.3558	3.3485	3.3369	3.3464	3.3710	3.3368	3.3334	3.3637
	-3	2.8856	2.8123	2.8586	2.8647	2.8674	2.8601	2.8645	2.8625
λ=0.15	0	388.6258	391.1241	390.6549	388.0637	389.4737	392.0942	388.2321	388.6258
	-0.2	50.2686	49.2512	48.9291	49.7754	51.0125	49.6702	49.9232	50.2686
	-0.4	19.0057	18.1276	17.6906	18.5638	19.6698	19.0051	18.6963	19.0057
	-0.6	11.2632	10.4499	10.2420	10.9317	10.7547	10.6792	11.0315	11.2632
	-0.8	9.8091	10.1634	9.9695	10.2254	10.1251	9.5637	10.2699	9.8091
	-1	7.9216	7.8636	7.8029	7.8387	8.0399	7.9807	7.8639	7.9216
	-1.5	4.2467	4.2951	4.1267	4.0328	4.5788	4.3936	4.0964	4.2467
	-2	3.5373	3.5728	3.5694	3.5836	3.5498	3.5564	3.5528	3.5549
	-2.5	3.3736	3.3502	3.3401	3.3545	3.3874	3.3389	3.3431	3.3793
	-3	2.8857	2.8719	2.8592	2.8689	2.8729	2.8653	2.8812	2.8746
λ=0.20	0	394.5009	392.0554	391.5438	393.6509	395.7984	393.1267	393.9047	394.5009
	-0.2	51.0561	52.6263	52.2143	50.3003	52.2123	53.4769	50.5258	51.0561
	-0.4	20.8655	21.6447	21.2697	20.1525	21.9581	21.4174	20.3452	20.8655
	-0.6	11.6126	12.2495	12.9577	11.0405	12.4878	12.0445	11.2126	11.6126
	-0.8	9.9723	10.3729	10.2062	10.3403	10.7783	10.3257	10.3044	9.9723
	-1	8.2169	8.5435	8.4605	8.0740	8.4228	8.7068	8.1274	8.2169
	-1.5	4.8264	5.2612	5.0354	4.5583	4.6172	4.5988	4.6377	4.8264
	-2	3.5396	3.5900	3.5890	3.5839	3.5675	3.5638	3.5602	3.5785
	-2.5	3.3788	3.3555	3.3800	3.3881	3.3895	3.3444	3.3442	3.3802
	-3	2.8863	2.8847	2.8603	2.8731	2.8804	2.8849	2.8885	2.8859

Table 7 - 15: ARL values for individual EWMA control charts for the two-parameter Lindley distribution (m=50) with scaled weighted variance, with α=0.0027, for various negative shifts

7.9.5 Example on the Two-Parameter Lindley Individual Shewhart-Type and EWMA Control Charts with Scaled Weighted Variance Using Simulated Data

This section contains the illustration of the proposed control charts by means of simulated data generated from the distribution of concern. The case of real data will be presented in section 7.9.6. For the same data set in Table 7-9, we construct the individual Shewhart-type and EWMA two-parameter Lindley control charts with scaled weighted variance which are presented in Figures 7-10 and 7-11, using the most commonly used value for the significance level α = 0.27%, as mentioned earlier.

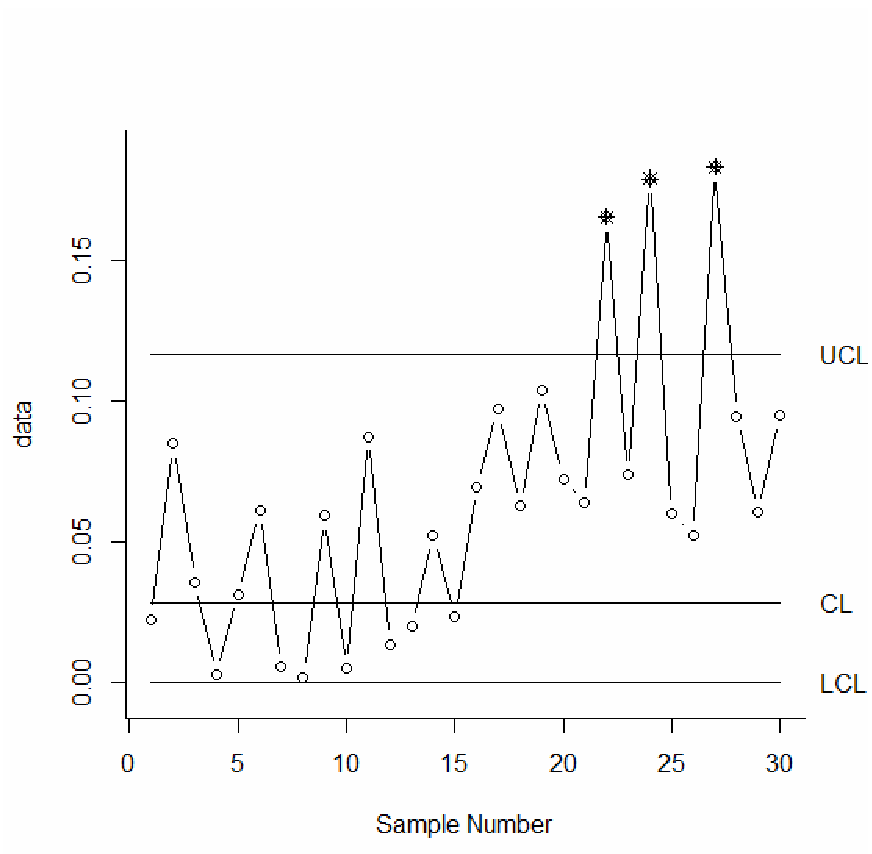

Figure 7 - 10: Individual two-parameter Lindley control chart with scaled weighted variance for the data set in Table 7-9 with a shift of one standard deviation unit in the process mean.

As we can see in the charts, there is an increasing trend after the first 15 observations and the control charts detect some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level. Compared to the charts in Figure 7-2 and Figure 7-3 the chart in Figure 7-10 detects the same out-of-control points but has narrower limits, while the EWMA in Figure 7-11 now presents the first out-of-control point one observation sooner than the EWMA with the skewness correction.

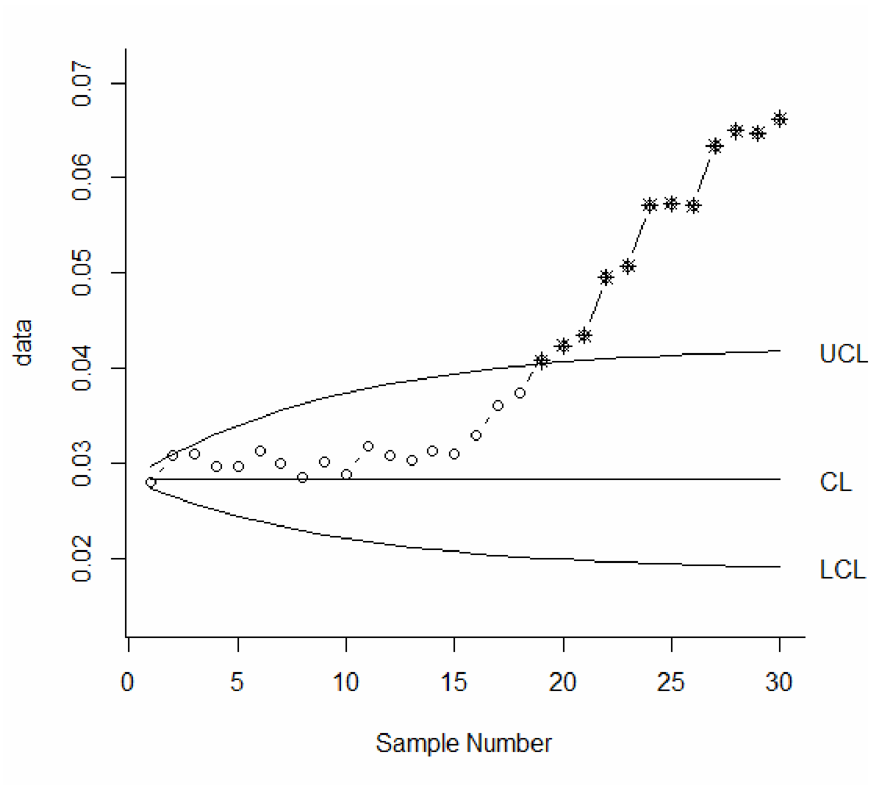

Figure 7 - 11: Individual EWMA two-parameter Lindley control chart with scaled weighted variance for the data set in Table 7-9 with a shift of one standard deviation unit in the process mean.

7.9.6 Application of the two-parameter Lindley individual Shewhart-type and EWMA control charts with scaled weighted variance to real data

This section deals with the illustration of the proposed control charts through application to the same real data sets as in Tables 7-10 and 7-11. For the first case of the waiting times dataset the two-parameter Lindley control chart with scaled weighted variance which can be seen in Figure 7-12, detects an out-of-control point which the control chart in Figure 7-5 did not detect, but the individual EWMA two-parameter Lindley control chart with scaled weighted variance presented in Figure 7-13 does not detect any out-of-control points probably due to the inertia effect we stated in Chapter 2. The weight to the present data given by the value of λ=0.08 is not high enough for the chart to respond quickly to the shift in the opposite direction after the previous low values, so this control chart did not detect the out-of-control point that the chart in Figure 7-12 did.

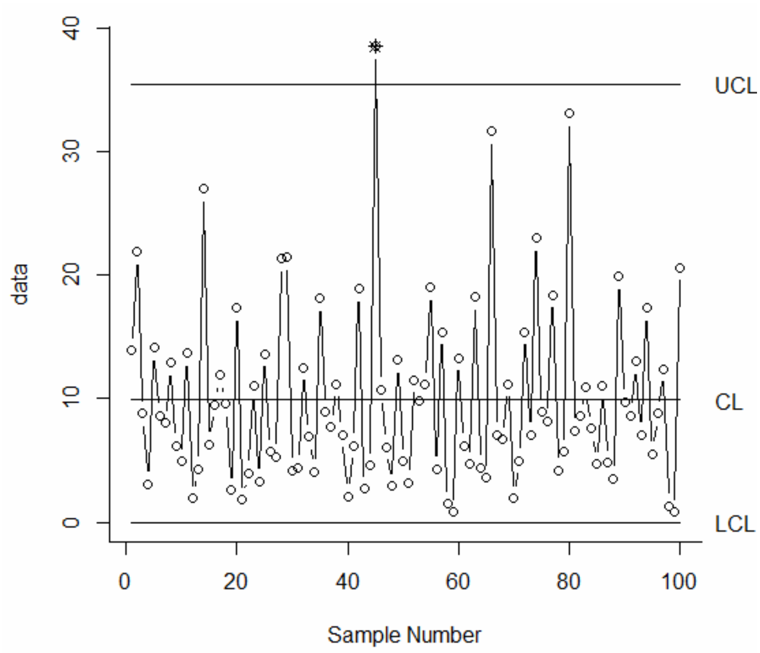

Figure 7 - 12: Individual two-parameter Lindley control chart with scaled weighted variance for the waiting times dataset

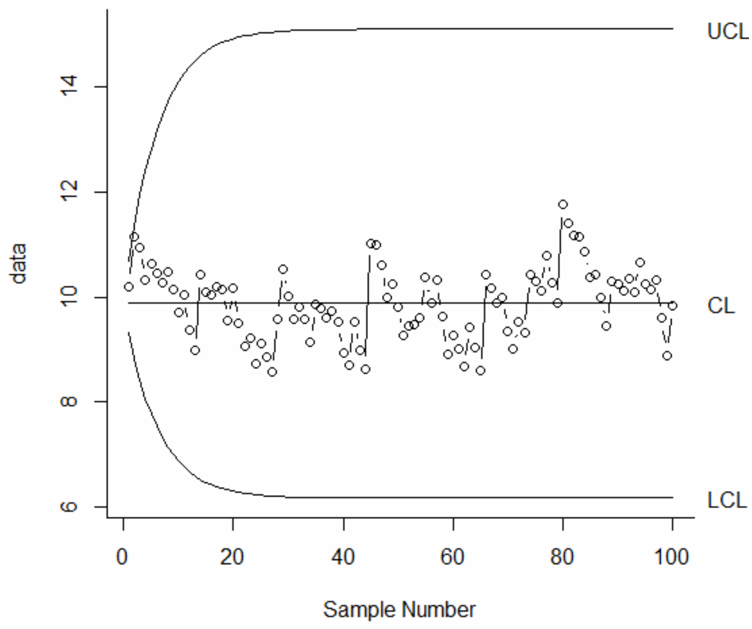

Figure 7 - 13: Individual EWMA two-parameter Lindley control chart with scaled weighted variance for the Waiting Times data set

For the second data set on the time intervals between failures of airplane air-conditioning equipment, the corresponding individual two-parameter Lindley and EWMA control charts with scaled weighted variance are presented in Figure 7-14 and Figure 7-15, respectively. Both charts have detected out-of-control points, which the corresponding control charts with the skewness correction had not detected.

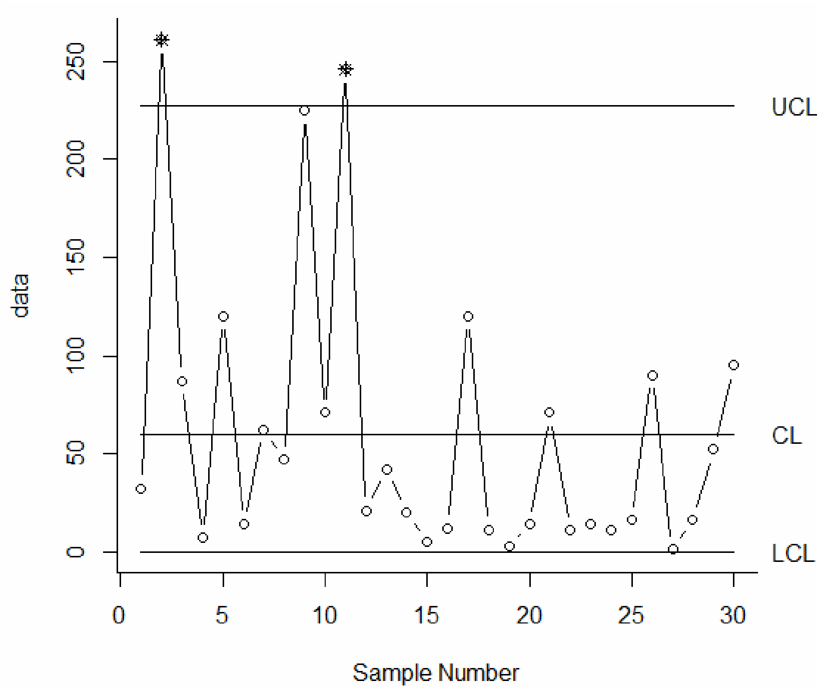

Figure 7 - 14: Individual two-parameter Lindley control chart with scaled weighted variance for the aircraft air-conditioning equipment failure data set

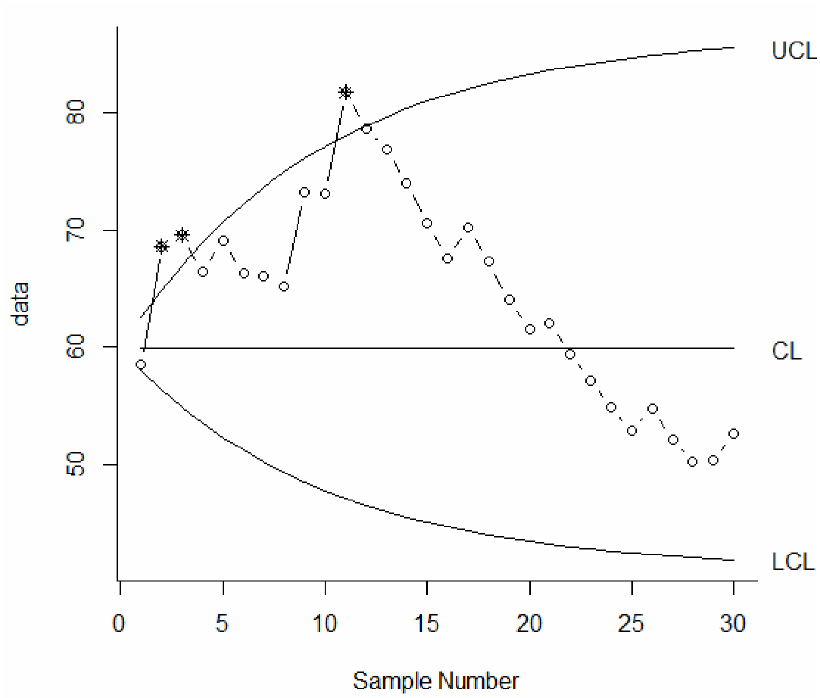

Figure 7 - 15: Individual EWMA two-parameter Lindley control chart with scaled weighted variance for the aircraft air-conditioning equipment failure dataset

7.10 Conclusions and Further Research

In this chapter probability-type, Shewhart-type and EWMA control charts have been constructed for monitoring individual observations from a process which is assumed to follow the two-parameter Lindley distribution for the theoretical scenario of known distributions' parameters. Two different methods for taking into account the distribution's skewness have been considered. The performance of the proposed control charts has been investigated for the cases of all the proposed control charts (probability-type, Shewhart-type and EWMA control charts with both skewness correction methods). Optimal design for the EWMA control chart has also been presented. The five types of proposed control charts have been illustrated with both simulated and real data.

The proposed control charts take into account the skewness of the distribution and this leads to a significant improvement of their performance as has been demonstrated along this chapter. The performance of the control charts seems to improve more when the scaled weighted variance method by Castagliola (2000) is used instead of the skewness correction method proposed by Chan and Cui (2003).

This study can also be applied to other Lindley-related distributions (generalizations, mixtures, transformations, etc.). Moreover, for future research, the whole analysis can be extended to include supplementary runs rules for the detection of small shifts. For this purpose it would also be useful to construct CUSUM control charts for the two-parameter Lindley distribution, as well.

CHAPTER 8

CONTROL CHARTS FOR INDIVIDUAL OBSERVATIONS FROM THE LOGARITHMIC DISTRIBUTION

8.1 Introduction

As discussed in chapter 4, Logarithmic distribution is a discrete distribution with various applications some of which are in ecology and biology, purchase studies, engineering and water resources, medicine, pharmacology, biochemistry, molecular biology, genetics, biotechnology, population growth and human ecology, agriculture, entomology, bacteriology, demography, science of accidents, environmental sciences, marine sciences, geosciences, soil science, meteorology and atmospheric sciences, physics and physical chemistry, applied chemistry, food science and technology, nanoscience and nanotechnology, computer science, telecommunications and others. Due to its variety of applications, it is of significant importance that control charts for detecting shifts in a process should be constructed when the quality characteristic of interest follows a Logarithmic distribution. Here we construct probability-type, Shewhart-type and EWMA control charts (and deal with the optimal choice of its parameter) for individual observations from the Logarithmic distribution, using two different methods for taking into account the distribution’s skewness, investigate the performance of all the proposed charts, compare them and illustrate them using examples with both simulated and real data. The whole analysis reveals the superiority of using skewness correction for the construction of the control charts against not using it, as well as the superiority of the scaled weighted variance as a method for considering the distribution’s skewness when constructing Shewhart-type and EWMA control charts.

More specifically, this chapter is organized as follows: Sections 8.2 and 8.3 discuss the construction of probability-type and Shewhart-type control charts with skewness correction as in Chan and Cui (2003), respectively, for monitoring individual observations from a Logarithmic distribution, while section 8.4 investigates the performance of those two charts and compares them, revealing the superiority of the Shewhart-type control charts over the probability-type ones. Sections 8.5 and 8.6 deal with the construction and performance investigation, respectively, of the EWMA control charts using the same skewness correction method, revealing the superiority of the proposed chart over the one without the skewness correction. Section 8.7 addresses the optimal design of the EWMA control charts of section 8.5. All the proposed control charts of the previous sections are illustrated in section 8.8 with both simulated and real data. Section 8.9 discusses the use of the scaled weighted variance method by Castagliola (2000) for the construction of Shewhart-type and EWMA control charts (subsections 8.9.1 and 8.9.3). The performances of these charts are investigated (subsections 8.9.2 and 8.9.4) and compared with the corresponding control charts of sections 8.3 and 8.5, revealing the superiority of the control charts with the scaled weighted variance method. This is also verified with the illustration of the proposed charts through application to the same simulated and real data as in section 8.8 (subsections 8.9.5 and 8.9.6, respectively).

8.2 Probability-Type Control Charts for Individual Observations from the Logarithmic Distribution

The control limits of the individual Logarithmic probability-type control charts will be derived in terms of the probability of type I error or false alarm rate, α, using our distribution of interest (see for example, Chang and Gan (1999) for the case of the modified geometric distribution). For this procedure, we need to use the cumulative probability of the Logarithmic distribution as presented in equation (4-2). The construction procedure is as follows.

For a significance level α, we have

$$P\left(X < LCL\right) \leq \frac{\alpha}{2}$$

and

$$P\left(X < LCL\right) = -\frac{1}{\ln\left(1-\theta\right)}\sum_{u=1}^{LCL}\frac{\theta^u}{u}, \quad LCL > 0, \quad 0 < \theta < 1,$$

from which we obtain

$$-\frac{1}{\ln\left(1-\theta\right)}\sum_{u=1}^{LCL}\frac{\theta^u}{u} \le \frac{\alpha}{2}.$$

Taking the maximum of the inequality above, we acquire

$$\sum_{u=1}^{LCL}\frac{\theta^u}{u} = -\frac{\alpha}{2}\ln\left(1-\theta\right) \tag{8-1}$$

and solving this equation we obtain the expression for LCL (see below). Similarly, for the upper control limit, we have

$$P\left(X > UCL\right) \ge \frac{\alpha}{2}$$

and

$$P\left(X > UCL\right) = 1 - P\left(X \le UCL\right) = 1 + \frac{1}{\ln\left(1-\theta\right)}\sum_{u=1}^{UCL}\frac{\theta^u}{u}, \quad 0 < \theta < 1,$$

from which we get that

$$1 + \frac{1}{\ln\left(1-\theta\right)}\sum_{u=1}^{UCL}\frac{\theta^u}{u} \ge \frac{\alpha}{2}.$$

Taking the minimum of the inequality above, we take

$$\sum_{u=1}^{UCL}\frac{\theta^u}{u} = \left(\frac{\alpha}{2} - 1\right)\ln\left(1-\theta\right) \tag{8-2}$$

and solving this equation we obtain the expression for UCL (see below).

For the computation of the sum required for finding the values of LCL and UCL, we will use the following equation we will use the following equation [Dwight (1934)]:

$$\int x^{2n+1}\ln|x^2 - \mathrm{a}^2|\,dx = \frac{1}{2n+2}\left\{\left(x^{2n+2} - \mathrm{a}^{2n+2}\right)\ln|x^2 - \mathrm{a}^2| - \sum_{k=1}^{n+1}\frac{1}{k}\mathrm{a}^{2n-2k+2}x^{2k}\right\}$$

For a = 1, and setting w = n + 1 and then y = x^2, the equation becomes

$$\sum_{k=1}^{w}\frac{y^k}{k} = \left(y^w - 1\right)\ln|y-1| - w\int y^{w-1}\ln|y-1|\,dy \tag{8-3}$$

Combining equations (8-1) and (8-3) we conclude that

$$\left(\theta^{LCL} - 1\right)\ln|\theta - 1| - LCL\int\theta^{LCL-1}\ln|\theta-1|\,d\theta = -\frac{\alpha}{2}\ln\left(1-\theta\right)$$

Differentiating with respect to θ and considering that c is a positive constant, we result in

$$\left(\theta^{LCL}-1\right)\frac{1}{|\theta-1|}=\frac{\alpha}{2}\frac{1}{1-\theta}+c$$

Considering that 0 < θ < 1 and for an appropriate value of c so that both sides of the equation above are negative, we get

$$\theta^{LCL}=1-\frac{\alpha}{2}\frac{1}{1-\theta}|\theta-1|$$

and since

$$0<\theta<1\Rightarrow|\theta-1|=1-\theta, \tag{8-4}$$

we will finally have

$$\theta^{LCL}\overset{(8\text{-}4)}{=}1-\frac{\alpha}{2}\frac{1}{1-\theta}\left(1-\theta\right)=1-\frac{\alpha}{2}\Rightarrow\ln\left(\theta^{LCL}\right)=\ln\left(1-\frac{\alpha}{2}\right)\Rightarrow$$

$$\Rightarrow LCL=\frac{\ln\left(1-\frac{\alpha}{2}\right)}{\ln\left(\theta\right)}$$

Similarly, for UCL, when combining equations (8-2) and (8-3), and then differentiating with respect to θ, we result in

$$\left(\theta^{UCL}-1\right)\frac{1}{|\theta-1|}=\left(1-\frac{\alpha}{2}\right)\frac{1}{1-\theta}+c\,.$$

Considering that 0 < θ < 1 and for an appropriate value of c so that both sides of the equation above are negative, we take

$$\theta^{UCL}=1-\left(1-\frac{\alpha}{2}\right)\frac{1}{1-\theta}|\theta-1|\overset{(8-4)}{=}1-\left(1-\frac{\alpha}{2}\right)=\frac{\alpha}{2}\Rightarrow$$

$$\Rightarrow\ln\left(\theta^{UCL}\right)=\ln\left(\frac{\alpha}{2}\right)\Rightarrow$$

$$\Rightarrow UCL=\frac{\ln\left(\frac{\alpha}{2}\right)}{\ln\left(\theta\right)}$$

Similarly for the central line we obtain

$$CL=\frac{\ln\left(1-0.5\right)}{\ln\left(\theta\right)}=\frac{\ln\left(0.5\right)}{\ln\left(\theta\right)}$$

As a result from all the above, the control limits of the chart in terms of the probability of type I error, α, are as follows.

$$UCL_\alpha = \frac{\ln\left(\frac{\alpha}{2}\right)}{\ln(\theta)}$$

$$CL_\alpha = \frac{\ln(0.5)}{\ln(\theta)} \quad , \quad 0 < \theta < 1 \qquad (8\text{-}5)$$

$$LCL_\alpha = \frac{\ln\left(1-\frac{\alpha}{2}\right)}{\ln(\theta)}$$

8.3 Shewhart-Type Control Charts for Individual Observations Coming from the Logarithmic Distribution

In this subsection, we discuss a different approach for the construction of individual Logarithmic control charts, based on the Shewhart-type individual control charts using the skewness correction as in Chan and Cui (2003). More specifically, following the general guidelines in equation (2-1), the construction procedure according to this method is as follows: the central line is placed at the mean of the Logarithmic distribution, which is computed using equation (4-3), while the control limits are placed around the mean at L times its standard deviation (the square root of the quantity computed by equation (4-4)) plus c_4^* times its standard deviation, where $c_4^*(x) = \frac{\frac{4}{3}\left[sk(x)\right]}{1+0.2\left[sk(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient computed from equation (4-5). This means that the skewness correction for the Logarithmic distribution will be

$$c_4^*(x) = \frac{4}{3}\frac{\left(1+\theta-3b\theta+2b^2\theta^2\right)(b\theta)^{1/2}(1-b\theta)^{3/2}}{b\theta(1-b\theta)^3+0.2\left(1+\theta-3b\theta+2b^2\theta^2\right)^2}, \text{ where } b = -\frac{1}{\ln(1-\theta)} \qquad (8\text{-}6)$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Logarithmic control chart are as follows.

$$UCL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}+\left[L+c_4^*\left(\bar{x}\right)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}$$

$$CL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}\qquad ,\quad 0<\theta<1$$

$$LCL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}+\left[-L+c_4^*\left(\bar{x}\right)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}$$

(8-7)

8.4 Performance Investigation for the Individual Logarithmic Control Charts

The performance of the individual logarithmic control charts is going to be investigated in this section using the ARL_0 and ARL_1 values, computed as follows:

$$ARL_0=\frac{1}{1-F_{in}\left(UCL\right)+F_{in}\left(LCL\right)} \tag{8-8}$$

where $F_{in}\left(x\right)$ is the cumulative distribution function of the Logarithmic distribution in equation (4-2) with in-control parameter and control limits as computed with equation (8-5) for the probability-type control charts or equations (8-7) and (8-6) for the Shewhart-type control charts and

$$ARL_1=\frac{1}{1-F_{out}\left(UCL\right)+F_{out}\left(LCL\right)} \tag{8-9}$$

where $F_{out}\left(x\right)$ is the cumulative distribution function for the distribution of concern with out-of-control parameter and same control limits as before. For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1=\mu_0+k\sigma$. Using this relationship, the new parameter of the distribution with the shifted mean will be computed by combining equations (4-3) and (4-4) and solving in terms of the distribution's parameter. The resulting value for the new parameter is given by $\theta_{new}=\frac{\sigma_{new}^2-\left(\mu_0+k\sigma\right)+\left(\mu_0+k\sigma\right)^2}{\left(\mu_0+k\sigma\right)^2+\sigma_{new}^2}$. Using the above formulas we obtain Tables

8-1 and 8-2, which show the in-control and out-of-control ARL values for the individual probability-type and individual Shewhart-type control chart, respectively, for the Logarithmic distribution for various values of the parameter θ of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the probability-type control charts we have chosen a significance level equal to the most commonly used value of 0.27%, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
-3	3.0807	3.2548	3.3255	3.4844	3.5482	3.6102	3.6973	3.7151
-2.8	4.1209	4.2828	4.3509	4.5059	4.5780	4.6280	4.6307	4.8075
-2.6	6.1432	6.3101	6.3779	6.4188	6.6093	6.6393	6.8184	6.8486
-2.4	8.1778	8.3412	8.4069	8.5434	8.6218	8.6806	8.7289	9.1975
-2.2	10.2146	10.3734	10.4378	10.5597	10.6460	10.6934	10.7322	10.8145
-2	12.2537	12.4086	12.5706	12.5977	12.6439	12.6817	12.7177	12.7553
-1.8	15.3048	15.4428	15.5054	15.5986	15.6284	15.6989	15.7791	16.2848
-1.6	16.3480	16.4819	16.5419	16.6210	16.6489	16.7840	16.9307	17.3128
-1.4	22.3728	22.5218	22.5702	22.6448	22.7577	22.7995	22.9610	23.4251
-1.2	30.4488	30.5932	30.6896	30.7295	30.8248	30.8360	31.0862	31.1046
-1	43.4843	43.6998	43.7160	43.7709	43.8235	43.8954	44.3717	44.5755
-0.8	60.7516	60.7548	60.7716	60.8420	60.8468	61.0793	61.2468	61.7148
-0.6	78.8036	78.8127	78.8215	78.8424	78.8715	78.9616	79.0400	79.3500
-0.4	121.8639	121.8673	121.8757	121.8812	121.9088	121.9391	121.9759	122.1254
-0.2	205.9314	205.9318	205.9368	205.9371	205.9543	205.9597	205.9753	206.0359
0	370.0648	370.1578	370.2671	370.3281	370.4475	370.6751	370.7929	371.1805
0.2	204.1930	203.8152	203.7025	203.4628	203.3642	203.3021	203.1890	203.0872
0.4	120.2073	119.8486	119.7378	119.5087	119.3996	119.3369	119.2195	119.1086
0.6	75.2437	74.8877	74.7772	74.5468	74.4357	74.3715	74.2489	74.1289
0.8	57.9396	57.8684	57.7863	57.7840	57.7530	57.6975	57.6843	57.6488
1	42.3319	41.9723	41.8699	41.6220	41.5048	41.4378	41.3048	41.1680
1.2	30.3784	30.0148	29.9009	29.6481	29.5391	29.4693	29.3309	29.1845
1.4	21.4244	21.0461	20.9507	20.6930	20.5710	20.4882	20.3557	20.2010
1.6	16.4693	16.0960	15.9790	15.7260	15.6014	15.5278	15.3795	15.2166
1.8	14.5124	14.1241	14.0157	13.7575	13.6303	13.5548	13.4019	13.2214
2	12.5535	12.1703	12.0503	11.7872	11.6686	11.5906	11.4231	11.2452
2.2	10.5916	10.2045	10.0531	9.8152	9.6842	9.6048	9.4431	9.2573
2.4	8.6395	8.2364	8.1240	7.8415	7.7073	7.6275	7.4620	7.2806
2.6	6.4640	6.2668	6.1430	5.8661	5.7300	5.6480	5.4898	5.2821
2.8	5.6963	5.3050	5.1701	4.8891	4.7512	4.6693	4.3955	4.2730
3	3.9512	3.9364	3.6954	3.4105	3.2710	3.1880	3.0122	2.8031

Table 8 - 1: ARL values for individual probability-type control charts for the Logarithmic distribution, with α = 0.0027.

k	θ=0.12, L=2.541	θ=0.26, L=2.7125	θ=0.39, L=2.7125	θ=0.45, L=3.0395	θ=0.54, L=2.542	θ=0.68, L=2.714	θ=0.73, L=2.715	θ=0.84, L=2.5405
-3	2.6021	2.6246	2.6459	2.6893	2.8212	3.0204	3.0728	3.1275
-2.8	3.6198	3.6327	3.6639	3.6869	3.7518	4.0280	4.0884	4.1288
-2.6	5.7072	5.7206	5.7517	5.7953	5.8204	6.0373	6.1084	6.1284
-2.4	7.5982	7.7093	7.7281	7.8412	8.0262	8.0871	8.1080	8.1289
-2.2	10.0484	10.0652	10.0857	10.1059	10.1262	10.1570	10.1680	10.1884
-2	12.1019	12.1228	12.1437	12.1643	12.1844	12.2048	12.2275	12.2459
-1.8	14.9739	15.1091	15.1201	15.1415	15.1621	15.1826	15.2064	15.2197
-1.6	15.9691	16.0053	16.0205	16.1052	16.1253	16.1689	16.1890	16.2548
-1.4	21.9822	22.0071	22.0284	22.0800	22.1036	22.1459	22.1680	22.1890
-1.2	30.0446	30.0701	30.0905	30.0976	30.1218	30.1439	30.1652	30.1825
-1	42.9868	43.0051	43.0275	43.0509	43.1045	43.1269	43.1597	43.1710
-0.8	59.9519	59.9739	60.0050	60.0535	60.0734	60.0951	60.1090	60.1464
-0.6	77.9766	77.9961	78.0150	78.0359	78.0530	78.0753	78.0899	78.1014
-0.4	120.9930	121.0127	121.0393	121.0486	121.0751	121.0826	121.0934	121.1080
-0.2	204.9877	205.0031	205.0263	205.0455	205.0633	205.0868	205.1012	205.1232
0	370.9368	370.8846	370.8284	370.8042	370.7725	370.7168	370.6981	370.6842
0.2	202.9324	202.8682	202.8168	202.7935	202.7554	202.7125	202.6953	202.6641
0.4	118.9301	118.8448	118.8052	118.7821	118.7578	118.7075	118.6915	118.6624
0.6	73.9096	73.8432	73.7951	73.7725	73.7595	73.7023	73.6873	73.6602
0.8	57.9005	57.8432	57.6048	57.4814	57.4052	57.2864	57.2775	57.1485
1	40.8916	40.8245	40.7786	40.7554	40.7375	40.6930	40.6896	40.6455
1.2	28.8846	28.8170	28.7719	28.7598	28.7325	28.6890	28.6862	28.6434
1.4	19.8795	19.8105	19.7541	19.7541	19.7284	19.6848	19.6634	19.6416
1.6	14.8752	14.8048	14.7512	14.7390	14.7248	14.6826	14.6609	14.6400
1.8	12.8695	12.7998	12.7548	12.7346	12.7216	12.6800	12.6688	12.6487
2	10.8642	10.7955	10.7530	10.7308	10.7189	10.6678	10.6670	10.6377
2.2	8.8615	8.7917	8.7596	8.7275	8.7168	8.6645	8.6459	8.6368
2.4	6.8482	6.7882	6.7368	6.7245	6.7148	6.6844	6.6442	6.6362
2.6	4.8453	4.7854	4.7542	4.7220	4.7133	4.6631	4.6432	4.6357
2.8	3.8426	3.7828	3.7521	3.7219	3.7198	3.6620	3.6424	3.6354
3	2.8402	2.7805	2.7502	2.7178	2.7108	2.6612	2.6418	2.6352

Table 8 - 2: ARL values for individual Shewhart-type control charts for the Logarithmic distribution

Comparison of Tables 8-1 and 8-2 reveals the improvement in the performance of the chart when the skewness corrected limits are used instead of the probability-based ones. The difference in ARL values between Shewhart-type and probability-type control charts is greater than 5% for all shift sizes of magnitude equal or greater than k=1.6. Comparison of the ARL values for positive and negative shifts shows that, although the control charts can detect both positive and negative shifts well, there are some slight differences with most values being a little higher for the negative shifts than for the corresponding positive ones. This holds for either the probability-type or the Shewhart-type control chart. The differences (in either direction) that are above 5% concern the shifts corresponding to large values of k for large values of the parameter θ of the logarithmic distribution for the probability-type control charts and values of k between 0.6 and 1.8 for the Shewhart-type control charts.

8.5 Construction of the EWMA Control Charts for Individual Observations from the Logarithmic distribution

When monitoring individual observations, besides Shewhart-type control charts we need to construct EWMA charts, too, as a better alternative (see Section 2.14.2). So it is useful to also construct EWMA control charts for the Logarithmic distribution. In order to do that, we need to remember the general form (2-3) for constructing EWMA control charts and the plotting statistic in equation (2-2), bearing in mind that the constant λ represents the weight assigned to each of the past values and needs to be smaller for detecting smaller shifts. The control limits in (2-3) will be constructed here using the skewness correction in Chan and Cui (2003), since the distribution of concern is asymmetric and, as also mentioned in Weiß and Atzmüller (2011), this is an easily applied method for taking the distribution's skewness into consideration and leads to a better ARL performance of the resulting control chart. In the next section, where we deal with the performance investigation of the constructed control chart, we will further demonstrate the need for this adjustment considering the asymmetry of the distribution and the improvement in the performance of the chart when using the skewness correction contrary to not using it but using the traditionally used symmetric EWMA control limits instead.

The construction procedure for the individual Logarithmic control charts will be the following: in equation (2-3) we will replace L by L plus c_4^*, where

$c_4^*(x) = \frac{\frac{4}{3}\left[\text{sk}(x)\right]}{1+0.2\left[\text{sk}(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient. EWMA control charts for individual observations from the Logarithmic distribution are constructed using the mean of the Logarithmic distribution, which is computed using equation (4-3), its standard deviation (the square root of the quantity computed by equation (4-4)) and the distribution's skewness coefficient computed from equation (4-5). This means that the skewness correction for the Logarithmic distribution will be

$$c_4^*(x) = \frac{4}{3}\frac{\left(1+\theta-3b\theta+2b^2\theta^2\right)(b\theta)^{1/2}(1-b\theta)^{3/2}}{b\theta(1-b\theta)^3+0.2\left(1+\theta-3b\theta+2b^2\theta^2\right)^2}, \text{ where } b = -\frac{1}{\ln(1-\theta)} \tag{8-10}$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Logarithmic EWMA control chart are as follows.

$$UCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \left[L + c_4^*(x)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}$$

$$LCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \left[-L + c_4^*(x)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$
(8-11)

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our Logarithmic distribution.

8.6 Performance Investigation for the EWMA Control Charts for Individual Observations from the Logarithmic Distribution

We will investigate the performance of the control chart constructed above, using the ARL, following Lucas and Saccucchi (1990). In other words, the ARL of the EWMA control chart will be computed through the Markov chain method and discretization of the control statistic. More specifically, according to this method, the region between the upper and lower control limits is divided into $2m+1$ subintervals. Each subinterval S_j ($j=1,2,\ldots,2m+1$) is taken to be represented by its midpoint s_j and then if δ is the half size of each subinterval, which means that $\delta = \frac{UCL - LCL}{2\left(2m+1\right)}$, then whenever $s_j - \delta < Z_i < s_j + \delta$ the process is in a transient state. Otherwise, the process is in the absorbing state. Therefore, the in-control transition probability from one transient state S_j to another transient state S_k is given by

$$\begin{aligned} p_{kj} &= P\left(Z_i \in S_k \,\middle|\, Z_{i-1} \in S_j\right) \\ &= P\left(s_k - \delta < Z_i < s_k + \delta \,\middle|\, Z_{i-1} = s_j\right) \\ &= P\left(s_k - \delta < \lambda X_i + \left(1-\lambda\right) Z_{i-1} < s_k + \delta \,\middle|\, Z_{i-1} = s_j\right) \\ &= P\left(\frac{s_k - \delta - \left(1-\lambda\right)s_j}{\lambda} < X_i < \frac{s_k + \delta - \left(1-\lambda\right)s_j}{\lambda}\right), \quad j,k = 1,2,\ldots,2m+1 \end{aligned} \tag{8-12}$$

The ith-stage transition probability matrix $\mathbf{P}^i$ is, then, defined as $\mathbf{P}^i = \begin{pmatrix} \mathbf{R}^i & \left(\mathbf{I} - \mathbf{R}^i\right)\mathbf{1} \\ \mathbf{0}^T & 1 \end{pmatrix}$, where $\mathbf{R}$ is the ($2m+1$, $2m+1$) matrix of the transient probabilities p_{kj} mentioned in (8-12) above and $\mathbf{0}^\mathrm{T}=(0,0,\ldots,0)$, i.e. $\mathbf{0}^\mathrm{T}$ is the transpose of $\mathbf{0}$ which is a vector of $2m+1$ zeros. The ith-stage transition probability matrix $\mathbf{P}^i$ contains the probabilities that the control statistic goes from one transient state to another in i steps and is used for the computation of the ARL of the EWMA control chart, which is given by

$$ARL = \mathbf{p}^T \left(\mathbf{I} - \mathbf{R}\right)^{-1} \mathbf{1} \tag{8-13}$$

where $\mathbf{p} = \left(p_{-m}, p_{-m+1}, \cdots p_{m-1}, p_m\right)^T$ is the vector of the initial probabilities related to the $2m+1$ transient states.

For the transient probabilities in (8-12) the cumulative distribution function for the Logarithmic distribution, i.e. equation (4-2), is going to be used with either in-control parameter for the case of computing the in-control ARL value or the out-of-control parameter for the case of the out-of-control ARL, with the asymptotic control limits as computed with equations (8-11) and (8-10) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \left[L + c_4^*(x)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \left[-L + c_4^*(x)\right]\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}}$$

(8-14)

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameter of the distribution with the shifted mean will be computed by combining equations (4-3) and (4-4) and solving in terms of its parameter, as for the Shewhart-type control chart.

Using those formulae we get Tables 8-3, 8-4, 8-5, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the Logarithmic distribution for various values of the parameter θ of the distribution of concern and for various values of k which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 8-3 contains the ARL values for λ=0.3 and L=6.876 (combination which gives in-control ARL value close to 370) for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ and L the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=180, above which ARL increases very slightly. Thus, the suggested value of m for the computation of ARL in the formulae above is m=180. Therefore, Tables 8-4 and 8-5 show the ARL values for m=180 for various values of L and λ for positive and negative shifts, respectively.

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some slight differences in ARL values between those two tables, with most of the differences being in favour of the ARL values for negative shifts. In general, for values of the parameter θ of the logarithmic distribution equal to or greater than 0.45 the ARL value is bigger for the negative shifts. This is sensible, because the larger the value of the parameter the easier it is to get out of control with a positive shift than for a negative one, and vice-versa. This is probably the reason that the differences (in either direction) are above 5% for large shifts for both very small and very large values of the parameter θ.

The need for using the skewness correction for the construction of the individual EWMA control charts for the Logarithmic distribution is justified by the fact that if we had used the traditional symmetric EWMA control limits without the skewness correction term $c_4^*(x)$ in equation (8-14) above, the ARL performance of the chart would have been worse, as can be seen when comparing the results in Table 8-6 for the case of not using the skewness correction term against the results in Table 8-4 for the case of using it. It should be noted that the ARL values in Table 8-6 have resulted from using the same values for λ and L as the ones in Table 8-4 for the shake of making comparisons between the two tables easier. The differences between the ARL values in Tables 8-4 and 8-6 are almost all higher than 5%. The only values for which the difference is less than 5% concern the values of k=±0.2 for all the values of the parameter θ and the values of k=±0.8 for values of θ equal to or greater than 0.45. Comparison is similar for the case of negative shifts so the corresponding table is omitted for space reasons.

m	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
80	0	370.7242	370.1580	370.0642	370.5489	370.2684	370.3737	370.5414	370.2848
	0.2	54.7255	54.5730	54.5431	54.4036	53.3154	53.3024	53.1895	52.0882
	0.5	17.9975	17.9604	17.6369	17.6223	17.5754	17.4355	17.4012	17.1727
	1	10.8128	10.6919	10.6886	10.5312	10.4403	10.4393	10.3080	9.1734
	1.5	7.8810	7.7591	7.7535	7.6079	7.5184	7.5121	7.3730	6.2208
	2	5.8441	5.8417	5.8288	5.6872	5.5784	5.4332	5.3784	4.2635
	2.5	4.9991	4.9046	4.8848	4.7371	4.6400	4.6370	4.4845	3.3017
	3	3.7031	3.6882	3.5343	3.5312	3.4575	3.3906	3.3357	3.0454
100	0	370.6871	370.1580	370.0642	370.7770	370.2684	370.4377	370.6481	370.3284
	0.2	54.7980	54.6887	54.6826	54.4822	53.3637	53.3023	53.1895	52.0882
	0.5	18.4818	18.4206	18.4087	18.1209	18.0617	18.0303	18.0073	18.0055
	1	10.9336	10.8272	10.7848	10.6239	10.5082	10.4400	10.3080	9.1735
	1.5	8.0378	7.9175	7.8684	7.7123	7.5909	7.5180	7.3731	6.2212
	2	6.0212	5.9975	5.9315	5.7937	5.5775	5.4648	5.4334	4.2642
	2.5	5.0916	5.0648	5.0050	4.8618	4.7281	4.6484	4.4848	3.3030
	3	3.7843	3.7009	3.6484	3.6226	3.5318	3.5202	3.3375	3.2486
120	0	371.1896	371.1684	371.0754	371.8030	371.2777	371.4481	371.6842	371.3378
	0.2	55.2030	54.8466	54.7237	54.4848	53.3734	53.3240	53.2122	52.1098
	0.5	18.7597	18.6205	18.6064	18.4507	18.3202	18.2573	18.2355	18.0428
	1	12.3910	12.0348	10.9323	10.6875	10.5719	10.5039	10.3717	9.2372
	1.5	8.5089	8.1448	8.0286	7.7848	7.6624	7.5799	7.4454	6.2846
	2	6.5172	6.2454	6.1254	5.8737	5.6693	5.5457	5.5144	4.3452
	2.5	5.6128	5.3350	5.2120	4.9543	4.8218	4.7321	4.5786	3.3954
	3	3.8957	3.8736	3.8706	3.7916	3.7754	3.6212	3.6093	3.4268
150	0	371.6893	371.6480	371.5442	372.2846	371.7573	371.9377	372.1737	371.8284
	0.2	55.7042	55.3377	55.2250	54.9950	53.8866	53.8252	53.7123	52.6120
	0.5	18.8460	18.7332	18.6845	18.5284	18.3621	18.3482	18.2488	18.1215
	1	12.8771	12.5209	12.4084	12.1737	12.0580	10.9901	10.8488	9.7234
	1.5	8.9935	8.6304	8.5142	8.2704	8.1480	8.0755	7.9312	6.7784
	2	7.0015	6.7288	6.6100	6.3572	6.1535	6.0288	5.9984	4.8286
	2.5	6.0227	5.7354	5.6228	5.3643	5.2322	5.1528	4.9889	3.8064
	3	4.3778	4.3643	4.3630	4.2848	4.2688	4.1228	4.1018	3.9193
180	0	371.9793	371.9580	371.8642	372.5936	372.0684	372.2377	372.4848	372.1284
	0.2	57.0042	55.6378	55.5250	55.2840	54.1866	54.1251	54.0123	52.9120
	0.5	18.8727	18.7504	18.7086	18.5734	18.4186	18.3552	18.2845	18.1439
	1	12.8203	12.7079	12.4842	12.3573	12.2893	12.1754	12.1573	10.0228
	1.5	9.2842	8.9312	8.8148	8.5710	8.4487	8.3757	8.2319	7.0789
	2	7.3012	7.0286	6.9096	6.6488	6.4526	6.3288	6.2884	5.1284
	2.5	6.4843	6.1252	5.9932	5.7345	5.6020	5.5214	5.3579	4.1759
	3	4.6875	4.6645	4.6625	4.5722	4.5484	4.4122	4.4012	4.2188
200	0	372.3793	372.3580	372.2642	372.9936	372.4684	372.6377	372.8737	372.5284
	0.2	57.3932	57.0278	55.9150	55.6840	54.5754	54.5151	54.4023	53.3010
	0.5	18.9878	18.8682	18.8246	18.7557	18.6288	18.5439	18.4355	18.2428
	1	12.8736	12.7578	12.6897	12.5770	12.5577	12.2207	12.1084	10.4232
	1.5	9.6953	9.3312	9.2150	8.9712	8.8488	8.7759	8.6320	7.4890
	2	7.7019	7.4303	7.3104	7.0573	6.8432	6.7304	6.6991	5.5289
	2.5	6.8934	6.5153	6.3935	6.1246	6.0021	5.9315	5.7590	4.5750
	3	5.0879	5.0648	5.0630	4.9825	4.9688	4.8124	4.8012	4.6190
220	0	372.5793	372.5580	372.4642	373.1936	372.6684	372.8487	373.0737	372.7284
	0.2	57.5932	57.2278	57.1250	55.8840	54.7754	54.7151	54.6023	53.5010
	0.5	18.9954	18.9382	18.9008	18.8272	18.7375	18.5939	18.5412	18.5175
	1	12.9572	12.8891	12.7754	12.7571	12.4201	12.3077	12.0730	10.6226
	1.5	9.8937	9.5306	9.4144	9.1705	9.0482	8.9753	8.8414	7.6884
	2	7.9015	7.6287	7.5099	7.2579	7.0527	6.9379	6.8986	5.7284
	2.5	7.0932	6.7160	6.5932	6.3353	6.2027	6.1222	5.9597	4.7757
	3	5.2881	5.2648	5.2633	5.1827	5.1684	5.0127	5.0017	4.8193
240	0	372.6893	372.6480	372.5442	373.2846	372.7573	372.9377	373.1737	372.8284
	0.2	57.7042	57.3378	57.2250	55.9950	54.8866	54.8252	54.7123	53.6120
	0.5	19.0828	18.9578	18.9361	18.8418	18.8275	18.7506	18.6930	18.5200
	1	12.9898	12.8759	12.8484	12.5206	12.4082	12.1735	12.0577	10.7231
	1.5	9.9935	9.6304	9.5141	9.2703	9.1480	9.0754	8.9312	7.7782
	2	8.0021	7.7304	7.6104	7.3573	7.1539	7.0305	6.9993	5.8400
	2.5	7.1931	6.8159	6.6930	6.4352	6.3027	6.2230	6.0596	4.8754
	3	5.3775	5.3643	5.3624	5.2843	5.2684	5.1221	5.1010	4.9186

Table 8 - 3: ARL values for individual EWMA control charts for the Logarithmic distribution (λ=0.3 and L=4.9802)

λ, L	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
λ=0.05 L=3.061	0	370.1814	370.1571	370.0643	370.7935	370.2684	370.4390	370.5755	370.3286
	0.2	55.1846	54.8161	54.7032	54.4826	53.3641	53.3026	53.1897	52.0884
	0.4	20.2082	19.8488	19.7379	19.5101	18.4008	18.3481	18.2207	17.1209
	0.6	14.9948	14.8893	14.7788	14.5489	14.4378	14.3734	14.2512	14.1228
	0.8	12.9320	12.8207	12.5780	12.4844	12.4081	12.2879	12.2808	12.1542
	1	10.3340	9.9757	9.8632	9.6268	9.5104	9.4421	9.3096	8.1750
	1.5	8.4508	8.0842	7.9646	7.7195	7.5961	7.5228	7.3779	6.2243
	2	6.4595	6.1822	6.0616	5.8041	5.5966	5.4843	5.4404	4.2698
	2.5	5.4646	5.2693	5.1454	4.8793	4.7542	4.6628	4.5070	3.6124
	3	3.8441	3.8175	3.8048	3.7350	3.7215	3.5488	3.5351	3.3593
λ=0.08 L=3.375	0	370.1807	370.1481	370.0642	370.7932	370.2686	370.4379	370.6953	370.3284
	0.2	55.1828	54.8157	54.7028	54.4825	53.3640	53.3025	53.1896	52.0882
	0.4	20.2072	19.8482	19.7282	19.5098	18.4005	18.3379	18.2206	17.1206
	0.6	14.9935	14.8884	14.7779	14.5484	14.4373	14.3730	14.2509	14.1424
	0.8	12.9306	12.8193	12.5971	12.4825	12.4075	12.2860	12.2803	12.1534
	1	10.3312	9.9736	9.8610	9.6253	9.5090	9.4409	9.3089	8.1737
	1.5	8.4445	8.0781	7.9615	7.7159	7.5936	7.5198	7.3780	6.2215
	2	6.4482	6.1721	6.0516	5.7970	5.5904	5.4875	5.4366	4.2648
	2.5	5.5343	5.2527	5.1280	4.8646	4.7327	4.6425	4.4804	3.6041
	3	3.8196	3.7935	3.7887	3.7061	3.7052	3.5379	3.5377	3.3391
λ=0.10 L=3.579	0	370.1801	370.1680	370.0642	370.7939	370.2673	370.4378	370.6841	370.3284
	0.2	55.1823	54.8153	54.7025	54.4823	53.3639	53.3023	53.1895	52.0882
	0.4	20.2064	19.8486	19.7379	19.5096	18.4002	18.3375	18.2203	17.1205
	0.6	14.9919	14.8877	14.7773	14.5489	14.4368	14.3725	14.2504	14.1222
	0.8	12.9193	12.8184	12.5964	12.4826	12.4068	12.2845	12.2796	12.1531
	1	10.3393	9.9719	9.8696	9.6242	9.5077	9.4399	9.3077	8.1733
	1.5	8.4401	8.0754	7.9575	7.7135	7.5796	7.5175	7.3734	6.2204
	2	6.4395	6.1645	6.0461	5.7937	5.5752	5.4620	5.4319	4.2628
	2.5	5.5224	5.2424	5.1204	4.8608	4.7243	4.6459	4.4843	3.6006
	3	3.8052	3.7816	3.7771	3.6969	3.6884	3.5279	3.5180	3.3341
λ=0.12 L=3.793	0	370.1795	370.1680	370.0752	370.7916	370.2684	370.4377	370.6848	370.3282
	0.2	55.1816	54.8148	54.7022	54.4821	53.3637	53.3023	53.1895	52.0882
	0.4	20.2057	19.8481	19.7375	19.5093	18.4001	18.3375	18.2203	17.1205
	0.6	14.9914	14.8869	14.7754	14.5373	14.4364	14.3723	14.2503	14.1422
	0.8	12.9182	12.8284	12.5755	12.4823	12.4064	12.2840	12.2793	12.1531
	1	10.3272	9.9702	9.8682	9.6228	9.5071	9.4395	9.3075	8.1732
	1.5	8.4362	8.0710	7.9557	7.7107	7.5984	7.5163	7.3727	6.2203
	2	6.4327	6.1597	6.0412	5.7878	5.5951	5.4699	5.4306	4.2626
	2.5	5.5122	5.2337	5.1230	4.8633	4.7212	4.6427	4.4812	3.6003
	3	3.8034	3.7726	3.7715	3.6934	3.6848	3.5250	3.5077	3.3336
λ=0.15 L=4.301	0	370.1781	370.1578	370.0642	370.7919	370.2681	370.4375	370.6845	370.3281
	0.2	55.1803	54.8140	54.7017	54.4816	53.3633	53.3019	53.1891	52.0882
	0.4	20.2041	19.8469	19.7366	19.5086	18.3995	18.3369	18.2196	17.1205
	0.6	14.9893	14.8753	14.7754	14.5463	14.4355	14.3714	14.2591	14.1422
	0.8	12.9360	12.8157	12.5730	12.4808	12.4048	12.2804	12.2775	12.1531
	1	10.3236	9.9682	9.8460	9.6208	9.5051	9.4373	9.3048	8.1732
	1.5	8.4395	8.0754	7.9514	7.7066	7.5753	7.5121	7.3688	6.2203
	2	6.4218	6.1505	6.0341	5.7812	5.5772	5.4431	5.4227	4.2625
	2.5	5.5964	5.2205	5.1026	4.8436	4.7123	4.6326	4.4800	3.6001
	3	3.7757	3.7595	3.7575	3.6889	3.6439	3.5102	3.4848	3.3352
λ=0.20 L=4.968	0	370.1775	370.1578	370.0648	370.7912	370.2680	370.4369	370.5735	370.3275
	0.2	55.1786	54.8140	54.7003	54.4808	53.3624	53.3018	53.1890	52.0872
	0.4	20.2020	19.8455	19.7348	19.5073	18.3980	18.3366	18.2193	17.1088
	0.6	14.9868	14.8824	14.7728	14.5446	14.4334	14.3709	14.2488	14.1284
	0.8	12.9335	12.8123	12.5715	12.4689	12.4042	12.2797	12.2771	12.1590
	1	10.3193	9.9639	9.8415	9.6175	9.5012	9.4363	9.3041	8.1687
	1.5	8.4218	8.0593	7.9532	7.7005	7.5771	7.5100	7.3662	6.2105
	2	6.4098	6.1409	6.0215	5.7712	5.5737	5.4419	5.4200	4.2578
	2.5	5.5797	5.2069	5.0840	4.8288	4.6957	4.6275	4.4669	3.4802
	3	3.7395	3.7248	3.6821	3.6482	3.6322	3.5048	3.4870	3.3082

Table 8 - 4: ARL values for individual EWMA control charts for the Logarithmic distribution (m=180) for various positive shifts

λ, L	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
λ=0.05 L=3.061	0	370.1814	370.1571	370.0643	370.7935	370.2684	370.4390	370.5755	370.3286
	-0.2	52.6218	53.7064	53.8126	53.8754	54.9891	55.2428	55.3712	55.8000
	-0.4	17.4893	18.5440	18.6848	18.7378	19.8618	20.1484	20.2873	20.8064
	-0.6	14.6828	14.7351	14.8484	14.9377	15.0717	15.4223	15.6125	15.9328
	-0.8	12.4373	12.5253	12.5416	12.5480	12.6930	12.7171	12.7800	12.9578
	-1	8.3481	9.3532	9.5024	9.6154	9.8684	10.1970	10.6159	10.9579
	-1.5	6.5751	7.5754	7.9160	8.1599	8.2263	8.2873	8.3934	8.4223
	-2	4.6264	5.6930	5.8448	6.0062	6.0937	6.2122	6.2755	6.3735
	-2.5	3.5128	4.8935	4.9312	4.9791	5.0975	5.1618	5.1893	5.3608
	-3	3.3419	3.4840	3.5402	3.6428	3.6887	3.8014	3.8736	3.9022
λ=0.08 L=3.375	0	370.1807	370.1481	370.0642	370.7932	370.2686	370.4379	370.6953	370.3284
	-0.2	52.5959	53.6805	53.7877	53.8486	54.9632	55.2168	55.3451	55.7731
	-0.4	17.4598	18.5364	18.6443	18.7093	19.8424	20.1286	20.2684	20.7734
	-0.6	14.6193	14.6816	14.7937	14.8648	15.0079	15.3580	15.5484	15.8437
	-0.8	12.4089	12.4254	12.5328	12.5593	12.6843	12.6987	12.7712	12.9370
	-1	8.3157	9.3199	9.4691	9.5721	9.8440	10.0868	10.5519	10.8484
	-1.5	6.5480	7.5484	7.8887	8.0757	8.1273	8.2603	8.2873	8.3202
	-2	4.6840	5.7515	5.8403	6.0222	6.1222	6.2127	6.2731	6.3579
	-2.5	3.5931	4.9373	4.9573	5.0182	5.1288	5.1802	5.2022	5.2640
	-3	3.2487	3.4848	3.5009	3.5428	3.6025	3.7548	3.8120	3.8401
λ=0.10 L=3.579	0	370.1801	370.1680	370.0642	370.7939	370.2684	370.4378	370.6841	370.3284
	-0.2	52.5432	53.6488	53.7550	53.8169	54.9304	55.1848	55.3121	55.7395
	-0.4	17.4416	18.5184	18.6261	18.6912	19.8141	20.1002	20.2489	20.7541
	-0.6	14.6250	14.6872	14.7996	14.8717	15.0127	15.3634	15.5521	15.8401
	-0.8	12.3737	12.4373	12.5487	12.5731	12.6991	12.7070	12.7861	12.9637
	-1	8.3273	9.3314	9.4806	9.5935	9.8452	10.0689	10.5486	10.8064
	-1.5	6.5248	7.5252	7.8643	8.0346	8.0884	8.2288	8.2315	8.2648
	-2	4.6159	5.6823	5.7579	5.9520	6.0234	6.1245	6.1771	6.2575
	-2.5	3.5350	4.8412	4.8780	4.9325	5.0436	5.0935	5.0962	5.1754
	-3	3.2489	3.5084	3.5157	3.5484	3.6169	3.7527	3.8244	3.8412
λ=0.12 L=3.793	0	370.1795	370.1680	370.0752	370.7916	370.2684	370.4377	370.6848	370.3282
	-0.2	52.5734	53.6489	53.7751	53.8481	54.9515	55.2048	55.3330	55.7598
	-0.4	17.4542	18.5310	18.6377	18.7037	19.8268	20.1226	20.2610	20.7548
	-0.6	14.6421	14.7143	14.8268	14.8988	15.0407	15.3900	15.5780	15.8693
	-0.8	12.3591	12.4579	12.5500	12.5964	12.7214	12.7236	12.8084	12.9860
	-1	8.3344	9.3373	9.4877	9.6006	9.8418	10.0543	10.5434	10.7822
	-1.5	6.5557	7.5541	7.8961	8.0486	8.1023	8.2275	8.2595	8.2750
	-2	4.6317	5.6981	5.7712	5.9341	6.0210	6.1244	6.1759	6.2516
	-2.5	3.5284	4.8407	4.8688	4.9175	5.0208	5.0725	5.0754	5.1482
	-3	3.2068	3.4842	3.4869	3.5312	3.5796	3.7377	3.7993	3.8264
λ=0.15 L=4.301	0	370.1781	370.1578	370.0642	370.7919	370.2681	370.4375	370.6845	370.3281
	-0.2	52.5541	53.6375	53.7357	53.8077	54.9312	55.1733	55.3020	55.7278
	-0.4	17.4809	18.5577	18.6443	18.7304	19.8432	20.1279	20.2868	20.7878
	-0.6	14.6464	14.7086	14.8210	14.8931	15.0348	15.3735	15.5701	15.8406
	-0.8	12.2680	12.4153	12.5284	12.5557	12.6806	12.6848	12.7577	12.9350
	-1	8.3620	9.3641	9.5153	9.6282	9.8787	10.0508	10.5543	10.7548
	-1.5	6.5173	7.5177	7.8482	7.9931	8.0373	8.1546	8.2075	8.2103
	-2	4.6431	5.7095	5.7509	5.9320	6.0126	6.1277	6.1607	6.2319
	-2.5	3.5536	4.8425	4.8714	4.9543	5.0282	5.0482	5.0712	5.2426
	-3	3.2205	3.5012	3.5317	3.5484	3.6063	3.7559	3.8206	3.8484
λ=0.20 L=4.968	0	370.1775	370.1578	370.0648	370.7912	370.2680	370.4369	370.5735	370.3275
	-0.2	52.6099	53.6935	53.8016	53.8632	54.9754	55.2284	55.3571	55.7815
	-0.4	17.4800	18.5468	18.6444	18.7193	19.8421	20.1269	20.2737	20.7726
	-0.6	14.6377	14.6999	14.8123	14.8845	15.0260	15.3730	15.5579	15.8280
	-0.8	12.2096	12.4007	12.5204	12.5468	12.6455	12.6817	12.7577	12.9357
	-1	8.3248	9.3289	9.4881	9.5907	9.8402	9.9880	10.4805	10.6906
	-1.5	6.5907	7.5910	7.9302	8.0348	8.0918	8.1884	8.2552	8.2693
	-2	4.6412	5.7277	5.7539	5.9359	6.0086	6.1053	6.1601	6.2245
	-2.5	3.5487	4.8412	4.8633	4.8960	4.9937	5.0284	5.0484	5.1228
	-3	3.1284	3.4124	3.4537	3.4641	3.5091	3.6844	3.7377	3.7577

Table 8 - 5: ARL values for individual EWMA control charts for the Logarithmic distribution (m=180) for various negative shifts

λ, L	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
λ=0.05 L=3.061	0	360.1812	360.1571	360.0643	360.7935	360.2684	360.4370	360.6845	360.3284
	0.2	57.7845	57.4160	57.3031	57.0726	55.9641	55.9026	55.7897	54.6884
	0.4	22.4079	22.0487	21.9378	21.7101	20.6008	20.5370	20.4207	19.3109
	0.6	16.7935	16.6893	16.5787	16.3488	16.2377	16.1733	16.0512	15.9328
	0.8	14.4317	14.3205	14.0879	12.9733	12.9080	12.7873	12.7807	12.6442
	1	12.5333	12.1753	12.0628	10.8264	10.7102	10.6419	10.5096	9.3737
	1.5	9.4481	9.0819	8.9648	8.7187	8.5954	8.5221	8.3777	7.2242
	2	6.9557	6.6893	6.5595	6.3025	6.0950	5.9732	5.9301	4.7596
	2.5	6.0489	5.6444	5.5418	5.2754	5.1422	5.0600	4.8963	3.7122
	3	4.0364	4.0120	4.0025	3.9348	3.9171	3.7364	3.7307	3.5488
λ=0.08 L=3.375	0	360.1802	360.1580	360.0642	360.7930	360.2684	360.4378	360.6842	360.3284
	0.2	57.7824	57.4154	57.3027	57.0723	55.9639	55.9024	55.7895	54.6882
	0.4	22.4064	22.0487	21.9370	21.7096	20.6004	20.5378	20.4204	19.3106
	0.6	16.7936	16.6878	16.5775	16.3480	16.2371	16.1728	16.0505	15.9322
	0.8	14.4286	14.3187	14.0864	12.9732	12.9072	12.7846	12.7797	12.6432
	1	12.5284	12.1720	12.0601	10.8243	10.7084	10.6406	10.5080	9.3734
	1.5	9.4400	9.0735	8.9593	8.7126	8.5912	8.5188	8.3739	7.2208
	2	6.9378	6.6445	6.5482	6.2848	6.0886	5.9648	5.9328	4.7535
	2.5	5.9309	5.6421	5.5217	5.2605	5.1286	5.0484	4.8848	3.7018
	3	4.0044	3.9841	3.9828	3.9015	3.8861	3.7300	3.7175	3.5357
λ=0.10 L=3.579	0	360.1791	360.1580	360.0642	360.7935	360.2684	360.4377	360.6848	360.3284
	0.2	57.7812	57.4148	57.3021	57.0720	55.9637	55.9023	55.7895	54.6882
	0.4	22.4053	22.0488	21.9372	21.7091	20.6001	20.5375	20.4203	19.3105
	0.6	16.7908	16.6864	16.5752	16.3482	16.2364	16.1723	16.0503	15.9322
	0.8	14.4275	14.3168	14.0843	12.9723	12.9063	12.7822	12.7793	12.6431
	1	12.5260	12.1693	12.0573	10.8226	10.7071	10.6393	10.5073	9.3732
	1.5	9.4335	9.0690	8.9537	8.7099	8.5782	8.5157	8.3725	7.2203
	2	6.9377	6.6440	6.5377	6.2863	6.0842	5.9593	5.9302	4.7525
	2.5	5.9042	5.6278	5.5075	5.2508	5.1203	5.0412	4.8806	3.7000
	3	3.9841	3.9712	3.9637	3.8902	3.8635	3.7240	3.7041	3.5332
λ=0.12 L=3.793	0	360.1773	360.1580	360.0642	360.7919	360.2681	360.4373	360.6844	360.3281
	0.2	57.7796	57.4141	57.3015	57.0716	55.9633	55.9019	55.7895	54.6882
	0.4	22.4031	22.0469	21.9364	21.7084	20.5993	20.5368	20.4202	19.3105
	0.6	16.7881	16.6842	16.5750	16.3461	16.2354	16.1712	16.0502	15.9321
	0.8	14.4259	14.3151	14.0846	12.9706	12.9048	12.7793	12.7786	12.6430
	1	12.5212	12.1648	12.0551	10.8203	10.7048	10.6371	10.5071	9.3732
	1.5	9.4244	9.0643	8.9373	8.7053	8.5737	8.5126	8.3719	7.2202
	2	6.9128	6.6482	6.5303	6.2786	6.0752	5.9519	5.9371	4.7524
	2.5	5.8840	5.6164	5.4868	5.2397	5.1093	5.0310	4.8789	3.6998
	3	3.9701	3.9544	3.9373	3.8754	3.8454	3.7215	3.6893	3.5328
λ=0.15 L=4.301	0	360.1739	360.1578	360.0642	360.7907	360.2680	360.4372	360.6848	360.3280
	0.2	57.7750	57.4125	57.3006	57.0712	55.9631	55.9018	55.7890	54.6882
	0.4	22.3989	22.0448	21.9351	21.7079	20.5990	20.5364	20.4193	19.3105
	0.6	16.7828	16.6824	16.5732	16.3453	16.2348	16.1708	16.0487	15.9321
	0.8	14.4221	14.3126	14.0825	12.9698	12.9041	12.7770	12.7717	12.6430
	1	12.5124	12.1619	12.0517	10.8186	10.7036	10.6360	10.5040	9.3732
	1.5	9.4093	9.0553	8.9328	8.7021	8.5712	8.5093	8.3648	7.2202
	2	6.8898	6.6341	6.5203	6.2733	6.0726	5.9377	5.9193	4.7523
	2.5	5.9512	5.5968	5.4827	5.2321	5.1031	5.0254	4.8648	3.6998
	3	3.9373	3.9348	3.9312	3.8695	3.7953	3.7028	3.6893	3.5327
λ=0.20 L=4.968	0	360.1682	360.1577	360.0648	360.7879	360.2644	360.4364	360.6816	360.3273
	0.2	57.7702	57.4093	57.2879	57.0703	55.9621	55.9007	55.7889	54.6872
	0.4	22.3933	22.0406	21.9315	21.7064	20.5973	20.5348	20.4193	19.3086
	0.6	16.7736	16.6848	16.5484	16.3432	16.2324	16.1682	16.0484	15.9390
	0.8	14.4148	14.3062	14.0795	12.9644	12.9004	12.7754	12.7516	12.6484
	1	12.4898	12.1526	12.0434	10.8148	10.6990	10.6312	10.5035	9.3680
	1.5	9.3789	9.0393	8.9377	8.6937	8.5728	8.5005	8.3648	7.2089
	2	6.8605	6.6105	6.4890	6.2622	6.0578	5.9350	5.9173	4.7351
	2.5	5.9128	5.5442	5.4539	5.2164	5.0844	5.0068	4.8619	3.6842
	3	3.9361	3.9053	3.8951	3.8454	3.7372	3.6993	3.6486	3.5028

Table 8 - 6: ARL values for individual EWMA control charts for the Logarithmic distribution (m=180) for various positive shifts for the case of not using the skewness correction term when constructing the control limits of the chart

Additionally, comparing the ARL values for the EWMA in Tables 8-4 and 8-5 with the ARL values for the Shewhart-type control chart in Table 8-1, we can see that the EWMA control chart performs better than the Shewhart-type control chart for smaller shifts, since for the case of small shifts, the EWMA out-of-control ARL values are smaller than the corresponding ARL values for the Shewhart-type charts. When it comes to large shifts, however, EWMA ARL values are slightly larger and, therefore, make Shewhart-type control charts preferable for those cases.

8.7 Optimal Choice for the Parameters of the EWMA Control Charts for Individual Observations from the Logarithmic distribution

When constructing an EWMA control chart, there are two parameters involved in the way the chart is going to perform, namely the constant λ which affects the weight we give to the past values of our observations and the value of L which affects the width of the chart's control limits. Therefore, we need to find the combination of the values of those two parameters which will lead us to the optimal performance of our control chart.

As presented in Section 6.7, the optimal design of control charts has been addressed a lot in relevant research by minimizing the out-of-control value of various performance criteria. Since all the study here has been based on ARL (which is the most commonly used performance criterion) the optimal design of the EWMA control chart will be done by minimizing the ARL. The algorithm applied here is as follows:

- Step 1: Set the desired in-control ARL value (e.g. ARL_0=370) and the size of the mean shift k to be detected (e.g. k = 0.5).
- Step 2: Set an initial value L = 1.
- Step 3: Vary the parameter λ (e.g. increasing by 0.01) so as $\lambda \in (0,1]$ and (using a nonlinear equation solver) find the value of λ for which the ARL_0 value in Step 1 is satisfied.
- Step 4: Calculate the ARL_1 value for the particular combination of λ and L resulting from Step 3. [The ARL_1 value is obtained as described in the previous section, using equation (8-12) for the computation of

the transient probabilities along with equation (4-2) for the cumulative distribution function of the Logarithmic distribution.]

- Step 5: Increase L by 0.01.
- Step 6: Repeat Steps 3-5 until the minimum ARL_1 value has been reached (i.e. until the ARL_1 value for L+0.01 is larger than the ARL_1 value for L).
- Step 7: Keep the combination of λ and L resulting from Step 6 for which the smallest ARL_1 value is obtained as the desired optimal one for the selected shift size in Step 1.
- Step 8: Repeat Steps 2-7 for all the desired values of shifts to be detected (e.g. k = {-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3}).

Application of this algorithm leads to Table 8-7 and Table 8-8 which present the optimal combination of values of the two parameters of concern (λ and L) of the EWMA chart with the corresponding ARL values for various values of the parameter θ of the Logarithmic distribution and various positive and negative values, respectively, and various values of k, which shows the shift of the process mean in terms of the process standard deviation which we want to be detected by the control chart we construct.

k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
0.2	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 59.0884)	(371.1581, 58.1897)	(371.2684, 57.3026)	(371.3285, 55.3642)	(371.438, 54.4727)	(371.6845, 53.7033)	(371.7936, 52.8162)	(372.1816, 52.1839)
0.4	(0.79, 3.82)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 30.1009)	(371.1581, 28.2208)	(371.2684, 27.3382)	(371.3285, 26.4009)	(371.438, 25.5103)	(371.6845, 24.7391)	(371.7936, 23.85)	(372.1816, 22.2084)
0.6	(0.81, 3.55)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 20.133)	(371.1581, 19.2512)	(371.2684, 18.3735)	(371.3285, 17.438)	(371.438, 16.5492)	(371.6845, 15.7792)	(371.7936, 14.8898)	(372.1816, 14.2452)
0.8	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 17.1545)	(371.1581, 16.281)	(371.2684, 15.4085)	(371.3285, 14.4748)	(371.438, 14.0885)	(371.6845, 12.9327)	(371.7936, 12.8212)	(372.1816, 12.2886)
1	(0.81, 3.55)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 12.1755)	(371.1581, 12.0301)	(371.2684, 10.4427)	(371.3285, 10.1541)	(371.438, 9.6275)	(371.6845, 8.8643)	(371.7936, 8.6979)	(372.1816, 8.3351)
1.2	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 10.8961)	(371.1581, 10.3384)	(371.2684, 9.7542)	(371.3285, 9.5467)	(371.438, 8.686)	(371.6845, 8.4071)	(371.7936, 8.0214)	(372.1816, 7.9828)
1.4	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 9.5161)	(371.1581, 9.366)	(371.2684, 8.9089)	(371.3285, 8.5814)	(371.438, 8.2037)	(371.6845, 7.9494)	(371.7936, 7.8643)	(372.1816, 7.3404)
1.6	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 8.9357)	(371.1581, 8.3929)	(371.2684, 7.8407)	(371.3285, 7.5152)	(371.438, 7.1405)	(371.6845, 6.9908)	(371.7936, 6.9785)	(372.1816, 6.8771)
1.8	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 8.4549)	(371.1581, 8.3191)	(371.2684, 7.7716)	(371.3285, 7.4481)	(371.438, 7.0763)	(371.6845, 6.9312)	(371.7936, 6.1506)	(372.1816, 6.0226)
2	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 6.4846)	(371.1581, 6.2445)	(371.2684, 6.0016)	(371.3285, 5.9801)	(371.438, 5.891)	(371.6845, 5.8704)	(371.7936, 5.6916)	(372.1816, 5.5663)
2.2	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 6.2819)	(371.1581, 5.9693)	(371.2684, 5.7308)	(371.3285, 5.1731)	(371.438, 4.8448)	(371.6845, 4.8084)	(371.7936, 4.7314)	(372.1816, 4.6082)
2.4	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 5.3097)	(371.1581, 5.1934)	(371.2684, 4.9592)	(371.3285, 4.7512)	(371.438, 4.6874)	(371.6845, 4.5453)	(371.7936, 4.4697)	(372.1816, 4.3481)
2.6	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 5.2272)	(371.1581, 4.9169)	(371.2684, 4.8867)	(371.3285, 4.6804)	(371.438, 4.5391)	(371.6845, 4.4809)	(371.7936, 4.3069)	(372.1816, 3.9859)
2.8	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.74, 4.04)	(0.02, 3.26)
	(371.0643, 5.1442)	(371.1581, 4.9397)	(371.2684, 4.6134)	(371.3285, 4.3987)	(371.438, 3.9397)	(371.6845, 3.9154)	(371.7936, 3.8428)	(372.1816, 3.7216)
3	(0.8, 3.68)	(0.61, 3.97)	(0.8, 3.95)	(0.72, 4.04)	(0.69, 4.04)	(0.76, 4.04)	(0.73, 4.04)	(0.02, 3.26)
	(371.0643, 3.9503)	(371.1581, 3.8619)	(371.2684, 3.8493)	(371.3285, 3.8261)	(371.438, 3.7594)	(371.6845, 3.6487)	(371.7936, 3.5775)	(372.1816, 3.4552)

Table 8 - 7: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the Logarithmic distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of positive shifts k (m=180)

k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
-0.2	(0.79, 3.82) (371.0643, 52.2416)	(0.61, 3.97) (371.1581, 53.1262)	(0.8, 3.95) (371.2684, 53.8434)	(0.72, 4.04) (371.3285, 54.5954)	(0.69, 4.04) (371.438, 55.4089)	(0.76, 4.04) (371.6845, 57.6627)	(0.74, 4.04) (371.7936, 58.7912)	(0.02, 3.26) (372.1816, 59.2201)
-0.4	(0.79, 3.82) (371.0643, 22.3174)	(0.61, 3.97) (371.1581, 23.8942)	(0.8, 3.95) (371.2684, 24.8019)	(0.72, 4.04) (371.3285, 25.867)	(0.69, 4.04) (371.438, 26.539)	(0.76, 4.04) (371.6845, 27.6765)	(0.74, 4.04) (371.7936, 28.8257)	(0.02, 3.26) (372.1816, 30.3359)
-0.6	(0.21, 3.75) (371.0643, 14.4005)	(0.66, 4.04) (371.1581, 14.9062)	(0.8, 3.95) (371.2684, 15.8751)	(0.72, 4.04) (371.3285, 16.7573)	(0.69, 4.04) (371.438, 17.6893)	(0.76, 4.04) (371.6845, 18.7301)	(0.74, 4.04) (371.7936, 19.9308)	(0.02, 3.26) (372.1816, 20.6094)
-0.8	(0.21, 3.75) (371.0643, 12.3007)	(0.57, 4.04) (371.1581, 12.8424)	(0.82, 4.04) (371.2684, 12.9569)	(0.72, 4.04) (371.3285, 14.2428)	(0.69, 4.04) (371.438, 14.4819)	(0.76, 4.04) (371.6845, 15.9037)	(0.74, 4.04) (371.7936, 16.4871)	(0.02, 3.26) (372.1816, 17.5902)
-1	(0.21, 3.75) (371.0643, 8.4509)	(0.08, 17.65) (371.1581, 8.7046)	(0.81, 4.04) (371.2684, 8.9538)	(0.72, 4.04) (371.3285, 9.8268)	(0.69, 4.04) (371.438, 10.1693)	(0.76, 4.04) (371.6845, 10.5312)	(0.02, 5.52) (371.7936, 12.8472)	(0.02, 3.26) (372.1816, 12.9378)
-1.2	(0.21, 3.75) (371.0643, 7.9901)	(0.13, 3.7) (371.1581, 8.1205)	(0.8, 3.95) (371.2684, 8.5786)	(0.72, 4.04) (371.3285, 8.7346)	(0.69, 4.04) (371.438, 9.5794)	(0.4, 4.04) (371.6845, 9.7875)	(0.41, 4.04) (371.7936, 10.8612)	(0.02, 3.26) (372.1816, 10.9344)
-1.4	(0.21, 3.75) (371.0643, 7.3903)	(0.13, 3.7) (371.1581, 7.8845)	(0.8, 3.95) (371.2684, 7.9557)	(0.72, 4.04) (371.3285, 8.539)	(0.52, 4.04) (371.438, 8.625)	(0.4, 4.04) (371.6845, 8.9806)	(0.41, 4.04) (371.7936, 9.571)	(0.02, 3.26) (372.1816, 9.6875)
-1.6	(0.21, 3.75) (371.0643, 6.9012)	(0.13, 3.7) (371.1581, 6.9878)	(0.8, 3.95) (371.2684, 6.9973)	(0.51, 4.04) (371.3285, 7.1732)	(0.47, 4.04) (371.438, 7.6148)	(0.4, 4.04) (371.6845, 7.8816)	(0.41, 4.04) (371.7936, 8.4614)	(0.02, 3.26) (372.1816, 8.9826)
-1.8	(0.21, 3.75) (371.0643, 6.1004)	(0.13, 3.7) (371.1581, 6.1822)	(0.6, 4.04) (371.2684, 6.9971)	(0.47, 3.99) (371.3285, 7.1284)	(0.47, 4.04) (371.438, 7.5716)	(0.4, 4.04) (371.6845, 7.7905)	(0.41, 4.04) (371.7936, 8.4508)	(0.02, 3.26) (372.1816, 8.6037)
-2	(0.21, 3.75) (371.0643, 5.57)	(0.68, 4.04) (371.1581, 5.7369)	(0.42, 4.04) (371.2684, 5.8969)	(0.47, 3.99) (371.3285, 8.9398)	(0.47, 4.04) (371.438, 5.9924)	(0.4, 4.04) (371.6845, 6.1804)	(0.41, 4.04) (371.7936, 6.2612)	(0.02, 3.26) (372.1816, 6.5178)
-2.2	(0.21, 3.75) (371.0643, 4.6251)	(0.61, 3.97) (371.1581, 4.7502)	(0.42, 4.04) (371.2684, 4.8248)	(0.47, 3.99) (371.3285, 4.8691)	(0.47, 4.04) (371.438, 5.1848)	(0.4, 4.04) (371.6845, 5.7802)	(0.41, 4.04) (371.7936, 5.9736)	(0.02, 3.26) (372.1816, 6.3543)
-2.4	(0.21, 3.75) (371.0643, 4.3502)	(0.61, 3.97) (371.1581, 4.4841)	(0.42, 4.04) (371.2684, 4.5916)	(0.47, 3.99) (371.3285, 4.6914)	(0.47, 4.04) (371.438, 4.7642)	(0.4, 4.04) (371.6845, 4.96)	(0.41, 4.04) (371.7936, 5.2003)	(0.02, 3.26) (372.1816, 5.3371)
-2.6	(0.21, 3.75) (371.0643, 3.99)	(0.4, 3.92) (371.1581, 4.3406)	(0.42, 4.04) (371.2684, 4.5064)	(0.47, 3.99) (371.3285, 4.5442)	(0.47, 4.04) (371.438, 4.6935)	(0.4, 4.04) (371.6845, 4.8903)	(0.41, 4.04) (371.7936, 4.9321)	(0.02, 3.26) (372.1816, 5.2355)
-2.8	(0.21, 3.75) (371.0643, 3.7345)	(0.4, 3.92) (371.1581, 3.8546)	(0.42, 4.04) (371.2684, 3.9362)	(0.47, 3.99) (371.3285, 3.9594)	(0.47, 4.04) (371.438, 4.4046)	(0.4, 4.04) (371.6845, 4.68)	(0.41, 4.04) (371.7936, 4.952)	(0.02, 3.26) (372.1816, 5.1577)
-3	(0.21, 3.75) (371.0643, 3.4628)	(0.4, 3.92) (371.1581, 3.5968)	(0.42, 4.04) (371.2684, 3.6861)	(0.47, 3.99) (371.3285, 3.7728)	(0.47, 4.04) (371.438, 3.8448)	(0.4, 4.04) (371.6845, 3.8601)	(0.41, 4.04) (371.7936, 3.8828)	(0.02, 3.26) (372.1816, 3.9648)

Table 8 - 8: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the Logarithmic distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of negative shifts k (m=180)

8.8 Examples on the Individual Logarithmic Probability-Type, Shewhart-Type and EWMA Control Charts

This section provides illustration of the proposed control charts by means of both simulated data generated from the distribution of concern and real data. The case of simulated data is presented in Subsection 8.8.1, while the real data case is covered in Subsection 8.8.2.

8.8.1 Examples with Simulated Data from the Logarithmic Distribution

The simulation process, for which the R programming language version 4.0.2 (R Core Team (2020)) has been used, is like this: Suppose we take a sample of n = 30 observations from a Logarithmic process as follows. First, we take a sample of 15 observations from a Logarithmic process with in-control θ value equal to 0.64. Now suppose that a shift of one standard deviation unit occurs in the process mean, and after that shift, we draw another set of 15 observations from the process. The resulting data set can be seen in Table 8-9. For this data set, we construct the individual probability-type Logarithmic control chart shown in Figure 8-1, using the most commonly used value for the significance level α = 0.27%, as mentioned in Section 8-2.

Data Set 1	1	3	2	1	2
	1	3	2	1	2
	1	2	1	1	3
	4	3	5	4	3
	2	5	4	3	5
	3	6	2	5	4

Table 8 - 9: Data from a Logarithmic process with in control θ = 0.64 and a shift of one standard deviation unit in the process mean due to an increasing shift after the first 15 observations (gray shading)

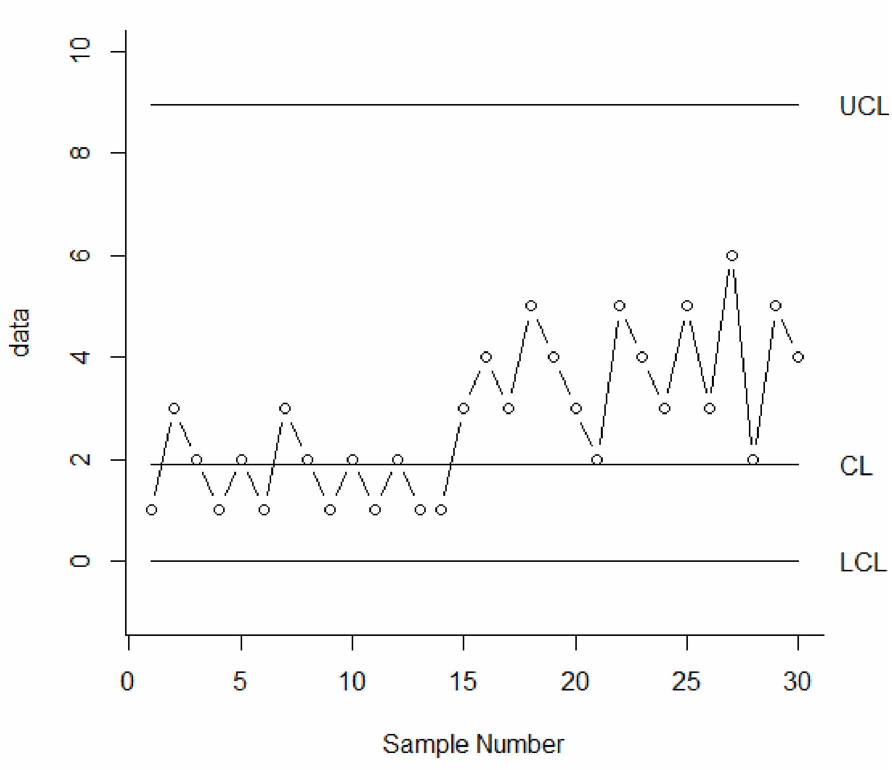

Figure 8 - 1: Individual probability-type Logarithmic control chart for the data set in Table 8-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations but control chart doe not detect any out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level.

For the same data with one standard deviation unit shift in Table 8-9, we now construct the Shewhart-type Logarithmic control chart shown in Figure 8-2, using L = 2.682 standard deviations (which gives a desired value of in-control ARL close to 370).

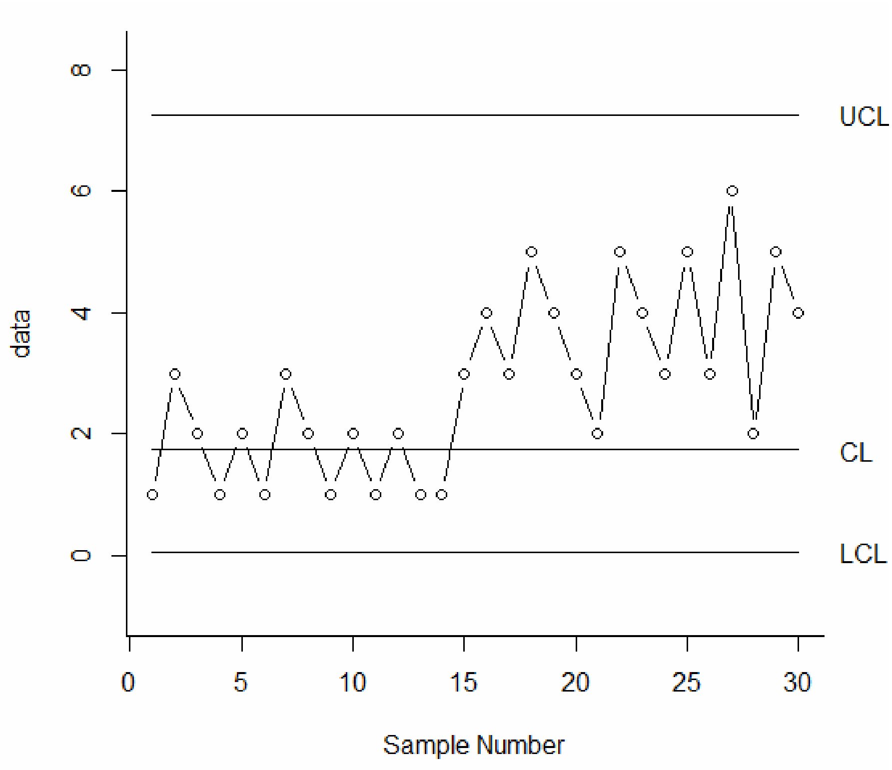

Figure 8 - 2: Individual Shewhart-type Logarithmic control chart for the data set in Table 8-9 with a shift of one standard deviation unit in the process mean

As we can see in the chart, there is an increasing trend after the first 15 observations, but still the control chart does not detect any out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level. Comparing this chart to the previous one (Figure 8-1), we observe similar behaviour of the probability-type chart to the Shewhart-type chart with skewness correction but the last 15 observations are closer to the upper control limit than they were with the probability-type control chart in the previous Figure.

Using the data set in Table 8-9 for the case of a shift of one standard deviation unit, we now construct the individual EWMA Logarithmic control chart shown in Figure 8-2, using $\lambda=0.05$ and $L = 2.64$ standard deviations (which gives a desired value of in-control ARL close to 370). As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 19th observation.

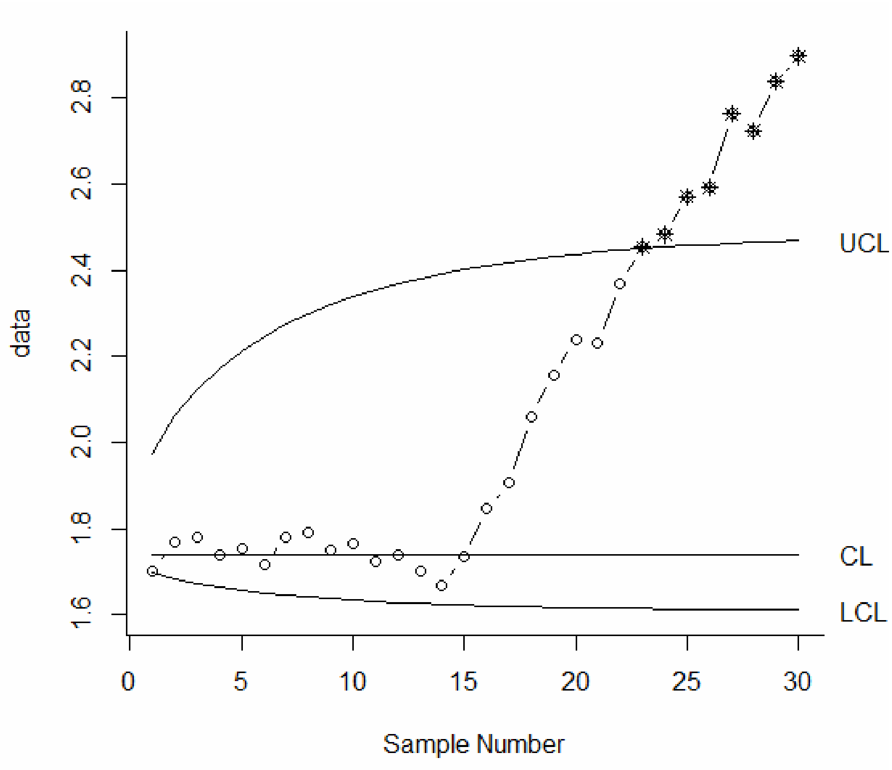

Figure 8 - 3: Individual EWMA Logarithmic control chart for the data set in Table 8-9 with a shift of one standard deviation unit in the process mean

Comparing Figure 8-3 with Figure 8-2 we can see now that, as expected, the EWMA control chart detects the one-standard deviation-unit shift and as expected presents out-of-control points quicker than the corresponding Shewhart-type control chart.

<u>8.8.2 Application of the Individual Logarithmic Probability-Type, Shewhart-Type and EWMA Control Charts to Real Data</u>

This section demonstrates the usefulness of the proposed control charts we have seen so far in this chapter through application to two real failure data sets. The first data set by Gaver and O'Muircheartaigh (1987), representing the number of pumps from several systems in a nuclear plant, is presented in Table 8-10.

Case No.	1	2	3	4	5	6	7	8	9	10
Failures	1	4	14	5	22	3	1	19	5	1

Table 8 - 10: Pump Failure Data Set

First of all, when dealing with any dataset, the normality assumption should be checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.05 which is an indication that normality assumption does not hold for our data. For the case of the Logarithmic distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.7591 with the presence of ties in our data and a p-value=0.5412 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the Logarithmic distribution fits our data well.

The value of the parameter of our assumed Logarithmic distribution being equal to 0.863 is going to be used for the construction of the individual probability-type control chart (along with the significance level value $\alpha = 0.27\%$) and for the Shewhart-type control chart for our data, in conjunction with the value of L=3.8682 standard deviations (for which in-control ARL is close to 370). The resulting control charts can be seen in Figure 8-4 and Figure 8-5 for the probability-type and Shewhart-type control chart, respectively, which show all the observations being inside the control limits. This is an indication that the number of pump failures is within the expected ranges.

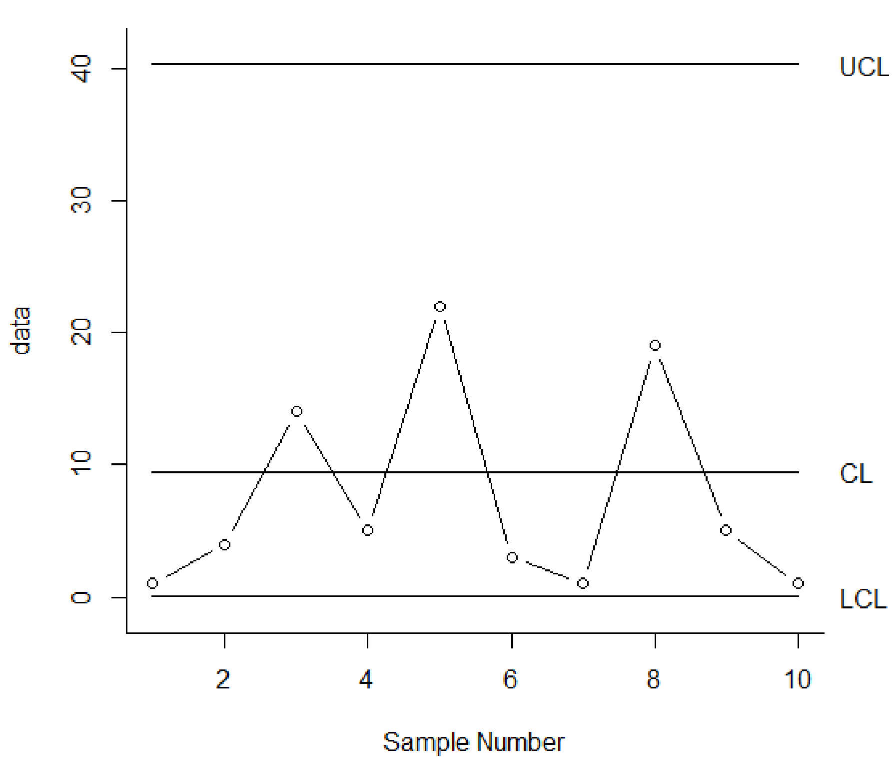

Figure 8 - 4: Individual probability-type control chart for the Pump Failure dataset assuming Logarithmic distribution for the data

For the construction of the individual EWMA control chart for our data, using the same parameter value of the assumed Logarithmic distribution in conjunction with the values of λ=0.05 and L=3.6877 standard deviations (for which in-control ARL is close to 370), the resulting control chart can be seen in Figure 8-6. This chart shows all the observations being inside the control limits, which, once again, is an indication that the number of pump failures is within the expected ranges.

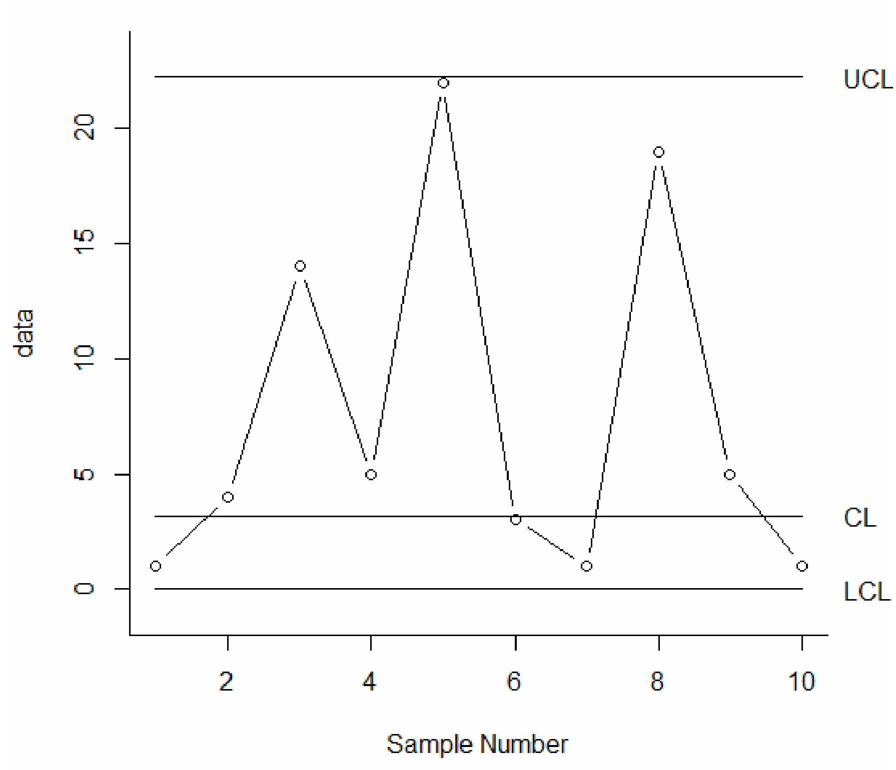

Figure 8 - 5: Individual Shewhart-type control chart for the Pump Failure dataset assuming Logarithmic distribution for the data

The second data set by Jelinski and Moranda (1972) represents the times between successive failures of a piece of software in days. The data set can be seen in Table 8-11. As far as the normality assumption is concerned, both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value<0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the Logarithmic distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value= 0.6649 with the presence of ties in our data and a p-value= 0.7937 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the Logarithmic distribution fits our data well. As we can see, there are a few outliers in our dataset. Let's see if our control charts can detect them.

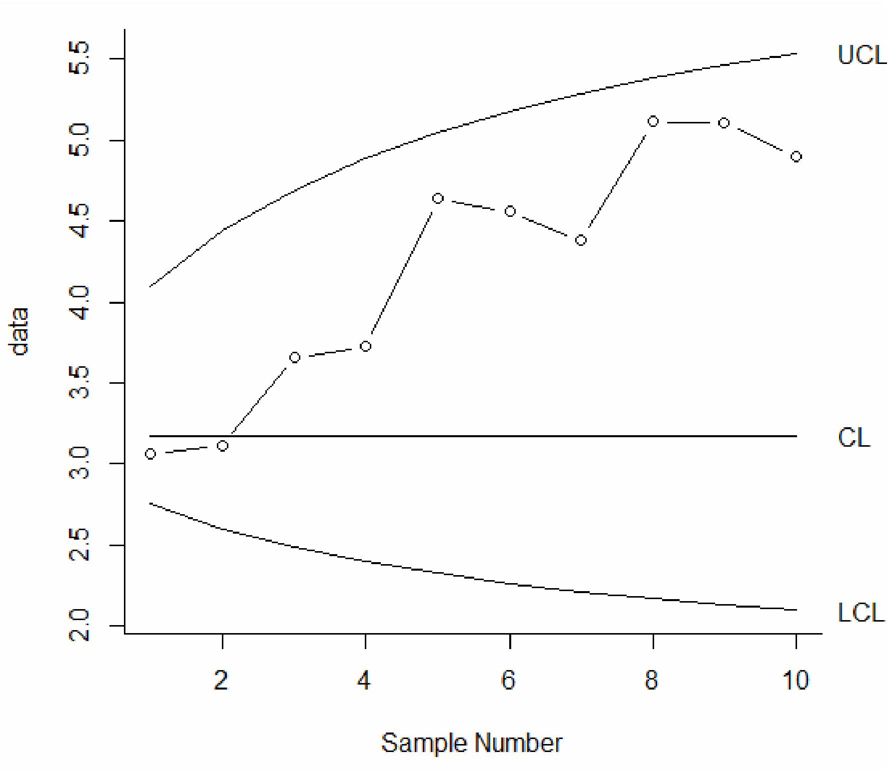

Figure 8 - 6: Individual EWMA control chart for the Pump Failure dataset assuming Logarithmic distribution for the data

9	12	11	4	7	2	5	8
5	7	1	6	1	9	4	1
3	3	6	1	11	33	7	91
2	1	87	47	12	9	135	258
16	35						

Table 8 - 11: Software Failure Data Set (days between successive failures)

The value of the parameter of our assumed Logarithmic distribution being equal to 0.9372 is going to be used for the construction of the individual probability-type control chart (along with the significance level value α = 0.27%) and for the Shewhart-type control chart for our data, in conjunction with the value of L=3.1846 standard deviations (for which in-control ARL is close to

370). The resulting control charts can be seen in Figure 8-7 and Figure 8-8 for the probability-type and Shewhart-type control chart, respectively. Both the control charts present an increasing trend and some out-of control points, which are an indication that the time between failures has increased and, therefore, the software has improved. The difference between the two charts is the number of out-of-control points detected by each one of them. The probability-type control chart detects only two out-of-control points, while the Shewhart-type control chart with the skewness correction presents more out-of-control points and detects the out-of-control shift sooner.

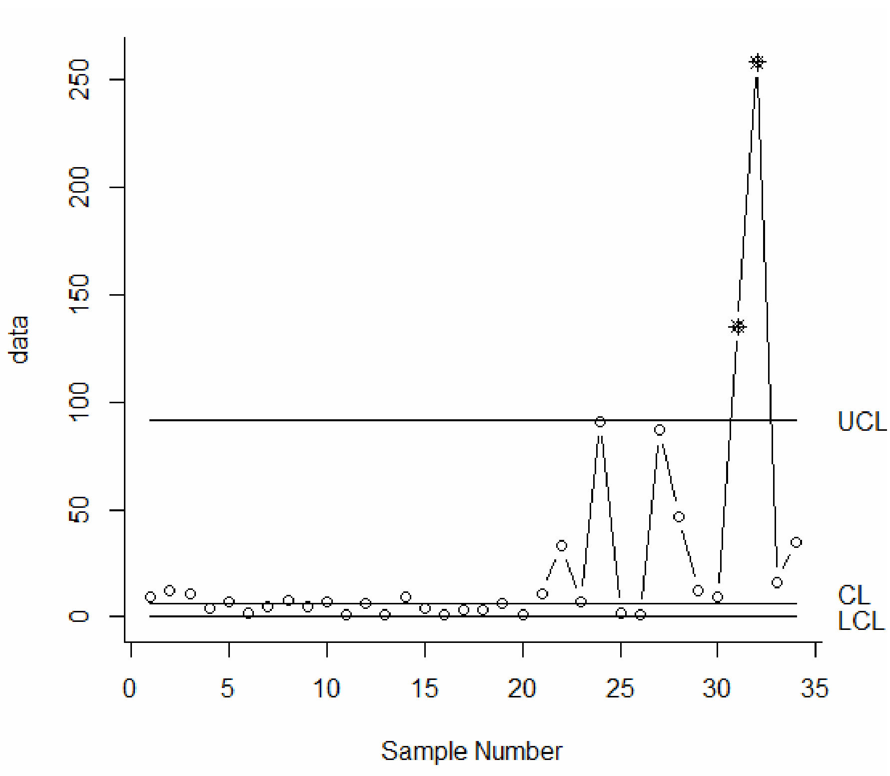

Figure 8 - 7: Individual probability-type control chart for the Software Failures dataset assuming Logarithmic distribution for the data

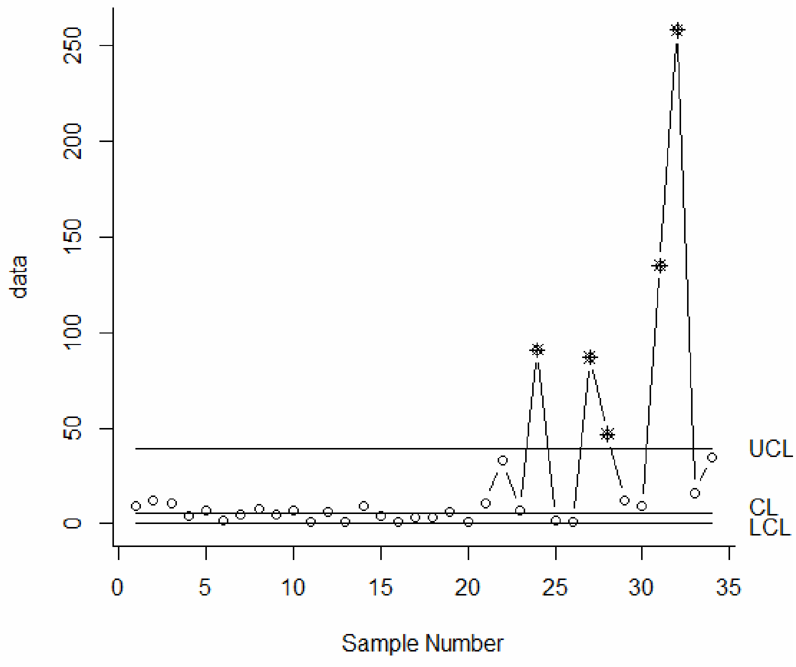

Figure 8 - 8: Individual Shewhart-type control chart for the Software Failures dataset assuming Logarithmic distribution for the data

For the construction of the individual EWMA control chart for our second dataset, using the same parameter value of the assumed Logarithmic distribution in conjunction with the values of $\lambda=0.05$ and L= 2.4182 standard deviations (for which in-control ARL is close to 370), the resulting control chart can be seen in Figure 8-9. This chart shows all the points from the 24th observation and on to be out-of-control, which, once again, is an indication of a quick detection that the time between failures of the software has shifted to an increased out-of-control level.

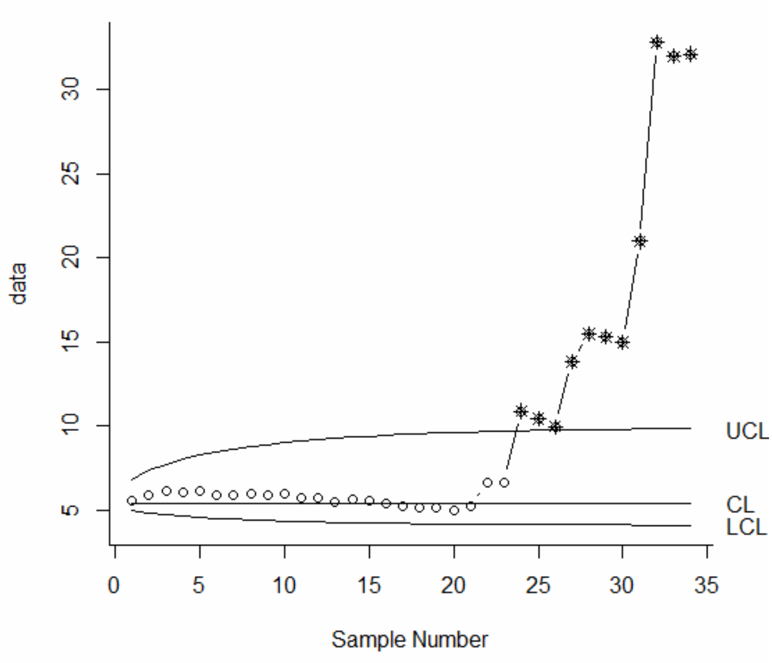

Figure 8 - 9: Individual EWMA control chart for the Software Failures dataset assuming Logarithmic distribution for the data

8.9 Control Charts for Individual Observations from the Logarithmic Distribution with the Scaled Weighted Variance Method

For the construction of the control charts for the Logarithmic distribution discussed in the previous sections, the skewness correction method in Chan and Cui (2003) has been used. Other methods for taking into consideration the distribution's skewness have also been proposed in the literature, such as the scaled weighted variance method described in Castagliola (2000), which was applied there only for continuous distributions. So it would be interesting to present an application of this method for a discrete distribution like the Logarithmic distribution. The use of this method will be presented in the following subsections for constructing control charts for individual observations from the Logarithmic distribution. Their performance will be investigated and compared with the performance of the control charts constructed for the Logarithmic distribution above.

8.9.1. Construction of Shewhart-type Control Charts for Individual Observations from a Process Following the Logarithmic distribution Using the Scaled Weighted Variance Method

The process to be followed according to the scaled weighted variance method by Castagliola (2000) is described below: the central line will be placed at the mean of the Logarithmic distribution, which is computed using equation (4-3), while the control limits will be placed around the mean at two different multiples of the standard deviation of the Logarithmic distribution, which is computed using equation (4-4). These multiples are functions of appropriate values of the quantiles of the standardized Normal distribution, the probability of type I error or false alarm rate, α, and the cumulative distribution function of the Logarithmic distribution, which is computed using equation (4-2). More specifically, the lower control limit is defined as $LCL = \mu - \sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)\sigma$, while the upper control limit is defined as $UCL = \mu + \sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)\sigma$.

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Logarithmic control chart are as follows.

$$UCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \sqrt{\frac{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}\right]}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}$$

$$CL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} \quad , \quad 0<\theta<1$$

$$LCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} - \sqrt{\frac{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1+\frac{\alpha}{\frac{4}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}$$

(8-16)

8.9.2. Performance Investigation for the Individual Logarithmic Control Charts Constructed With the Scaled Weighted Variance Method

In order to investigate the performance of the proposed chart we will use the ARL_0 and ARL_1 values computed with equations (8-4) and (8-5) with $F_{in}(x)$ being the cumulative distribution function of the Logarithmic distribution in equation (4-2) with in-control parameter, $F_{out}(x)$ being the cumulative distribution function for the particular distribution with out-of-control parameter given by $\theta_{new} = \frac{\sigma_{new}^2 - (\mu_0 + k\sigma) + (\mu_0 + k\sigma)^2}{(\mu_0 + k\sigma)^2 + \sigma_{new}^2}$. Using the above formulas we obtain Table 8-12 which shows the in-control and out-of-control ARL values for the individual control chart with scaled weighted variance for the Logarithmic distribution for various values of the parameter θ of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the probability-type control charts we have chosen a significance level equal to the most commonly used value of 0.27%, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

Comparison of Tables 8-12 and 8-2 reveals the improvement in the performance of the chart when using the scaled weighted variance instead of the skewness correction method. The difference in ARL values between those two control charts is greater than 5% for all shift sizes of magnitude equal or greater than k=±1.6 and k=-0.6. Comparison of the ARL values for positive and negative shifts shows that, although the control chart can detect both positive and negative shifts well, there are some differences with almost half of the values being higher for the negative shifts than for the corresponding positive ones. The differences (in either direction) that are above 5% concern the shifts corresponding to large values of k (larger than or equal to ±2.8) combined with the smallest and largest values of the parameter θ of the logarithmic distribution. Moreover, the differences between the ARL values for positive and negative shifts are higher for larger shift magnitudes.

k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
-3	2.0146	2.0233	2.0321	2.0344	2.0410	2.0505	2.1003	2.2001
-2.8	3.0148	3.0234	3.0323	3.0370	3.0412	3.0510	3.1012	3.2004
-2.6	4.0150	4.0235	4.0324	4.0373	4.0416	4.0512	4.1019	4.2007
-2.4	6.0152	6.0236	6.0328	6.0398	6.0423	6.0512	6.1022	6.2012
-2.2	8.0154	8.0241	8.0334	8.0434	8.0524	8.0575	8.1032	8.2017
-2	10.0164	10.0248	10.0335	10.0440	10.0543	10.0648	10.1036	10.2019
-1.8	12.0196	12.0254	12.0337	12.0448	12.0544	12.0795	12.1037	12.2023
-1.6	14.0200	14.0264	14.0342	14.0464	14.0546	14.0934	14.1040	14.2025
-1.4	19.0212	19.0268	19.0368	19.0488	19.0548	19.1042	19.1064	19.2032
-1.2	28.0228	28.0284	28.0412	28.0481	28.0553	28.1046	28.1080	28.2035
-1	40.0230	40.0335	40.0414	40.0484	40.0557	40.1050	40.1201	40.2048
-0.8	57.0289	57.0340	57.0423	57.0488	57.0559	57.1064	57.1223	57.2151
-0.6	73.0303	73.0354	73.0446	73.0515	73.0596	73.1227	73.1252	73.2200
-0.4	118.0336	118.0377	118.0488	118.0606	118.0612	118.1223	118.1225	118.2684
-0.2	202.0421	202.0482	202.0625	202.0682	202.0735	202.1263	202.1804	202.2891
0	379.0543	379.1052	379.2030	379.2122	379.0486	379.0414	379.0336	379.0284
0.2	202.3904	202.2097	202.1224	202.0637	202.0542	202.0480	202.0412	202.0355
0.4	118.3735	118.2093	118.1221	118.0634	118.0557	118.0486	118.0406	118.0350
0.6	73.3752	73.2090	73.1227	73.0630	73.0554	73.0481	73.0402	73.0346
0.8	57.3686	57.2086	57.1222	57.0626	57.0548	57.0487	57.0397	57.0341
1	40.3605	40.2084	40.1209	40.0621	40.0545	40.0482	40.0393	40.0336
1.2	28.3520	28.2079	28.1204	28.0617	28.0540	28.0468	28.0378	28.0331
1.4	19.3428	19.2075	19.1200	19.0612	19.0535	19.0462	19.0373	19.0326
1.6	14.3332	14.2070	14.1095	14.0607	14.0530	14.0457	14.0377	14.0320
1.8	12.3228	12.2064	12.1091	12.0602	12.0525	12.0452	12.0372	12.0315
2	10.3126	10.2062	10.1086	10.0597	10.0520	10.0446	10.0364	10.0309
2.2	8.2893	8.2057	8.1080	8.0593	8.0514	8.0440	8.0360	8.0303
2.4	6.2861	6.2052	6.1075	6.0575	6.0508	6.0435	6.0354	6.0288
2.6	4.2712	4.2048	4.1069	4.0571	4.0503	4.0428	4.0348	4.0284
2.8	3.2544	3.2042	3.1064	3.0575	3.0487	3.0423	3.0343	3.0288
3	2.2348	2.2036	2.1057	2.0548	2.0481	2.0418	2.0339	2.0284

Table 8 - 12: ARL values for individual control charts with scaled weighted variance for the Logarithmic distribution, with $\alpha = 0.0027$.

We also observe that the higher the value of the θ parameter the larger the ARL value for the negative shifts. This makes sense if one considers that the values of the logarithmic distribution with a higher θ parameter value are also higher, which makes it more possible for them to get out of control with a positive shift and less possible to come out of control with a negative shift. Similarly smaller values of the θ parameter result in smaller observations which are easier to get out of control with a negative shift than a positive one.

8.9.3. Construction of the EWMA Control Charts For Individual Observations from the Logarithmic Distribution Using the Scaled Weighted Variance Method

The scaled weighted variance method is going to be used here for the construction of EWMA charts, as well. This is going to improve the performance of the chart compared with the previously used skewness correction method, as we will prove in the next subsection. The method we will apply here is the following: in equation (2-3) we will replace L by $\sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)$ for the lower control limit and $\sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)$ for the upper control limit, where μ is the mean of the Logarithmic distribution, which is computed using equation (4-3), and $F_X(x)$ is its cumulative distribution function given by equation (4-2). For the construction of the EWMA control charts we will also need the standard deviation of the Logarithmic distribution computed from equation (4-4).

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Logarithmic EWMA control chart are as follows.

$$UCL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}+\sqrt{\frac{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}\right]}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}$$

$$LCL=-\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta}-\sqrt{\frac{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1+\frac{\alpha}{\frac{4}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

(8-17)

The plotting statistic will be the one in equation (2-2) with x_i being the observations from our Logarithmic distribution.

8.9.4. Performance Investigation for the Individual EWMA Logarithmic Control Charts Constructed With the Scaled Weighted Variance Method

For the investigation of the performance of the control chart we just constructed, we will use the ARL value as presented in equation (8-13). For the transient probabilities in (8-12) the cumulative distribution function for the Logarithmic distribution, i.e. equation (4-2), is going to be used with either in-control parameter for the case of computing the in-control ARL value or the out-of-control parameter for the case of the out-of-control ARL, with the asymptotic control limits as computed with equation (8-17) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} + \sqrt{\frac{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}\right]}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = -\frac{1}{\ln(1-\theta)}\frac{\theta}{1-\theta} - \sqrt{\frac{1+\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}{-\frac{1}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}}\Phi^{-1}\left(1+\frac{\alpha}{\frac{4}{\ln(1-\theta)}\sum_{u=1}^{x}\frac{\theta^u}{u}}\right)\sqrt{-\frac{1}{\ln(1-\theta)}\frac{\theta}{(1-\theta)^2}\left(1+\frac{\theta}{\ln(1-\theta)}\right)}\sqrt{\frac{\lambda}{2-\lambda}}$$

(8-18)

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameter of the distribution with the shifted mean will be computed by combining equations (4-3) and (4-4) and solving in terms of its parameter, as for the Shewhart-type control chart.

Using those formulae we get Tables 8-13, 8-14, and 8-15, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the Logarithmic distribution for various values of the parameter θ of the distribution of concern and for various values of k (which shows the shift of the process mean in terms of the process standard deviation). More specifically, Table 8-13 contains the ARL values for λ=0.3 for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=180, above which ARL increases very slightly. As a result, the suggested value of m for the computation of ARL in the formulae above is m=180. Therefore, Tables 8-14 and 8-15 show the ARL values for m=180 for various values of λ for positive and negative shifts, respectively.

m	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
80	0	371.1476	371.0129	370.9164	370.7986	370.7231	370.6496	370.6179	370.5158
	0.2	52.1475	52.0121	51.9153	51.7787	51.7230	51.6479	51.5954	51.4707
	0.5	17.1474	16.9121	16.7229	16.5764	16.4676	16.4100	16.2944	16.0870
	1	10.1238	9.8878	9.6902	9.5607	9.4437	9.3744	9.2765	9.0750
	1.5	7.1404	6.8698	6.6969	6.5623	6.4503	6.3468	6.2827	6.0817
	2	6.1408	5.8390	5.6972	5.5457	5.4503	5.3472	5.2580	5.0822
	2.5	5.1255	4.8426	4.6705	4.5229	4.4248	4.3404	4.2399	4.0637
	3	4.0889	3.8258	3.6386	3.4917	3.4156	3.3310	3.2171	3.0545
100	0	370.4258	370.3522	370.3206	370.2961	370.2271	370.1855	370.0870	370.0647
	0.2	51.3984	51.3206	51.3205	51.2635	51.2064	51.1559	51.0647	51.0647
	0.5	17.2576	17.2470	16.9283	16.9277	16.7809	16.7803	16.6212	16.5714
	1	10.1686	10.1678	9.9154	9.8321	9.7672	9.7006	9.6060	9.5520
	1.5	7.1427	6.9120	6.7181	6.5929	6.5029	6.4325	6.2891	6.1605
	2	6.1427	5.9124	5.7182	5.5907	5.4971	5.4052	5.2896	5.1610
	2.5	5.1464	4.9002	4.7120	4.5718	4.5002	4.3975	4.2876	4.1276
	3	4.1260	3.8878	3.7006	3.5625	3.4571	3.3695	3.2783	3.1284
120	0	371.1457	370.8438	370.7025	370.5426	370.4649	370.3523	370.2700	370.0870
	0.2	52.1456	51.8436	51.7022	51.5226	51.4556	51.3432	51.2628	51.0870
	0.5	16.4681	16.4258	16.4095	16.3390	16.2917	16.2274	16.1658	16.0647
	1	9.4000	9.3869	9.3292	9.3087	9.2169	9.2153	9.1264	9.0527
	1.5	7.3697	7.2515	7.0531	7.0527	6.8693	6.7931	6.6444	6.6453
	2	6.3205	6.2517	5.9968	5.9964	5.8308	5.7767	5.6247	5.6246
	2.5	5.2554	4.9269	4.7799	4.6185	4.5634	4.4463	4.2939	4.1846
	3	4.2462	3.9173	3.7703	3.6084	3.5121	3.4022	3.2842	3.1556
150	0	371.1933	370.9274	370.7786	370.5968	370.5094	370.4122	370.2944	370.1658
	0.2	52.1801	51.9019	51.7124	51.5961	51.5094	51.4100	51.2886	51.1658
	0.5	17.1799	16.9001	16.7121	16.5727	16.4671	16.3796	16.2700	16.1559
	1	10.1679	9.8318	9.7000	9.5528	9.4538	9.3319	9.2508	9.1264
	1.5	6.5374	6.5041	6.4605	6.3743	6.3359	6.2575	6.2356	6.1233
	2	5.5178	5.4610	5.4206	5.3342	5.3341	5.2226	5.2016	5.1238
	2.5	5.4230	5.3739	5.1241	5.0574	4.9149	4.8716	4.7215	4.6708
	3	4.3967	4.3645	4.0877	4.0480	3.8905	3.8236	3.6915	3.6402
180	0	371.3252	371.0574	370.7984	370.6493	370.5654	370.4556	370.3521	370.2409
	0.2	52.3251	52.0016	51.7984	51.6296	51.5649	51.4373	51.3206	51.2064
	0.5	17.3057	17.0014	16.7810	16.6295	16.5643	16.4350	16.2949	16.1856
	1	10.2714	9.9889	9.7644	9.6155	9.5120	9.4084	9.2829	9.1736
	1.5	7.2512	6.9226	6.7733	6.6124	6.4964	6.4047	6.2891	6.1605
	2	6.1751	5.8961	5.7080	5.6128	5.4971	5.3936	5.2652	5.1512
	2.5	4.5951	4.5644	4.4643	4.4363	4.3974	4.2934	4.2690	4.1549
	3	3.5859	3.5323	3.4156	3.4027	3.3310	3.2598	3.2525	3.1284
200	0	371.4230	371.1475	371.1262	370.8761	370.7986	370.6721	370.6709	370.5676
	0.2	52.4061	52.1457	52.1257	51.8743	51.7787	51.6706	51.6490	51.5676
	0.5	17.3764	17.0989	16.8368	16.6706	16.5645	16.4674	16.3781	16.2409
	1	10.3634	10.0865	9.8233	9.6583	9.5292	9.4436	9.3401	9.2289
	1.5	7.3693	7.0539	6.7933	6.6435	6.5174	6.4325	6.3327	6.2012
	2	6.3216	6.0527	5.7938	5.6438	5.5045	5.4210	5.3158	5.2016
	2.5	5.3246	5.0012	4.7777	4.6285	4.5010	4.4090	4.2939	4.1846
	3	4.3148	3.9907	3.7685	3.6190	3.4555	3.3998	3.2846	3.1754
220	0	370.4556	370.3986	370.3520	370.2949	370.2700	370.1856	370.1560	370.0870
	0.2	51.4258	51.3414	51.3205	51.2628	51.2628	51.1560	51.1560	51.0647
	0.5	17.4256	17.2565	17.1277	16.9278	16.9150	16.7812	16.7196	16.5727
	1	10.4124	10.1678	10.1245	9.8635	9.8318	9.7002	9.6872	9.5607
	1.5	7.4179	7.1298	6.8700	6.6445	6.5673	6.5041	6.3746	6.2832
	2	6.4181	6.1277	5.8388	5.6470	5.5628	5.4623	5.3751	5.2361
	2.5	5.4074	5.0991	4.8353	4.6497	4.5638	4.4648	4.3510	4.2399
	3	4.3962	4.0889	3.8259	3.6386	3.5324	3.4454	3.3289	3.2171
240	0	371.4063	371.0979	370.7988	370.6487	370.5426	370.4374	370.3391	370.2064
	0.2	52.4060	52.0457	51.7789	51.6485	51.5222	51.4128	51.3205	51.2064
	0.5	16.5019	16.4671	16.4123	16.4100	16.2949	16.2886	16.1864	16.1658
	1	9.4899	9.3863	9.3863	9.3292	9.2580	9.2154	9.1444	9.1264
	1.5	7.3199	6.9957	6.8309	6.6241	6.5169	6.4605	6.3338	6.2012
	2	6.3701	6.0528	5.8706	5.6470	5.5595	5.5046	5.3751	5.2361
	2.5	5.4054	5.0971	4.9000	4.7012	4.5944	4.5417	4.4122	4.2690
	3	4.4121	4.1251	3.9061	3.7128	3.6077	3.5575	3.4276	3.2842

Table 8 - 13: ARL values for individual EWMA control charts with scaled weighted variance with $\alpha = 0.0027$ for the Logarithmic distribution (λ=0.3) for various values of m.

λ	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
λ=0.05	0	371.4718	371.1812	371.1695	370.9650	370.8152	370.7650	370.7010	370.6561
	0.2	52.4497	52.1789	52.1264	51.9439	51.7928	51.7389	51.6740	51.6305
	0.4	17.4127	17.0910	16.9146	16.7040	16.5938	16.5821	16.4535	16.3014
	0.6	14.8543	14.5256	14.3684	14.1554	14.0570	14.0459	13.9267	13.1774
	0.8	12.7755	12.4421	12.3008	12.0872	12.0224	11.9792	11.8919	11.6470
	1	10.2416	9.9065	9.7716	9.5580	9.4769	9.0714	8.9512	7.8147
	1.5	7.2083	6.8704	6.7409	6.5269	6.4512	6.4122	6.2916	6.1657
	2	6.1786	5.8356	5.7098	5.4945	5.4373	5.3867	5.2678	4.1476
	2.5	4.4212	4.3633	4.3581	4.3021	4.2422	4.1895	4.1212	3.0883
	3	3.3894	3.3281	3.3278	3.2672	3.2151	3.1580	3.1083	3.0653
λ=0.08	0	371.5122	371.1933	371.1769	371.0016	370.8123	370.7756	370.6972	370.6742
	0.2	52.4807	52.1748	52.1548	51.9745	51.7912	51.7468	51.6712	51.6338
	0.4	17.4358	17.1028	16.9369	16.7099	16.6122	16.5969	16.4560	16.3026
	0.6	14.8739	14.5368	14.3868	14.1597	14.0784	14.0482	13.9274	13.1776
	0.8	12.7887	12.4491	12.3149	12.0910	12.0390	11.9820	11.8931	11.6483
	1	10.2527	9.9122	9.7835	9.5612	9.4905	9.0691	8.9515	7.8152
	1.5	7.2175	6.8749	6.7507	6.5293	6.4535	6.4223	6.2921	6.1657
	2	6.1865	5.8392	5.7178	5.4963	5.4358	5.3939	5.2682	4.1477
	2.5	4.4229	4.3636	4.3621	4.3010	4.2425	4.1887	4.1212	3.0882
	3	3.3908	3.3318	3.3274	3.2644	3.2153	3.1577	3.1083	3.0652
λ=0.10	0	371.5418	371.2121	371.1735	371.0292	370.8127	370.8034	370.6950	370.6945
	0.2	52.5081	52.1728	52.1714	52.0006	51.7900	51.7719	51.6490	51.6422
	0.4	17.4568	17.1253	16.9564	16.7271	16.6241	16.6175	16.4835	16.3232
	0.6	14.8887	14.5449	14.4012	14.1735	14.0907	14.0632	13.9507	13.1953
	0.8	12.7985	12.4544	12.3246	12.1000	12.0475	11.9912	11.9125	11.6612
	1	10.2608	9.9164	9.7915	9.5684	9.4955	9.0681	8.9632	7.8219
	1.5	7.2242	6.8782	6.7571	6.5350	6.4606	6.4256	6.2993	6.1708
	2	6.1920	5.8417	5.7230	5.5006	5.4352	5.3964	5.2737	4.1515
	2.5	4.4283	4.3655	4.3609	4.2998	4.2464	4.1886	4.1219	3.0880
	3	3.3947	3.3331	3.3264	3.2659	3.2180	3.1576	3.1200	3.0651
λ=0.12	0	371.5670	371.2307	371.1701	371.0527	370.8229	370.8101	370.7016	370.6914
	0.2	52.5293	52.1889	52.1681	52.0224	51.7902	51.7887	51.6485	51.6445
	0.4	17.4750	17.1283	16.9749	16.7424	16.6273	16.6220	16.4917	16.3240
	0.6	14.8988	14.5507	14.4124	14.1812	14.0939	14.0643	13.9586	13.1958
	0.8	12.8059	12.4588	12.3324	12.1059	12.0498	11.9929	11.9160	11.6615
	1	10.2648	9.9198	9.7978	9.5730	9.4968	9.0643	8.9659	7.8222
	1.5	7.2290	6.8808	6.7621	6.5385	6.4620	6.4263	6.3012	6.1708
	2	6.1959	5.8437	5.7269	5.5032	5.4339	5.3970	5.2751	4.1515
	2.5	4.4294	4.3659	4.3604	4.2997	4.2475	4.1883	4.1226	3.0880
	3	3.3954	3.3334	3.3263	3.2658	3.2187	3.1575	3.1200	3.0651
λ=0.15	0	371.5991	371.2553	371.1652	371.0812	370.8380	370.8081	370.7225	370.6873
	0.2	52.5539	52.2031	52.1633	52.0422	51.8001	51.7870	51.6842	51.6442
	0.4	17.4942	17.1412	16.9920	16.7537	16.6431	16.6383	16.4946	16.3353
	0.6	14.9120	14.5616	14.4257	14.1894	14.1120	14.0753	13.9623	13.2038
	0.8	12.8127	12.4638	12.3400	12.1094	12.0643	11.9980	11.9202	11.6655
	1	10.2730	9.9236	9.8038	9.5757	9.4985	9.0658	8.9680	7.8239
	1.5	7.2339	6.8837	6.7647	6.5405	6.4631	6.4316	6.3021	6.1726
	2	6.1999	5.8458	5.7304	5.5046	5.4321	5.4010	5.2757	4.1527
	2.5	4.4315	4.3688	4.3576	4.2988	4.2479	4.1865	4.1285	3.0870
	3	3.3973	3.3353	3.3245	3.2653	3.2189	3.1565	3.1204	3.0647
λ=0.20	0	371.6543	371.2928	371.1582	371.1212	370.8778	370.8053	370.7490	370.6808
	0.2	52.5973	52.2336	52.1563	52.0828	51.8310	51.7846	51.7038	51.6575
	0.4	17.5231	17.1600	17.0207	16.7729	16.6486	16.6484	16.5199	16.3388
	0.6	14.9310	14.5716	14.4436	14.2021	14.1295	14.0820	13.9778	13.2043
	0.8	12.8249	12.4708	12.3524	12.1189	12.0756	12.0042	11.9286	11.6680
	1	10.2815	9.9287	9.8122	9.5828	9.5009	9.0704	8.9718	7.8252
	1.5	7.2403	6.8873	6.7738	6.5456	6.4618	6.4374	6.3072	6.1742
	2	6.2049	5.8484	5.7356	5.5081	5.4353	5.4051	5.2793	4.1537
	2.5	4.4305	4.3715	4.3567	4.2960	4.2503	4.1864	4.1285	3.0882
	3	3.3994	3.3371	3.3239	3.2635	3.2203	3.1564	3.1208	3.0659

Table 8 - 14: ARL values for individual EWMA control charts with scaled weighted variance with $\alpha = 0.0027$ for the Logarithmic distribution (m=180) for various positive shifts.

λ	k	θ=0.12	θ=0.26	θ=0.39	θ=0.45	θ=0.54	θ=0.68	θ=0.73	θ=0.84
λ=0.05	0	371.4718	371.1812	371.1695	370.9650	370.8152	370.7650	370.7010	370.6561
	-0.2	51.5415	51.6255	51.7323	51.7926	51.9157	52.2106	52.4704	52.9078
	-0.4	16.5174	16.5942	16.7007	16.7643	16.9037	17.1515	17.4049	17.7760
	-0.6	13.7393	13.7684	13.8745	13.9641	14.0859	14.3438	14.5129	14.8912
	-0.8	11.6269	11.7379	11.8565	11.9461	12.0852	12.3414	12.4887	12.8882
	-1	8.2093	8.2427	8.3540	8.4419	9.0574	9.3789	9.5056	9.5823
	-1.5	6.5074	6.5305	6.6277	6.6912	6.7312	6.8234	6.9642	7.1812
	-2	4.5676	4.6363	4.6454	4.7031	5.2589	5.4820	5.4954	5.5284
	-2.5	3.4459	3.5358	3.5631	3.6142	4.1308	4.2439	4.2729	4.2946
	-3	3.0924	3.1247	3.1293	3.1788	3.2310	3.2391	3.2672	3.3121
λ=0.08	0	371.5122	371.1933	371.1769	371.0016	370.8123	370.7756	370.6972	370.6742
	-0.2	51.5415	51.6255	51.7326	51.7930	51.9165	52.2187	52.5099	52.9552
	-0.4	16.5174	16.5942	16.7014	16.7644	16.9044	17.1583	17.4106	17.7893
	-0.6	13.7468	13.7709	13.8748	13.9642	14.0865	14.3473	14.5169	14.9312
	-0.8	11.6401	11.7379	11.8567	11.9463	12.0858	12.3431	12.4890	12.8964
	-1	8.2106	8.2467	8.3681	8.4697	9.0640	9.4329	9.5151	9.6520
	-1.5	6.5089	6.5344	6.6495	6.6940	6.7709	6.8714	7.0223	7.2037
	-2	4.6028	4.6390	4.6495	4.7189	5.3039	5.4821	5.5068	5.5486
	-2.5	3.4782	3.5410	3.5682	3.6190	4.1717	4.2475	4.2745	4.3006
	-3	3.1090	3.1250	3.1548	3.1849	3.2314	3.2398	3.2841	3.3843
λ=0.10	0	371.5418	371.2121	371.1735	371.0292	370.8127	370.8034	370.6950	370.6945
	-0.2	51.5416	51.6259	51.7327	51.7942	51.9186	52.2239	52.5406	52.9783
	-0.4	16.5174	16.5942	16.7015	16.7647	16.9062	17.1712	17.4147	17.7992
	-0.6	13.7627	13.7714	13.8749	13.9645	14.0879	14.3498	14.5190	14.9645
	-0.8	11.6523	11.7379	11.8568	11.9468	12.0872	12.3445	12.4900	12.9023
	-1	8.2129	8.2525	8.3796	8.4741	9.0653	9.4432	9.5178	9.6553
	-1.5	6.5107	6.5489	6.6535	6.7076	6.7780	6.8827	7.0775	7.2083
	-2	4.6031	4.6394	4.6863	4.7499	5.3325	5.4897	5.5161	5.5719
	-2.5	3.5191	3.5437	3.5799	3.6212	4.1829	4.2496	4.2840	4.3122
	-3	3.1205	3.1282	3.1644	3.1936	3.2343	3.2455	3.2858	3.3879
λ=0.12	0	371.5670	371.2307	371.1701	371.0527	370.8229	370.8101	370.7016	370.6914
	-0.2	51.5416	51.6260	51.7330	51.7943	51.9196	52.2278	52.5681	52.9971
	-0.4	16.5174	16.5942	16.7015	16.7649	16.9071	17.1719	17.4181	17.8065
	-0.6	13.7627	13.7759	13.8750	13.9647	14.0886	14.3546	14.5207	15.0022
	-0.8	11.6548	11.7379	11.8568	11.9469	12.0879	12.3465	12.4904	12.9065
	-1	8.2142	8.2570	8.3857	8.4839	9.1494	9.4438	9.5287	9.7020
	-1.5	6.5162	6.5512	6.6891	6.7082	6.7864	6.8922	7.1009	7.2318
	-2	4.6287	4.6408	4.6899	4.7610	5.4535	5.4918	5.5176	5.6586
	-2.5	3.5246	3.5455	3.5882	3.6269	4.2234	4.2498	4.2853	4.3129
	-3	3.1225	3.1283	3.1708	3.1968	3.2358	3.2512	3.2883	3.3887
λ=0.15	0	371.5991	371.2553	371.1652	371.0812	370.8380	370.8081	370.7225	370.6873
	-0.2	51.5416	51.6260	51.7332	51.7948	51.9202	52.2312	52.5984	53.0189
	-0.4	16.5218	16.5993	16.7017	16.7653	16.9075	17.1740	17.4219	17.8512
	-0.6	13.7627	13.7900	13.8751	13.9651	14.0890	14.3569	14.5225	15.0144
	-0.8	11.6578	11.7379	11.8568	11.9471	12.0883	12.3490	12.4912	12.9107
	-1	8.2169	8.2642	8.3937	8.4842	9.1730	9.4465	9.5434	9.7647
	-1.5	6.5171	6.5544	6.6893	6.7101	6.7880	6.8968	7.1269	7.2341
	-2	4.6302	4.6436	4.6937	4.7621	5.4724	5.4947	5.5237	5.7244
	-2.5	3.5322	3.5479	3.5886	3.6405	4.2326	4.2525	4.2907	4.3393
	-3	3.1231	3.1286	3.1733	3.2120	3.2365	3.2549	3.2901	3.3948
λ=0.20	0	371.6543	371.2928	371.1582	371.1212	370.8778	370.8053	370.7490	370.6808
	-0.2	51.5416	51.6262	51.7333	51.7952	51.9214	52.2365	52.6564	53.0559
	-0.4	16.5222	16.5997	16.7019	16.7656	16.9085	17.1749	17.4269	17.9509
	-0.6	13.7627	13.7928	13.8751	13.9653	14.0897	14.3588	14.5245	15.0256
	-0.8	11.6853	11.7379	11.8569	11.9472	12.0890	12.3575	12.4928	12.9162
	-1	8.2324	8.2912	8.4148	8.4843	9.1841	9.4563	9.5505	9.7756
	-1.5	6.5275	6.5548	6.6899	6.7150	6.7893	6.8968	7.1296	7.2637
	-2	4.6309	4.6454	4.7012	4.7645	5.4815	5.4952	5.5256	5.7491
	-2.5	3.5328	3.5564	3.6104	3.6471	4.2393	4.2573	4.2919	4.3412
	-3	3.1241	3.1291	3.1747	3.2127	3.2386	3.2644	3.2918	3.4022

Table 8 - 15: ARL values for individual EWMA control charts with scaled weighted variance with $\alpha = 0.0027$ for the Logarithmic distribution (m=180) for various negative shifts.

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some differences in ARL values between those two tables, with almost half of the ARLs for negative shifts being larger than the corresponding ones for the positive shifts. This is valid for the larger values of the parameter θ, which makes sense since a larger θ value gives a larger observation which is less possible to get out of control with a negative shift than with a positive one, and vice-versa. This is probably the reason that the differences (in either direction) are above 5% for shifts larger than ±0.6 for the smallest and largest values of θ.

Additionally, comparing the ARL values for the EWMA in Tables 8-14 (8-15) and 8-4 (8-5), we can see that the EWMA control chart with the scaled weighted variance performs better than the corresponding one with the skewness correction method, since the in-control ARL values with the scaled weighted variance are larger and the out-of-control ARL values are smaller than the corresponding ones with the skewness correction method. All the differences are significant since they are larger than 5%.

8.9.5 Example on the Logarithmic individual Shewhart-type and EWMA control charts with scaled weighted variance using simulated data

This section presents the illustration of the proposed control charts using simulated data generated from the distribution of concern. The case of real data will be presented in section 8.9.6. For the same dataset in Table 8-9, we construct the individual Shewhart-type Logarithmic control charts with scaled weighted variance presented in Figure 8-10, using the most commonly used value for the significance level $\alpha = 0.27\%$, as mentioned earlier.

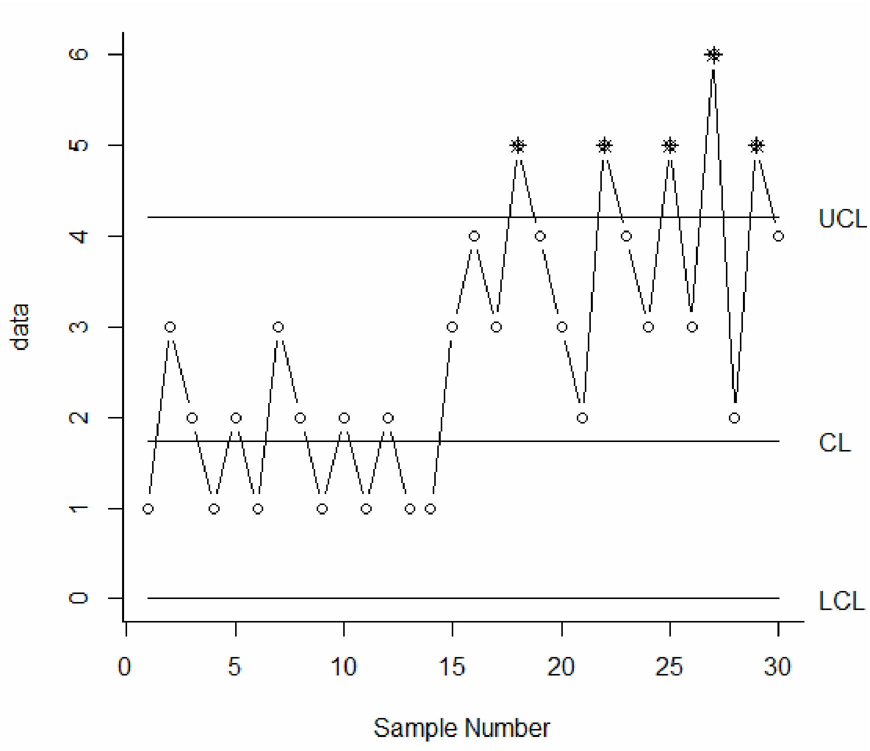

Figure 8 - 10: Individual Logarithmic control chart with scaled weighted variance for the data set in Table 8-9 with a shift of one standard deviation unit in the process mean

As we can see the control chart detects some out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level, which the Shewhart-type chart with skewness correction had not detected.

Using the same data set, we now construct the individual EWMA Logarithmic control chart with scaled weighted variance shown in Figure 8-11, using λ=0.05. As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 19th observation.

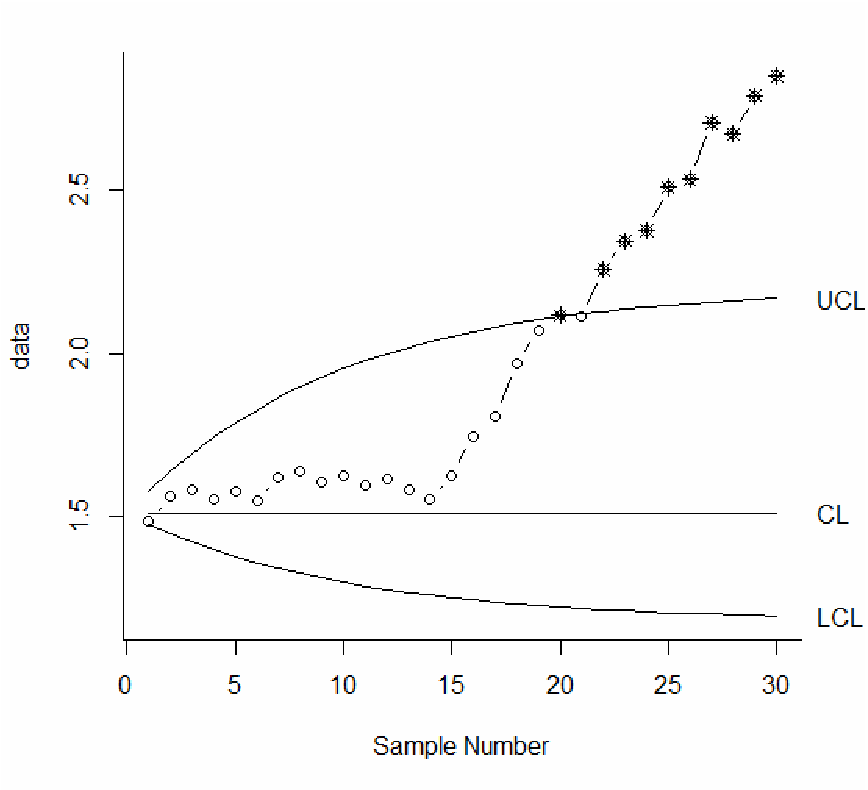

Figure 8 - 11: Individual EWMA Logarithmic control chart with scaled weighted variance for the data set in Table 8-9 with a shift of one standard deviation unit in the process mean

Comparing Figure 8-11 with Figure 8-3 for the skewness correction we can see that the EWMA control chart detects the one-standard deviation-unit shift quickly and presents out-of-control points quicker than the corresponding EWMA control chart with skewness correction.

8.9.6 Application of the Logarithmic individual Shewhart-type and EWMA control charts with scaled weighted variance to real data

This section contains the illustration of the proposed control charts through application to the same real datasets as earlier (Tables 8-10 and 8-11) and for the same values of the parameter of our assumed Logarithmic distribution. As we can see, for the first case of the pump failure data, the resulting control charts

(Figures 8-12 and 8-13) present out-of-control points that the corresponding control charts with skewness correction had not detected.

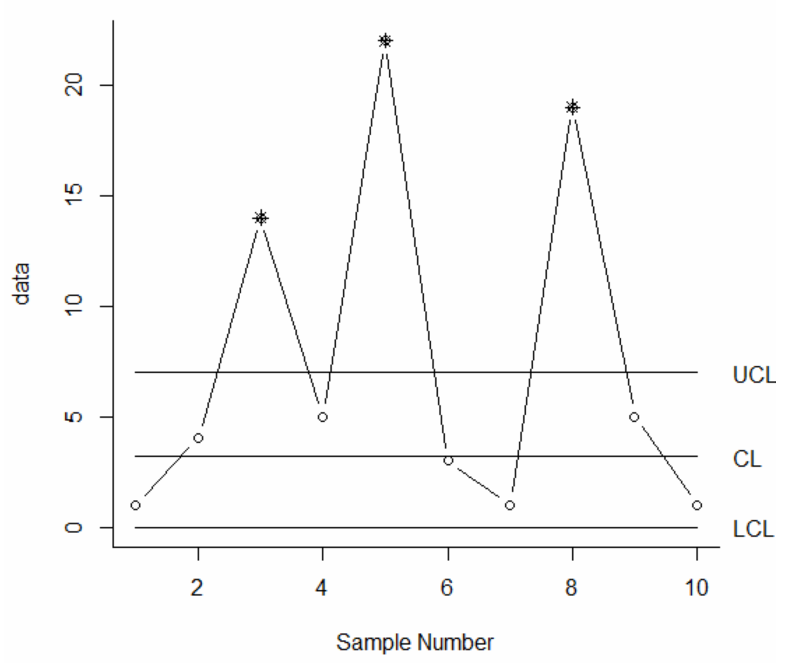

Figure 8 - 12: Individual control chart with scaled weighted variance for the Pump Failure dataset assuming Logarithmic distribution for the data

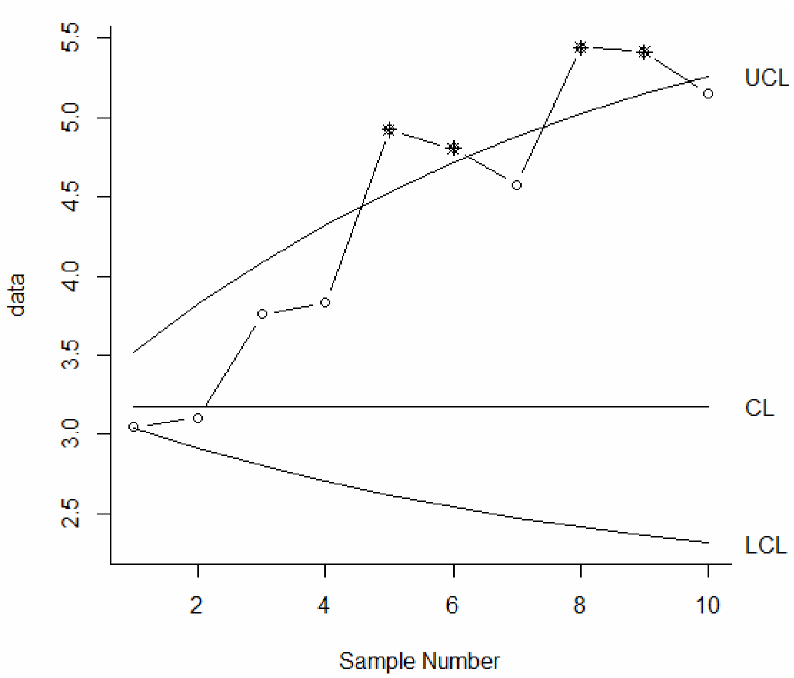

Figure 8 - 13: Individual EWMA control chart with scaled weighted variance for the Pump Failure dataset assuming Logarithmic distribution for the data

For the second data set by Jelinski and Moranda (1972) representing the times between successive failures of a piece of software in days, the resulting individual logarithmic and individual logarithmic EWMA control charts with scaled weighted variance are presented in Figure 8-14 and Figure 8-15, respectively. For the EWMA the value of $\lambda=0.05$ was chosen. The charts once again, present more out-of-control points than the corresponding ones with the skewness correction.

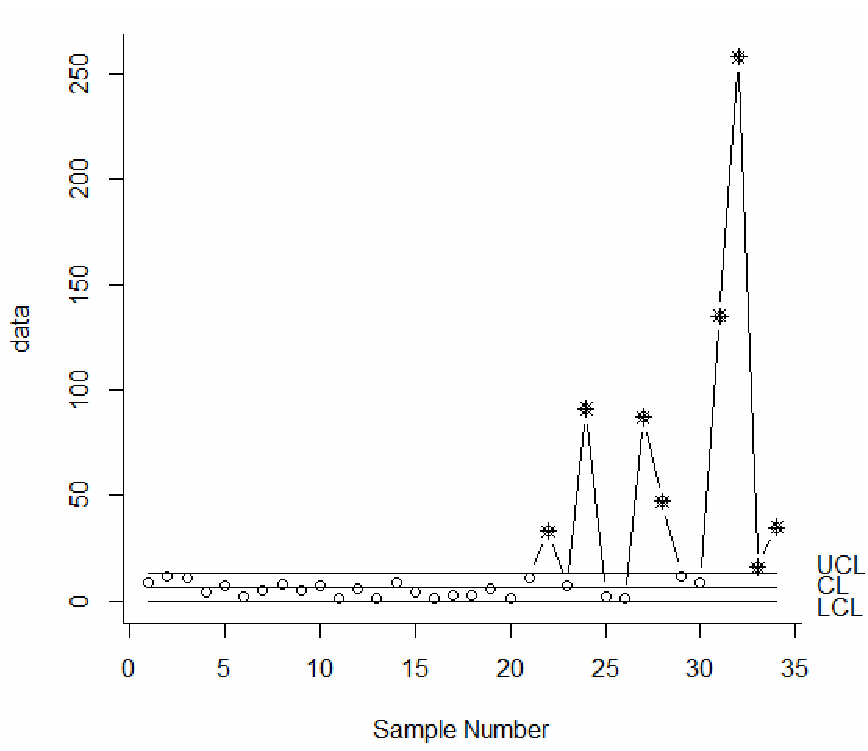

Figure 8 - 14: Individual control chart with scaled weighted variance for the Software Failures dataset assuming Logarithmic distribution for the data

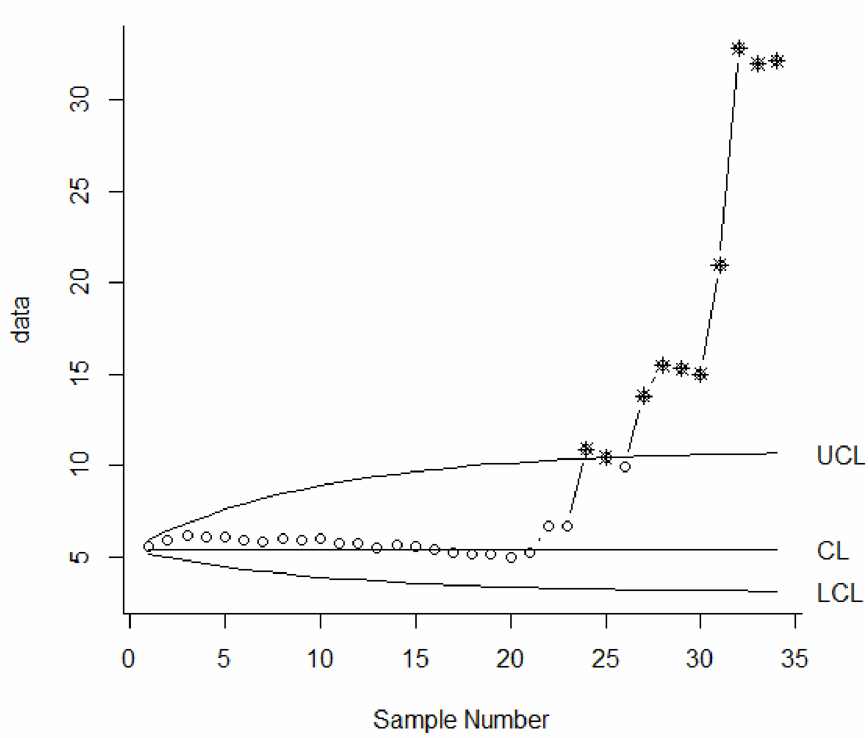

Figure 8 - 15: Individual EWMA control chart with scaled weighted variance for the Software Failures dataset assuming Logarithmic distribution for the data

8.10 Conclusions and Further Research

In this chapter probability-type, Shewhart-type and EWMA control charts have been constructed for monitoring individual observations from a process which is assumed to follow the Logarithmic distribution for the theoretical scenario of known distributions' parameters. Two different methods for taking into account the distribution's skewness have been considered. The performance of the proposed control charts has been investigated for the cases of all the proposed control charts (probability-type, Shewhart-type and EWMA control charts with both skewness correction methods). Optimal design for the EWMA control chart has also been presented. The five types of proposed control charts have been illustrated with both simulated and real data.

The proposed control charts take into account the skewness of the distribution and this leads to a significant improvement of their performance as has been demonstrated along this chapter. The performance of the control charts seems to improve more when the scaled weighted variance method by Castagliola (2000) is used instead of the skewness correction method proposed by Chan and Cui (2003).

This study can also be applied to other Logarithmic-related distributions (generalizations, mixtures, transformations, etc.). Moreover, for future research, the whole analysis can be extended to include supplementary runs rules for the detection of small shifts. For this purpose it would also be useful to construct CUSUM control charts for the Logarithmic distribution, as well.

CHAPTER 9

CONTROL CHARTS FOR INDIVIDUAL OBSERVATIONS FROM THE PARETO DISTRIBUTION

9.1 Introduction

As presented in Chapter 5, Pareto distribution is a continuous distribution with various applications some of which are in finance and actuarial sciences, reliability and engineering, life testing and survival analysis, ecology, meteorology, sociology, demography, agriculture, hydrology, geosciences, computer science and communications, computing and data transmission, medicine, biology, sociology, astronomy and astrophysics, and modelling of industrial accidents, injuries in road accidents, athletic events etc. Due to its variety of applications, some control charts for detecting shifts in a process have been constructed under the assumption that the quality characteristic of interest is Pareto distributed, as indicated in section 2.29.13. As it was presented, there, however, most of the control charts were concentrated on monitoring a (function of a) parameter of the distribution or were constructed for the Pareto II distribution. Here we present control charts for observations from the Pareto I distribution. More specifically, we construct probability-type, Shewhart-type and EWMA control charts (and deal with the optimal choice of its parameters) for individual observations from the Pareto I distribution using two different methods for taking into consideration the distribution's skewness for the construction of the Shewhart-type and EWMA charts, investigate their performance and illustrate them using examples with both simulated and real data (same for all charts for easy comparisons). The whole analysis reveals the superiority of using skewness correction for the construction of the control charts against not using it and the superiority of the scaled weighted variance method for taking into account the distribution's skewness during the construction of the proposed control charts. More specifically, the chapter is outlined as follows:

Sections 9.2 and 9.3 describe the construction of probability-type and Shewhart-type control charts (with the skewness correction method proposed by Chan and Cui (2003)), respectively, for individual observations from the Pareto distribution, with both control charts' performances investigated and compared with each other in section 9.4. Sections 9.5 and 9.6 present the construction and performance investigation, respectively, of the corresponding EWMA charts with the same skewness correction method as for the Shewhart-type charts. Section 9.7 addresses the optimal design of the EWMA chart considered in Section 9.5. Section 9.8 provides illustration of all the proposed charts of the previous sections through application to both simulated and real data. Section 9.9 discusses control charts for individual observations from the Pareto distribution using the scaled weighted variance method proposed by Castagliola (2000) for taking into account the distribution's skewness. More specifically, subsections 9.9.1 and 9.9.2 present the construction and performance investigation of Shewhart-type control charts, while subsections 9.9.3 and 9.9.4 address the construction and performance investigation of EWMA charts. Both charts are illustrated through application to simulated and real data (the same data of Section 9.8 for easy comparison). Both performance investigation and examples reveal the superiority of the scaled weighted variance method for taking into account the distribution's skewness.

9.2 Probability-Type Control Charts for Individual Observations from the Pareto Distribution

The control limits of the probability-type control charts for individual Pareto distributed observations will be obtained in terms of the probability of type I error or false alarm rate, α, for the Pareto distribution (as, for example, in Chang and Gan (1999) for the case of the modified geometric distribution). In order to do that we need to use the cumulative probability of the Pareto distribution as presented in equation (5-2). The construction procedure is as follows.

For a significance level α, we have

$$P\left(X < LCL\right) = \frac{\alpha}{2}$$

and

$$P(X < LCL) = 1 - \left(\frac{r}{LCL}\right)^d, \quad LCL \geq r, \quad r \geq d > 0,$$

from which we obtain

$$1 - \left(\frac{r}{LCL}\right)^d = \frac{\alpha}{2} \Rightarrow LCL = \frac{r}{\left(1 - \frac{\alpha}{2}\right)^{1/d}},$$

Similarly, for the upper control limit, we have

$$P(X > UCL) = \frac{\alpha}{2}$$

and

$$P(X > UCL) = 1 - P(X \leq UCL) = \left(\frac{r}{UCL}\right)^d, \quad UCL \geq r, \quad r \geq d > 0,$$

from which we get that

$$\left(\frac{r}{UCL}\right)^d = \frac{\alpha}{2} \Rightarrow UCL = \frac{r}{\left(\frac{\alpha}{2}\right)^{1/d}}$$

Similarly for the central line we obtain

$$CL = \frac{r}{(0.5)^{1/d}}$$

As a result from all the above, the control limits of the chart in terms of the probability of type I error, α, are as follows.

$$UCL_\alpha = \frac{r}{\left(\frac{\alpha}{2}\right)^{1/d}}$$

$$CL_\alpha = \frac{r}{(0.5)^{1/d}} \quad , \quad r \geq d > 0 \tag{9-1}$$

$$LCL_\alpha = \frac{r}{\left(1 - \frac{\alpha}{2}\right)^{1/d}}$$

9.3 Shewhart-Type Control Charts for Individual Pareto Observations

Now a different approach is considered for the construction of control charts for individual observations from the Pareto distribution, based on the Shewhart-type individual control charts using the skewness correction as in Chan and Cui (2003). More specifically, the construction will be as follows: the central line will be placed at the mean of the Pareto distribution, which is computed using equation (5-3), and the control limits will be placed around the mean at L times its standard deviation (the square root of the quantity computed by equation (5-4)) plus c_4^* times its standard deviation, where $c_4^*(x) = \frac{\frac{4}{3}\left[\mathrm{sk}(x)\right]}{1+0.2\left[\mathrm{sk}(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient computed from equation (5-5). This means that the skewness correction for the Pareto distribution will be

$$c_4^*(\bar{x}) = \frac{\frac{8}{3}\frac{d+1}{d-3}\sqrt{\frac{d-2}{d}}}{1+0.2\left[\frac{2(d+1)}{d-3}\sqrt{\frac{d-2}{d}}\right]^2} \Rightarrow c_4^*(\bar{x}) = \frac{8(d+1)(d-3)\sqrt{d(d-2)}}{3d(d-3)^2+2.4(d-2)(d+1)^2} \qquad (9\text{-}2)$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Pareto control chart are as follows.

$$\begin{aligned} UCL &= dr(d-1)^{-1} + \left[L + c_4^*(\bar{x})\right]\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}} \\ CL &= dr(d-1)^{-1} \qquad\qquad ,d>3 \qquad (9\text{-}3) \\ LCL &= dr(d-1)^{-1} + \left[-L + c_4^*(\bar{x})\right]\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}} \end{aligned}$$

9.4 Performance Investigation for the Individual Pareto Control Charts

In order to investigate the performance of the above proposed control charts we can use again the ARL_0 and ARL_1 values, with the following formulae:

$$ARL_0 = \frac{1}{1 - F_{in}\left(UCL\right) + F_{in}\left(LCL\right)} \qquad (9\text{-}4)$$

with $F_{in}\left(x\right)$ being the cumulative distribution function of the Pareto distribution from equation (5-2) with in-control parameters and control limits as computed from equation (9-1) for the probability-type control charts or equations (9-3) and (9-2) for the Shewhart-type control charts and

$$ARL_1 = \frac{1}{1 - F_{out}\left(UCL\right) + F_{out}\left(LCL\right)} \qquad (9\text{-}5)$$

with $F_{out}\left(x\right)$ being the cumulative distribution function for our distribution with out-of-control parameters and same control limits as before. For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (5-3) and (5-4) in terms of the distribution's two parameters. The resulting values for them are given by $d_{new} = 1 + \frac{\sqrt{\sigma^2 + \left(\mu_0 + k\sigma\right)^2}}{\sigma}$ and $r_{new} = \left(\mu_0 + k\sigma\right)\frac{\sqrt{\sigma^2 + \left(\mu_0 + k\sigma\right)^2}}{\sigma + \sqrt{\sigma^2 + \left(\mu_0 + k\sigma\right)^2}}$.

Using the above formulas we obtain Table 9-1 and Table 9-2, which show the in-control and out-of-control ARL values for the individual probability-type and individual Shewhart-type control chart, respectively, for the Pareto distribution for various values of the two parameters d and r of the specific distribution and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the probability-type control charts we have chosen a significance level equal to the most commonly used value of 0.27%, which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

Comparing Tables 9-1 and 9-2 we observe that the performance of the chart improves significantly when using the skewness corrected limits instead of the probability based ones. The difference in ARL values between Shewhart-type and probability-type control charts is greater than 5% for all shift sizes except k=±0.2 where it is slightly smaller than 5%. Comparison of the ARL values for positive and negative shifts shows that, although the control charts can detect both positive and negative shifts well, there are some slight differences with most values being a little smaller for the negative shifts than for the corresponding positive ones. This holds for either the probability-type or the Shewhart-type control chart. The only differences that are above 5% concern shift sizes of k between 2.4 and 2.8 for the Shewhart-type control chart, while for the probability-type one, they concern shift sizes of k between 2.2 and 2.6 for large values of the distribution's parameters and k between 1.6 and 2 for small values of the distribution's parameters.

k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=112	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
-3	3.5488	3.5502	3.5507	3.5512	3.5514	3.5516	3.5518	3.5520
-2.8	6.0596	6.0617	6.0625	6.0632	6.0634	6.0636	6.0640	6.0642
-2.6	9.0733	9.0761	9.0772	9.0781	9.0785	9.0787	9.0792	9.0795
-2.4	10.0907	10.0943	10.0957	10.0970	10.0973	10.0978	10.0986	10.0988
-2.2	12.1227	12.1275	12.1293	12.1210	12.1216	12.1220	12.1228	12.1233
-2	14.1412	14.1472	14.1496	14.1518	14.1525	14.1532	14.1543	14.1548
-1.8	16.1782	16.1861	16.1891	16.1919	16.1928	16.1937	16.1951	16.1957
-1.6	20.2273	20.2375	20.2414	20.2450	20.2463	20.2473	20.2493	20.2501
-1.4	36.2842	36.3072	36.3122	36.3169	36.3186	36.3199	36.3224	36.3235
-1.2	60.3978	60.4046	60.4122	60.4173	60.4195	60.4212	60.4244	60.4259
-1	75.5244	75.5468	75.5553	75.5735	75.5464	75.5587	75.5728	75.5748
-0.8	105.0368	105.0571	105.0789	105.0900	105.0930	105.0972	105.1040	105.1057
-0.6	154.1010	154.1444	154.1614	154.1775	154.1831	154.1878	154.1961	154.2000
-0.4	177.8437	177.9140	177.9301	177.9557	177.9750	177.9824	177.9957	178.0020
-0.2	235.0884	235.2328	235.2895	235.3433	235.3625	235.3782	235.4062	235.4193
0	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704	370.3704
0.2	235.1834	233.4402	231.0369	230.8157	229.6684	228.7348	227.0694	226.2891
0.4	181.8882	179.5125	177.9693	175.6481	173.8848	172.3789	169.7082	168.4545
0.6	154.6412	152.9515	150.1855	148.8450	146.6015	144.7815	141.5476	140.0379
0.8	108.5382	107.1980	105.8678	103.0302	100.6184	98.6646	95.1995	93.6846
1	78.8640	77.0837	75.6477	72.7324	70.3128	68.3493	64.8710	63.2528
1.2	57.1601	55.7700	53.5376	51.8500	48.5045	46.6098	44.2609	42.7053
1.4	43.1498	41.7001	39.8699	37.5798	35.3798	33.6048	30.4825	28.9197
1.6	32.7261	30.5575	28.2469	27.4579	25.4484	23.8109	20.9435	19.6154
1.8	25.9308	23.2296	21.4978	19.2593	17.4362	15.9688	14.0871	12.8931
2	19.9364	18.7779	17.6431	16.9642	15.3428	14.0375	12.8439	10.6844
2.2	14.0288	12.4103	10.8634	10.7353	9.3052	9.1526	9.1403	8.2105
2.4	10.5975	10.4632	9.4840	8.8646	7.6168	6.6157	5.8612	5.0527
2.6	9.3732	9.1230	8.9034	7.7733	6.6934	5.8278	5.3126	4.6152
2.8	6.4845	6.4804	6.3254	6.1797	5.2517	4.5086	4.4082	4.0107
3	3.9046	3.8688	3.4682	3.4097	3.3793	3.3708	3.3022	3.2621

Table 9 - 1: ARL values for individual probability-type control charts for the Pareto distribution, with $\alpha = 0.0027$.

k	d=25 r=37	d=42 r=68	d=57 r=93	d=86 r=112	d=105 r=154	d=128 r=185	d=210 r=250	d=300 r=310
-3	2.5486	2.5501	2.5507	2.5512	2.5514	2.5516	2.5518	2.5520
-2.8	4.0595	4.0616	4.0624	4.0632	4.0634	4.0636	4.0640	4.0642
-2.6	5.0632	5.0761	5.0771	5.0781	5.0785	5.0787	5.0793	5.0795
-2.4	6.0806	6.0843	6.0857	6.0870	6.0874	6.0878	6.0884	6.0888
-2.2	8.0936	8.0974	8.0993	8.1009	8.1015	8.1020	8.1028	8.1035
-2	10.1410	10.1482	10.1495	10.1517	10.1525	10.1532	10.1543	10.1548
-1.8	12.1780	12.1860	12.1890	12.1918	12.1928	12.1937	12.1951	12.1957
-1.6	19.2272	19.2375	19.2414	19.2448	19.2462	19.2482	19.2493	19.2501
-1.4	30.2840	30.3072	30.3121	30.3169	30.3184	30.3199	30.3223	30.3235
-1.2	40.3975	40.4045	40.4120	40.4172	40.4193	40.4212	40.4244	40.4259
-1	63.5241	63.5364	63.5452	63.5534	63.5553	63.5587	63.5728	63.5748
-0.8	92.7364	92.7669	92.7787	92.7899	92.7939	92.7971	92.8030	92.8054
-0.6	114.1005	114.1441	114.1612	114.1772	114.1820	114.1877	114.1960	114.2000
-0.4	165.8430	165.9126	165.9398	165.9646	165.9748	165.9824	165.9957	166.0020
-0.2	225.0869	225.2419	225.2888	225.3428	225.3621	225.3779	225.4060	225.4193
0	370.7502	370.7503	370.7504	370.7546	370.7784	370.8261	370.8445	370.8482
0.2	230.9932	232.2808	230.6861	229.2268	228.3742	227.6487	226.4033	225.8172
0.4	180.4442	177.5910	174.5730	171.7534	170.7579	169.9348	168.5048	167.8451
0.6	130.0618	122.3279	119.4069	116.6810	115.7315	114.9369	113.5787	112.9501
0.8	108.3741	101.3882	98.6824	96.1084	95.2230	94.7937	94.2214	93.6284
1	75.5399	69.4464	67.2007	65.1260	64.3950	63.7987	62.7845	62.2822
1.2	52.1833	47.4084	45.6419	44.0375	43.4682	43.0215	42.2251	41.8646
1.4	35.8427	32.2425	30.9294	29.7284	29.3133	28.9752	28.3751	28.1234
1.6	24.4873	21.8497	20.8987	20.0325	19.7339	19.4910	19.0684	18.8731
1.8	15.0754	14.3675	12.7549	12.1995	12.0084	10.8436	10.5732	10.4599
2	12.3872	10.9654	10.4548	9.9964	9.8484	9.7101	9.4873	9.3953
2.2	10.1648	8.9871	8.5571	8.1876	8.0573	7.9518	7.7886	7.6842
2.4	6.1631	5.4444	5.1901	4.9612	4.8828	4.8193	4.7096	4.6480
2.6	5.0332	4.4487	4.2424	4.0571	3.9937	3.9526	3.8439	3.8130
2.8	4.1048	3.6319	3.4657	3.3164	3.2645	3.2243	3.1530	3.1203
3	2.9542	2.6207	2.5041	2.3998	2.3643	2.3357	2.2861	2.2633

Table 9 - 2: ARL values for individual Shewhart-type control charts for the Pareto distribution

9.5 Construction of the EWMA Control Charts for Individual Observations from the Pareto Distribution

When dealing with individual observations, besides Shewhart-type control charts we also construct EWMA charts, which (as mentioned in Section 2.14.2) are a better alternative in that case. So it is useful to construct EWMA control charts for individual observations from the Pareto distribution. For that purpose, we need to remember the general instructions for constructing an EWMA chart as summarized in equation (2-3) and the plotting statistic in equation (2-2), with the value of the constant λ being the weight assigned to each of the past values and chosen to be smaller when we are interested in detecting smaller shifts.

The construction of the individual Pareto control charts is going to be done based on the EWMA control charts (2-3) using the skewness correction as in Chan and Cui (2003), since the distribution of concern is asymmetric and, as also mentioned in Weiß and Atzmüller (2011), this is an easily applied method for taking the distribution's skewness into consideration and leads to a better ARL performance of the resulting control chart. In the next section, where we deal with the performance investigation of the constructed control chart, we will further demonstrate the need for this adjustment considering the asymmetry of the distribution and the improvement in the performance of the chart when using the skewness correction contrary to not using it but using the traditionally used symmetric EWMA control limits instead.

The method for constructing this control chart is the following: in equation (2-3) we replace L by L plus c_4^*, where $c_4^*(x)=\dfrac{\frac{4}{3}\left[\mathrm{sk}(x)\right]}{1+0.2\left[\mathrm{sk}(x)\right]^2}$ is the skewness correction and sk(X) is the distribution's skewness coefficient. EWMA control charts for individual observations from the Pareto distribution are constructed using the mean of the Pareto distribution, which is computed using equation (5-3), its standard deviation (the square root of the quantity computed by equation (5-4)) and the distribution's skewness coefficient computed from equation (5-5). This means that the skewness correction for the mean of the Pareto distribution is

$$c_4^*(x)=\frac{8(d+1)(d-3)\sqrt{d(d-2)}}{3d(d-3)^2+2.4(d-2)(d+1)^2} \tag{9-6}$$

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Pareto EWMA control chart are as follows.

$$UCL = dr(d-1)^{-1}+\left[L+c_4^*(\bar{x})\right]\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = dr(d-1)^{-1} \qquad ,d>3 \tag{9-7}$$

$$LCL = dr(d-1)^{-1}+\left[-L+c_4^*(\bar{x})\right]\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

The plotting statistic will be the one in equation (2-2) with x_i being the observations from the Pareto distribution.

9.6 Performance Investigation for the EWMA Control Charts for Individual Observations from the Pareto Distribution

For the investigation of the performance of the proposed control chart above, we will use the ARL, based on the method by Lucas and Saccucchi (1990). In other words, the ARL of the EWMA control chart will be computed with the Markov chain method and discretization of the control statistic. More specifically, according to this method, the region between the upper and lower control limits is divided into $2m+1$ subintervals. Each subinterval S_j ($j=1,2,\ldots,2m+1$) is taken to be represented by its midpoint s_j and then if δ is the half size of each subinterval, which means that $\delta = \dfrac{UCL-LCL}{2(2m+1)}$, then whenever $s_j - \delta < Z_i < s_j + \delta$ the process is in a transient state. Otherwise, the process is in the absorbing state. Therefore, the in-control transition probability from one transient state S_j to another transient state S_k is given by

$$\begin{aligned} p_{kj} &= P\left(Z_i \in S_k \,\middle|\, Z_{i-1} \in S_j\right) \\ &= P\left(s_k - \delta < Z_i < s_k + \delta \,\middle|\, Z_{i-1} = s_j\right) \\ &= P\left(s_k - \delta < \lambda X_i + (1-\lambda) Z_{i-1} < s_k + \delta \,\middle|\, Z_{i-1} = s_j\right) \\ &= P\left(\frac{s_k - \delta - (1-\lambda)s_j}{\lambda} < X_i < \frac{s_k + \delta - (1-\lambda)s_j}{\lambda}\right), \quad j,k = 1,2,\ldots,2m+1 \end{aligned} \tag{9-8}$$

The ith-stage transition probability matrix $\mathbf{P}^i$ is, then, defined as $\mathbf{P}^i = \begin{pmatrix} \mathbf{R}^i & (\mathbf{I}-\mathbf{R}^i)\mathbf{1} \\ \mathbf{0}^T & 1 \end{pmatrix}$, where $\mathbf{R}$ is the $(2m+1,\ 2m+1)$ matrix of the transient probabilities p_{kj} mentioned in (9-8) above and $\mathbf{0}^T=(0,0,\ldots,0)$, i.e. $\mathbf{0}^T$ is the transpose of $\mathbf{0}$ which is a vector of $2m+1$ zeros. The ith-stage transition probability matrix $\mathbf{P}^i$ contains the probabilities that the control statistic goes from one transient state to another in i steps and is used for the computation of the ARL of the EWMA control chart, which is given by

$$ARL = \mathbf{p}^T \left(\mathbf{I}-\mathbf{R}\right)^{-1} \mathbf{1} \tag{9-9}$$

where $\mathbf{p} = \left(p_{-m}, p_{-m+1}, \ldots p_{m-1}, p_m\right)^T$ is the vector of the initial probabilities related to the $2m+1$ transient states.

For the transient probabilities in (9-8) the cumulative distribution function for the Pareto distribution, i.e. equation (5-2), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equations (9-7) and (9-6) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = dr\left(d-1\right)^{-1} + \left[L + c_4^*\left(\bar{x}\right)\right]\sqrt{dr^2\left(d-1\right)^{-2}\left(d-2\right)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL = dr\left(d-1\right)^{-1} \qquad ,d>3 \qquad (9\text{-}10)$$

$$LCL = dr\left(d-1\right)^{-1} + \left[-L + c_4^*\left(\bar{x}\right)\right]\sqrt{dr^2\left(d-1\right)^{-2}\left(d-2\right)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}}$$

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (5-3) and (5-4) in terms of its two parameters, as for the Shewhart-type control chart.

Using those formulae we get Tables 9-3, 9-4 and 9-5, which show the in-control and out-of-control ARL values for the individual EWMA control chart for the Pareto distribution for various values of the two parameters d and *r* of the distribution of concern and for various values of *k* which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 9-3 contains the ARL values for λ=0.3 and L=4.2802 (combination which gives in-control ARL value close to 370) for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ and L the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=150, above which ARL increases very slightly. Consequently, the suggested value of m for the computation of ARL in the formulae above is m=150. Therefore, Tables 9-4 and

9-5 show the ARL values for m=150 for various values of L and λ for positive and negative shifts, respectively.

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some differences in ARL values between those two tables, with most of the differences being in favour of the ARL values for negative shifts. The only cases for which the ARL values for negative shifts are bigger are for small values of lambda (up to 0.10). For lamda values larger than 0.10 the ARL values for positive shifts are higher than the ones for negative shifts.

The need for using the skewness correction for the construction of the individual EWMA control charts for the Pareto distribution is justified by the fact that if we had used the traditional symmetric EWMA control limits without the skewness correction term $c_4^*(x)$ in equation (18) above, the ARL performance of the chart would have been worse, as can be seen when comparing the results in Table 9-6 for the case of not using the skewness correction term against the results in Table 9-4 for the case of using it. It should be noted that the ARL values in Table 9-6 have resulted from using the same values for λ and L as the ones in Table 9-4 for the shake of making comparisons between the two tables easier. The difference between the ARL values in Tables 9-4 and 9-6 are all higher than 10%. Comparison is similar for the case of negative shifts so the corresponding table is omitted for space reasons.

Additionally, comparing the ARL values for the EWMA in Tables 9-4 and 9-5 with the ARL values for the Shewhart-type control chart in Table 9-2, we can see that the EWMA control chart performs better than the Shewhart-type control chart for almost all shifts, since the EWMA out-of-control ARL values are smaller than the corresponding ARL values for the Shewhart-type charts. When it comes to large shifts of magnitude 3 standard deviation units, however, EWMA ARL values are slightly larger only for positive.

m	k	d=25 r=37	d=42 r=68	d=57 r=93	d=86 r=112	d=105 r=154	d=128 r=184	d=210 r=250	d=300 r=310
10	0	370.3249	370.1734	370.1244	370.4785	370.4629	370.4507	370.4287	370.4203
	0.2	60.2597	60.6483	60.6136	60.5837	60.5736	60.5657	60.5520	60.5458
	0.5	14.1478	15.8559	18.8582	22.3207	27.8594	31.5967	34.5804	37.6919
	1	6.4380	6.4179	6.4121	6.4050	6.403	6.4012	6.3984	6.3971
	1.5	4.2496	4.2422	4.2412	4.2409	4.241	4.2412	4.2416	4.2418
	2	4.1553	4.1418	4.1279	4.1248	4.1226	4.1241	4.1218	4.1212
	2.5	2.9261	2.9319	2.935	2.9384	2.9397	2.8408	2.9428	2.9438
	3	2.8390	2.8473	2.8412	2.8454	2.8482	2.8486	2.8612	2.8622
20	0	372.8154	373.5748	373.4971	373.4315	373.4098	373.3928	373.3637	373.3506
	0.2	61.6806	61.9365	61.8849	61.8422	62.2148	62.1985	62.1709	62.1586
	0.5	14.1500	15.8597	18.8891	22.4071	28.4054	31.9773	35.1028	38.3823
	1	7.4449	7.5837	7.9093	7.8825	7.8739	7.8683	7.8562	7.8412
	1.5	5.1283	5.3847	5.5755	5.541	5.5466	5.5432	5.5375	5.535
	2	4.9619	5.0777	5.0618	5.0485	5.0441	5.0408	5.0348	5.0322
	2.5	3.5227	3.6714	3.6602	3.6404	3.6472	3.6446	3.6402	3.6382
	3	3.1273	3.235	3.2336	3.1593	3.0408	3.0057	3.0912	3.0451
50	0	376.4743	375.8694	375.6494	376.7775	376.6843	376.6121	375.4807	376.4369
	0.2	63.1951	62.8206	62.6296	63.6885	64.6048	63.5394	63.429	63.3798
	0.5	14.1521	15.8684	18.9584	25.4187	28.4619	32.3134	36.3387	40.6634
	1	9.5690	9.1343	10.2641	10.0939	10.038	9.9954	9.9204	9.8873
	1.5	7.2930	6.9997	6.8862	7.4706	7.4219	7.2828	7.3724	7.343
	2	5.5708	6.9178	6.8014	6.7862	6.7528	6.7264	7.3187	7.2895
	2.5	4.4104	4.553	4.4884	4.1814	4.4128	4.3962	4.3691	4.3546
	3	3.3133	3.2875	3.2752	3.3543	3.2287	3.2412	3.218	3.079
80	0	379.8438	380.0651	379.468	380.8988	380.1476	380.9664	380.645	380.5326
	0.2	66.473	67.0368	66.4626	67.5274	68.6968	67.5233	66.3782	67.2609
	0.5	14.1548	15.8736	18.9994	25.5808	28.72	32.867	37.1145	43.0096
	1	12.9936	12.5612	12.3591	12.2791	12.1491	12.0486	12.8754	12.7984
	1.5	8.7563	10.4123	8.0539	10.1268	9.9829	10.8791	10.7046	10.6271
	2	8.0250	8.1923	7.9957	7.8276	7.2648	8.2122	8.1201	8.0785
	2.5	4.7750	4.8275	4.8463	4.4212	4.6804	4.6310	4.7766	4.5746
	3	3.3574	3.3015	3.3218	3.4881	3.2848	3.2871	3.2409	3.2128
120	0	382.5175	382.3686	384.1016	383.1883	382.8531	383.5751	384.8227	384.568
	0.2	68.7734	69.3108	70.2784	70.0997	70.7898	69.5977	71.0907	70.8589
	0.5	14.1559	15.8804	22.6024	25.608	29.3161	33.418	37.3591	43.6922
	1	15.4494	15.3053	14.6463	16.2296	15.9785	15.7845	15.4643	15.3231
	1.5	12.6085	12.5006	10.9642	12.9444	12.7502	12.5996	12.3462	12.2437
	2	8.8927	8.8310	8.6804	8.9639	8.4206	8.8081	8.6908	8.6379
	2.5	5.0100	5.6918	5.5459	4.6848	5.2460	5.5419	5.5807	4.7591
	3	3.4573	3.395	3.5591	3.6912	3.4284	3.3246	3.2737	3.2645
150	0	384.2239	386.5796	385.1288	387.373	385.8407	386.4388	386.7839	386.0365
	0.2	70.5153	71.5145	72.6398	73.1801	72.3037	72.9716	72.3399	72.0671
	0.5	14.1577	15.9015	22.7188	25.8141	29.6804	34.1542	39.5554	43.8253
	1	16.9408	18.6601	17.5451	19.7698	18.6539	20.3257	18.7867	18.5526
	1.5	12.2134	14.2683	12.5088	14.4206	14.1628	14.9637	14.631	14.4841
	2	9.0667	8.8644	8.822	10.0023	8.6487	9.0408	8.7816	9.1538
	2.5	5.5612	5.6926	5.5704	5.6486	5.4084	5.5755	5.6012	5.4612
	3	3.5090	3.5400	3.7306	3.8208	3.482	3.4801	3.5093	3.4282
200	0	387.447	388.2507	389.6179	389.1504	388.8049	389.4549	389.103	388.5301
	0.2	73.1508	74.9996	75.4294	74.8515	75.7082	76.5914	75.5420	74.1564
	0.5	14.1597	19.214	23.3741	26.4671	32.882	36.3196	40.7514	44.7805
	1	22.5858	22.7572	23.0163	24.3722	24.7106	24.4037	21.8757	25.246
	1.5	16.2569	16.0478	17.6469	17.9284	14.7109	16.8002	17.8221	16.2505
	2	12.3286	9.7895	9.6321	10.8232	9.2857	9.7366	9.4044	10.464
	2.5	6.1657	6.1475	5.9915	7.6160	5.9785	5.9303	5.9601	6.0182
	3	3.8975	3.8469	3.9399	3.9309	3.8899	3.8759	3.8542	3.9593
240	0	388.4575	391.5186	391.5157	390.2163	389.5464	390.7034	390.2602	390.9128
	0.2	73.1754	75.4085	75.5103	74.9469	77.1557	77.7566	76.2458	75.7910
	0.5	14.1608	19.4126	23.9368	27.4073	34.6230	38.1640	41.0873	47.4505
	1	23.8308	23.8382	24.3542	24.9451	24.9974	24.6088	22.0255	25.3442
	1.5	16.9938	17.6424	18.5751	18.2018	15.5393	17.3372	18.5917	17.4301
	2	12.4228	10.0785	9.846	10.844	9.7027	10.2025	10.7507	10.6497
	2.5	6.3920	6.3641	6.2022	8.0253	6.2669	6.2242	6.1504	6.2414
	3	3.9073	3.9527	3.9826	3.9822	3.9737	3.9182	3.9805	3.9689

Table 9 - 3: ARL values for individual EWMA control charts for the Pareto distribution (λ=0.3 and L=4.2802)

λ, L	k	d=25 r=37	d=42 r=68	d=57 r=93	d=86 r=112	d=105 r=154	d=128 r=184	d=210 r=250	d=300 r=310
λ=0.05 L=2.0355	0	371.0686	372.2257	372.3168	372.5054	372.1251	371.8435	372.5403	372.3044
	0.2	60.9335	61.3753	61.4515	63.3798	63.0688	62.8168	63.4250	63.2101
	0.4	15.1390	15.5543	15.6407	15.8120	15.5393	15.3280	15.8912	15.7070
	0.6	12.4804	12.0728	12.0710	12.1016	12.8840	12.7145	12.0243	12.8800
	0.8	8.7751	8.9702	9.0216	9.1079	8.9888	8.8953	9.1535	9.0730
	1	6.3109	6.3460	6.2750	6.2272	6.1390	6.0693	6.1500	6.0930
	1.5	4.4522	4.3461	4.3275	4.3170	4.2848	4.3400	4.2848	4.2750
	2	4.0555	4.0040	3.9982	4.0016	3.9861	3.9737	4.0017	3.9912
	2.5	3.2793	3.2668	3.2390	3.2231	3.2245	3.2171	3.2041	3.1980
	3	2.8437	2.7061	2.6939	2.6812	2.8757	2.6812	2.6682	2.6645
λ=0.08 L=2.2624	0	370.4842	370.7579	372.7135	372.5057	372.0361	372.8279	372.1437	371.8464
	0.2	63.1757	63.0248	63.5782	63.3593	62.9842	63.7219	63.1709	63.8057
	0.4	15.0033	15.4289	15.3570	16.0682	15.7553	15.5143	15.7799	15.5730
	0.6	12.5791	12.1462	12.9577	12.8035	12.1628	12.9519	12.1910	12.0084
	0.8	8.4028	8.1820	8.0226	8.2868	8.1248	8.0084	8.1828	8.0777
	1	8.1239	8.0370	9.1668	9.0843	8.9715	9.1842	9.0213	8.9578
	1.5	5.4807	5.4145	5.3914	5.3604	5.4130	5.3968	5.4024	5.3793
	2	4.0890	4.0512	4.0271	4.0234	4.0325	4.0191	3.9957	4.0305
	2.5	3.0341	3.0231	3.0321	3.0187	3.0109	3.0289	3.0182	3.0128
	3	2.6488	2.6260	2.6218	2.6099	2.6179	2.6135	2.6079	2.6042
λ=0.10 L=2.5995	0	370.9905	373.8230	372.2101	375.0144	373.4530	373.0275	372.3300	372.0275
	0.2	60.8019	62.9125	61.5730	63.1695	62.6962	62.3361	61.7534	61.4844
	0.4	15.4884	17.0339	15.9055	15.6489	16.8413	16.5570	16.0704	15.8464
	0.6	12.1021	12.5522	12.5575	14.0202	12.3032	14.0484	12.5957	12.4018
	0.8	8.7157	9.5357	8.9073	10.0518	10.8010	10.6066	10.2796	10.1344
	1	7.7246	8.4026	7.9357	8.3457	8.1932	8.0739	7.8706	7.7795
	1.5	5.8480	5.7060	6.5024	6.3605	6.3123	6.2739	6.4395	6.4048
	2	4.1690	4.1751	4.1215	4.1446	4.1270	4.1228	4.0880	4.0786
	2.5	3.3484	3.3040	3.3107	3.2842	3.2736	3.2759	3.2860	3.2893
	3	2.8245	2.8279	2.8082	2.8148	2.8082	2.8028	2.7935	2.7891
λ=0.12 L=3.1037	0	371.6042	372.6375	372.5218	373.9336	373.5260	373.2069	373.9198	373.7154
	0.2	62.1015	62.1257	62.7357	63.6827	63.2840	64.0288	63.4697	64.6481
	0.4	14.9310	15.0096	15.0516	15.7251	15.6861	15.4044	15.4879	16.2260
	0.6	14.6826	12.7936	14.0540	14.0270	14.3968	14.1548	12.9396	14.2786
	0.8	10.2007	10.2239	10.1687	10.0687	9.8436	10.3935	10.0606	9.9131
	1	9.0259	9.1227	9.1840	9.1939	9.4860	9.3253	9.5140	9.3984
	1.5	6.2060	6.1714	6.1528	6.1412	6.0781	6.0280	6.0788	6.0373
	2	4.9324	4.8880	4.8427	4.8259	4.8122	4.8451	4.8093	4.8235
	2.5	3.6912	3.6884	3.6875	3.6880	3.6848	3.6468	3.6431	3.6450
	3	2.9771	2.9500	2.9357	2.9315	2.9348	2.9360	2.9164	2.9109
λ=0.15 L=3.2512	0	371.5590	372.4697	373.7539	373.0068	373.5302	373.1688	373.1214	373.8128
	0.2	63.4884	63.6420	64.7217	64.9357	64.4886	64.1639	64.1053	64.8208
	0.4	16.0108	16.2301	16.2823	16.3968	17.4687	17.1434	16.6075	17.6282
	0.6	12.8993	12.9775	15.0414	15.3425	14.9752	16.0971	14.5148	14.2663
	0.8	12.3548	12.9324	14.1602	14.3154	14.0050	12.7573	14.3984	14.1808
	1	9.8242	9.4884	9.5245	9.7506	9.5410	9.3784	9.6816	9.5439
	1.5	6.6378	6.4809	6.5517	6.5460	6.4805	6.6378	6.5454	6.4840
	2	5.0978	5.0918	5.0053	5.0189	5.0464	5.0332	4.9828	4.9734
	2.5	3.8060	3.8090	3.7939	3.7554	3.7720	3.7510	3.7516	3.7578
	3	3.0930	3.0457	3.0455	3.0332	3.0255	3.0195	3.0214	3.0163
λ=0.20 L=3.9786	0	372.4402	371.0686	372.7930	372.8423	372.7573	372.3402	371.6871	372.8643
	0.2	64.3964	63.4610	62.8841	64.4840	64.3953	64.0289	64.4451	64.3914
	0.4	14.6188	15.3008	15.3572	14.9712	17.0968	17.6863	17.1286	17.1906
	0.6	14.3508	14.2048	14.3648	14.4060	12.6848	15.8004	14.2889	14.8708
	0.8	10.4488	12.0810	10.3487	10.6133	10.2395	10.4646	10.0982	12.0703
	1	8.8404	10.4643	10.2506	10.5789	10.3772	10.0348	9.6933	9.9371
	1.5	6.5982	7.4577	7.0014	6.6306	6.5090	8.6027	8.3322	8.2217
	2	6.4334	6.7846	6.6148	6.4663	6.4164	6.3759	7.0030	6.9625
	2.5	4.5289	4.5990	4.5324	4.4645	4.4543	4.4371	4.6484	4.6331
	3	3.3575	3.3753	3.3457	3.3188	3.3093	3.3019	3.2886	3.2793

Table 9 - 4: ARL values for individual EWMA control charts for the Pareto distribution (m=150) for various positive shifts

λ, L	k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=112	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
λ=0.05 L=2.0355	0	371.0686	372.2257	372.3168	372.5054	372.1251	371.8435	372.5403	372.3044
	-0.2	61.5107	61.9370	62.0895	61.6233	61.9641	61.8428	61.6091	62.0128
	-0.4	14.8954	16.0704	16.0378	15.7715	15.9339	15.8697	15.7323	15.9368
	-0.6	12.2709	12.3648	12.4021	12.2572	12.4091	12.3684	12.2881	12.3968
	-0.8	10.1777	10.8150	10.7934	10.6930	10.7371	10.7212	10.6845	10.7312
	-1	9.5712	9.5781	9.5981	9.5364	9.5984	9.5712	9.5516	9.6125
	-1.5	6.2316	6.2052	6.1993	6.1881	6.1781	6.1702	6.1757	6.1693
	-2	5.4875	5.4457	5.4284	5.4157	5.4148	5.4101	5.4018	5.4036
	-2.5	3.9936	3.9621	3.9371	3.9373	3.9341	3.9309	3.9371	3.9345
	-3	2.3643	2.5010	2.4844	2.4871	2.4844	2.4846	2.4805	2.4899
λ=0.08 L=2.2624	0	370.4842	370.7579	372.7135	372.5057	372.0361	372.8279	372.1437	371.8464
	-0.2	60.5452	60.5703	60.4481	61.6039	61.4844	61.3726	61.5000	61.4173
	-0.4	14.0640	14.0353	14.1014	14.0528	14.1254	14.0918	14.8048	14.7548
	-0.6	12.1721	12.1288	12.0889	12.1054	12.0750	12.1037	12.0626	12.0989
	-0.8	9.5228	9.4684	9.4575	9.4318	9.4399	9.4253	9.4289	9.4173
	-1	8.7320	8.7048	8.7090	8.6870	8.6912	8.6815	8.6896	8.6819
	-1.5	5.5932	5.5442	5.5484	5.5436	5.5375	5.5328	5.5353	5.5316
	-2	4.8443	4.8448	4.8286	4.8222	4.8235	4.8212	4.8187	4.8170
	-2.5	3.4072	3.3900	3.3732	3.3793	3.3775	3.3757	3.3733	3.3734
	-3	2.0844	2.1757	2.1703	2.1648	2.1631	2.1617	2.1593	2.1571
λ=0.10 L=2.5995	0	370.9905	373.8230	372.2101	375.0144	373.4530	373.0275	372.3300	372.0275
	-0.2	59.9330	59.9606	59.6448	60.6224	60.5214	60.4419	60.7532	60.6864
	-0.4	12.3055	12.8444	12.9026	12.9686	12.9315	12.8844	12.8205	12.7916
	-0.6	10.8912	10.8481	10.8700	10.7968	10.7723	10.8423	10.8079	10.7933
	-0.8	9.1800	9.0804	9.0789	9.0842	9.0708	9.0593	9.0399	9.0812
	-1	8.3489	8.3228	8.3212	8.2840	8.3154	8.3084	8.2848	8.2801
	-1.5	5.1806	5.1593	5.3142	5.3005	5.3068	5.3031	5.2869	5.2841
	-2	4.5784	4.5712	4.5480	4.5442	4.5422	4.5407	4.5571	4.5550
	-2.5	3.2102	3.1954	3.1915	3.1882	3.1869	3.1848	3.1840	3.1842
	-3	2.0233	2.1028	2.0964	2.0912	2.0893	2.0878	2.0843	2.0845
λ=0.12 L=3.1037	0	371.6042	372.6375	372.5218	373.9336	373.5260	373.2069	373.9198	373.7154
	-0.2	59.2282	59.1900	59.2191	59.2701	59.1961	59.2698	59.9317	59.8888
	-0.4	12.8105	12.2593	12.2537	12.2724	12.2361	12.2559	12.2060	12.2402
	-0.6	10.2712	10.2330	10.2328	10.2448	10.2264	10.2373	10.2121	10.2322
	-0.8	8.6187	8.5737	8.5777	8.5488	8.8487	8.8281	8.8284	8.8420
	-1	8.1500	8.1284	8.1068	8.0902	8.0870	8.0809	8.0816	8.0848
	-1.5	4.8986	5.0095	5.0037	4.9982	4.9953	4.9969	4.9952	4.9934
	-2	4.3687	4.3548	4.4484	4.4442	4.4424	4.4412	4.4397	4.4377
	-2.5	3.1223	3.1273	3.1228	3.1970	3.1952	3.1937	3.1912	3.1901
	-3	2.0355	2.0178	2.0122	2.0075	2.0060	2.0048	2.0027	2.0017
λ=0.15 L=3.2512	0	371.5590	372.4697	373.7539	373.0068	373.5302	373.1688	373.1214	373.8128
	-0.2	57.5799	57.6033	57.5140	57.5716	57.5150	57.5486	57.4815	57.5721
	-0.4	12.2527	12.5543	12.5537	12.5271	12.5489	12.5271	12.5317	12.5154
	-0.6	9.6821	9.6446	9.6393	9.6309	9.6350	9.6250	9.6303	9.6225
	-0.8	8.3530	8.3044	8.3012	8.2844	8.2875	8.2812	8.2842	8.2884
	-1	7.6030	7.5716	7.5484	7.7193	7.7128	7.7127	7.7064	7.7093
	-1.5	4.7141	4.6984	4.6934	4.6895	4.6884	4.6871	4.6848	4.6848
	-2	4.2488	4.2359	4.3164	4.3128	4.3105	4.3091	4.3073	4.3063
	-2.5	3.1253	3.0973	3.0916	3.0869	3.0843	3.0841	3.0821	3.0812
	-3	1.6215	1.6214	1.6216	1.6219	1.6220	1.6221	1.6224	1.6225
λ=0.20 L=3.9786	0	372.4402	371.0686	372.7930	372.8423	372.7373	372.3402	371.6871	372.8643
	-0.2	57.3446	57.2334	57.3612	57.2548	57.2222	57.4255	57.3750	57.3537
	-0.4	12.5448	12.5034	12.5536	12.5072	12.4840	12.4800	12.5543	12.5468
	-0.6	9.0842	9.0350	9.0481	9.0251	9.0172	9.0122	9.0442	9.0393
	-0.8	7.7105	7.6826	7.6845	7.6825	7.6484	7.6441	7.6487	7.6844
	-1	7.1817	7.1612	7.1517	7.1552	7.1528	7.1509	7.1488	7.1464
	-1.5	4.4375	4.4245	4.4219	4.4184	4.4172	4.4163	4.4842	4.4843
	-2	4.1289	4.1263	4.1225	4.1090	4.1079	4.1075	4.1061	4.1054
	-2.5	2.6152	2.6168	2.6175	2.6184	2.6189	2.6191	2.6196	2.6199
	-3	1.3487	1.3524	1.3541	1.3557	1.3544	1.3548	1.3577	1.3571

Table 9 - 5: ARL values for individual EWMA control charts for the Pareto distribution (m=150) for various negative shifts

λ, L	k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=112	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
λ=0.05 L=2.0355	0	362.6182	364.0573	364.1686	364.3712	364.9842	364.6895	364.4452	364.1759
	0.2	72.7362	73.0621	73.0351	73.0684	72.7771	72.5519	72.9796	72.7848
	0.4	18.8077	19.2536	20.6937	20.8441	20.6210	20.4548	20.8909	20.7355
	0.6	17.2012	17.1073	17.0401	17.0257	16.9054	16.8106	16.9690	16.8887
	0.8	14.0863	14.2848	14.3404	14.4030	14.3251	14.2634	15.3457	15.2822
	1	12.9573	12.9044	12.8705	12.8488	12.7957	12.7535	12.8061	12.7706
	1.5	9.1023	9.0398	9.0284	9.0222	9.0548	9.0362	9.0036	9.0550
	2	7.1222	7.1279	7.0932	7.0691	7.0544	7.0464	7.0412	7.0327
	2.5	5.4819	5.4371	5.4163	5.4021	5.3995	5.3936	5.3732	5.3737
	3	3.9964	3.9734	3.9680	3.9630	3.9577	3.9552	3.9548	3.9537
λ=0.08 L=2.2624	0	364.3440	364.3691	368.6142	368.1697	368.7057	368.2595	368.5737	368.2191
	0.2	73.4425	73.9720	73.6079	75.0501	73.6444	75.0579	73.4873	75.0701
	0.4	19.5484	20.7357	20.9722	20.7151	21.0121	20.8175	20.8934	20.7342
	0.6	17.4841	17.2062	17.3486	17.2757	17.3731	17.2715	17.2842	17.2021
	0.8	14.5939	14.5415	14.4018	14.4288	14.3482	14.4287	14.3127	14.3723
	1	12.5442	12.4861	12.4880	12.4469	12.4801	12.4290	12.4488	12.4127
	1.5	8.6428	8.6259	8.6120	8.6321	8.6121	8.6241	8.5953	8.5964
	2	6.6430	6.6100	6.6025	6.5906	6.5784	6.5706	6.5727	6.5753
	2.5	5.0688	5.0544	5.0545	5.0412	5.0354	5.0339	5.0257	5.0219
	3	3.9061	3.8825	3.8754	3.8714	3.8687	3.8646	4.0284	4.0260
λ=0.10 L=2.5995	0	364.8935	368.2284	369.1284	368.3775	369.5209	368.9591	368.0462	368.6433
	0.2	73.8444	73.4526	73.9345	75.0212	75.5443	75.2184	73.6424	75.2154
	0.4	20.3937	20.5089	19.8219	20.1268	19.8961	20.6800	20.3263	20.1697
	0.6	16.8454	16.8417	16.9645	17.1273	16.9759	16.8648	16.6889	16.5934
	0.8	14.3506	14.3088	14.0680	14.1273	14.0600	12.9978	14.1806	14.1263
	1	12.3737	12.3409	12.3702	12.2284	12.1812	12.3187	12.2464	12.2124
	1.5	8.2805	8.2590	8.2601	8.2127	8.1975	8.5571	8.5284	8.5152
	2	6.4841	6.4593	6.4555	6.4306	6.4219	6.4379	6.4250	6.4190
	2.5	4.8457	4.9984	4.9818	4.9770	4.9714	4.9648	4.9708	4.9648
	3	3.8484	3.8484	3.8480	3.8464	3.8433	3.8406	3.8259	3.8237
λ=0.12 L=3.1037	0	368.4840	366.7542	368.4163	368.7730	369.1784	368.6330	368.8232	369.5028
	0.2	73.8640	75.0075	73.3759	73.6017	73.8172	73.4872	73.6484	75.0148
	0.4	20.9642	20.9645	21.0235	20.6826	20.7935	20.5786	20.6431	20.8648
	0.6	16.6828	16.6434	17.4303	17.2330	17.2700	17.4012	17.1888	17.3080
	0.8	14.0337	12.9775	12.9550	12.9336	12.8648	12.9195	12.9195	12.8681
	1	12.2591	12.2003	12.1759	12.1573	12.1093	12.1284	12.1248	12.1018
	1.5	8.3073	8.2793	8.2548	8.2302	8.2319	8.2180	8.2040	8.2126
	2	6.3372	6.3231	6.3041	6.2898	6.2877	6.2812	6.2734	6.2690
	2.5	4.8082	4.7846	4.7772	4.7598	4.7542	4.7573	4.7548	4.7517
	3	3.7127	3.7057	3.6986	3.6972	3.6937	3.6937	3.6891	3.6931
λ=0.15 L=3.2512	0	368.6882	369.5457	368.6843	369.2786	368.5773	369.2882	369.6148	369.1219
	0.2	75.0412	73.7370	73.9754	75.3406	73.8820	75.3245	73.6843	75.2062
	0.4	20.4054	21.1206	21.2848	20.9869	21.1625	20.9318	20.9908	20.8126
	0.6	17.1869	17.0122	16.8440	16.9121	17.0354	17.8486	16.9398	16.8425
	0.8	14.8052	14.8093	15.7160	15.9322	15.6448	15.2535	15.9375	15.8798
	1	10.9072	12.0348	12.0339	10.9541	10.9684	10.9373	10.9328	10.9693
	1.5	8.0431	8.0173	7.9880	7.9934	7.9778	7.9822	7.9778	7.9682
	2	6.0532	6.1914	6.1817	6.1635	6.1644	6.1593	6.1522	6.1481
	2.5	4.6455	4.6321	4.6230	4.6164	4.6143	4.6120	4.6082	4.6106
	3	3.5703	3.6488	3.6420	3.6354	3.6333	3.6314	3.6280	3.6264
λ=0.20 L=3.9786	0	366.6871	369.7277	369.1909	372.2057	369.2500	370.5393	369.3996	368.9148
	0.2	75.2600	75.5364	75.0371	75.7954	75.2461	75.8284	75.1464	73.8400
	0.4	20.9364	21.2688	20.4416	21.3218	21.0290	20.8031	20.4250	20.2571
	0.6	16.8415	17.1686	16.7170	17.1263	16.9802	17.8481	16.6407	17.4128
	0.8	12.6812	14.9573	14.1863	12.9306	12.8452	12.7777	14.1206	14.0689
	1	10.6221	12.8648	12.9707	12.8264	12.5775	12.5373	10.9143	12.8793
	1.5	8.8702	8.8168	7.8445	7.7935	7.7778	9.7541	9.7354	9.8836
	2	7.8751	7.9644	7.9370	7.2377	7.2322	7.2369	7.9375	7.9331
	2.5	5.4461	5.4284	5.4190	5.5124	5.5084	5.5054	5.4898	5.4872
	3	3.8455	3.8288	3.8223	3.8161	3.8128	3.8121	3.8088	3.8073

Table 9 - 6: ARL values for individual EWMA control charts for the Pareto distribution (m=150) for various positive shifts for the case of not using the skewness correction term when constructing the control limits of the chart

9.7 Optimal Choice for the Parameters of the EWMA Control Charts for Individual Observations from the Pareto Distribution

When constructing an EWMA control chart, there are two parameters involved in the way the chart is going to perform, namely the constant λ which affects the weight we give to the past values of our observations and the value of L which affects the width of the chart's control limits. Therefore, we need to find the combination of the values of those two parameters which will lead us to the optimal performance of our control chart.

As discussed in Section 6.7, a lot of research has been done on optimal design of control charts by minimizing the out-of-control value of various performance criteria. Since all the study here has been based on ARL (which is the most commonly used performance criterion) the optimal design of the EWMA control chart will be done by minimizing the ARL. The algorithm applied here is as follows:

- Step 1: Set the desired in-control ARL value (e.g. ARL_0=370) and the size of the mean shift k to be detected (e.g. k = 0.5).
- Step 2: Set an initial value L = 1.
- Step 3: Vary the parameter λ (e.g. increasing by 0.01) so as $\lambda \in (0,1]$ and (using a nonlinear equation solver) find the value of λ for which the ARL_0 value in Step 1 is satisfied.
- Step 4: Calculate the ARL_1 value for the particular combination of λ and L resulting from Step 3. [The ARL_1 value is obtained as described in the previous section, using equation (9-8) for the computation of the transient probabilities along with equation (5-2) for the cumulative distribution function of the Pareto distribution.]
- Step 5: Increase L by 0.01.
- Step 6: Repeat Steps 3-5 until the minimum ARL_1 value has been reached (i.e. until the ARL_1 value for L+0.01 is larger than the ARL_1 value for L).
- Step 7: Keep the combination of λ and L resulting from Step 6 for which the smallest ARL_1 value is obtained as the desired optimal one for the selected shift size in Step 1.

- Step 8: Repeat Steps 2-7 for all the desired values of shifts to be detected (e.g. k = {-3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3}).

Application of this algorithm yields Table 9-7 and Table 9-8 which present the optimal combination of values of the two parameters of concern (λ and L) of the EWMA chart with the corresponding ARL values for various values of the parameters *d* and *r* of the Pareto distribution and various positive and negative values, respectively, of *k*, which shows the shift of the process mean in terms of the process standard deviation which we want to be detected by the control chart we construct.

k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=122	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
0.2	(0.73, 3.99)	(0.73, 4.21)	(0.75, 4.07)	(0.73, 4.84)	(0.73, 4.34)	(0.75, 4.57)	(0.72, 2.82)	(0.75, 4.98)
	(369.8335, 60.7545)	(369.9863, 60.832)	(369.2348, 61.2441)	(370.6121, 61.5754)	(369.398, 61.1227)	(369.9551, 61.4099)	(369.6978, 61.4814)	(369.6884, 61.1759)
0.4	(0.73, 4.98)	(0.75, 4.25)	(0.75, 4.07)	(0.75, 4.75)	(0.73, 4.32)	(0.75, 4.59)	(0.75, 4.83)	(0.73, 4.98)
	(369.4259, 15.2706)	(369.6205, 15.289)	(369.2357, 15.061)	(369.8235, 15.9093)	(369.8148, 15.5072)	(369.2773, 15.6428)	(369.0123, 15.0036)	(369.6884, 15.1028)
0.6	(0.75, 4.99)	(0.72, 2.12)	(0.73, 2.97)	(0.72, 2.75)	(0.72, 3.38)	(0.73, 3.54)	(0.72, 2.82)	(0.72, 3.01)
	(369.1803, 12.5445)	(369.6287, 12.3102)	(369.1517, 12.2571)	(369.5157, 12.1226)	(369.7338, 12.1824)	(369.4372, 12.1935)	(369.6978, 12.2109)	(369.1641, 12.3277)
0.8	(0.73, 3.99)	(0.75, 4.25)	(0.75, 4.07)	(0.73, 4.84)	(0.73, 4.34)	(0.75, 4.57)	(0.75, 4.81)	(0.75, 4.98)
	(369.8335, 8.3142)	(369.6205, 8.2545)	(369.2357, 8.7502)	(370.6121, 8.6224)	(369.398, 8.4526)	(369.9551, 8.4254)	(370.8107, 8.6682)	(369.6884, 8.8603)
1	(0.73, 3.99)	(0.75, 4.25)	(0.75, 4.07)	(0.73, 4.84)	(0.75, 4.32)	(0.75, 4.57)	(0.75, 4.81)	(0.75, 4.98)
	(369.8335, 6.7017)	(369.6205, 6.0457)	(369.2357, 6.846)	(370.6121, 6.8333)	(369.8148, 6.3428)	(369.9551, 6.6845)	(370.8107, 6.8643)	(369.6884, 6.0023)
1.2	(0.75, 4.99)	(0.75, 4.21)	(0.75, 4.02)	(0.75, 4.75)	(0.73, 4.34)	(0.75, 4.57)	(0.75, 4.81)	(0.73, 3.99)
	(369.1803, 5.7062)	(369.9863, 5.7389)	(369.3446, 5.5393)	(370.6121, 5.0961)	(369.398, 5.0838)	(369.9551, 5.9124)	(370.8107, 5.1071)	(369.3178, 5.7733)
1.4	(0.02, 1.31)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)
	(371.0987, 5.0937)	(371.8489, 5.0103)	(371.8982, 4.9822)	(371.9791, 4.9632)	(371.1951, 4.9861)	(371.3207, 4.9754)	(371.2844, 4.9682)	(371.262, 4.9723)
1.6	(0.01, 1)	(0.77, 20)	(0.77, 20)	(0.02, 1.3)	(0.02, 1.31)	(0.75, 20)	(0.02, 1.3)	(0.75, 20)
	(371.2209, 4.1936)	(369.0535, 4.2752)	(369.2183, 4.034)	(371.9791, 4.4457)	(371.1951, 4.4046)	(369.3245, 4.2214)	(371.2844, 4.3795)	(369.3171, 4.2121)
1.8	(0.03, 1.61)	(0.02, 1.3)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)
	(371.9642, 4.1252)	(371.8489, 4.0935)	(372.8936, 4.0535)	(372.9375, 4.0099)	(372.6463, 3.9937)	(372.4575, 3.9825)	(372.1512, 3.961)	(17.9918, 3.951)
2	(0.03, 1.61)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)
	(371.9642, 3.6289)	(371.8489, 3.6339)	(371.8982, 3.6464)	(371.9791, 3.6873)	(371.1951, 3.6846)	(371.3207, 3.6934)	(371.2844, 3.6841)	(371.262, 3.6354)
2.2	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)
	(371.2209, 3.4552)	(372.9195, 3.378)	(372.8936, 3.3557)	(372.9375, 3.3548)	(372.6863, 3.3517)	(372.4575, 3.3505)	(372.1512, 3.3507)	(17.9918, 3.3524)
2.4	(0.03, 1.61)	(0.03, 1.6)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)
	(371.9642, 3.1262)	(371.1248, 3.0937)	(371.8982, 3.1282)	(371.9791, 3.1572)	(371.1951, 3.1734)	(371.3207, 3.1689)	(371.2844, 3.1864)	(371.262, 3.1452)
2.6	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.01, 1)	(0.04, 1.87)
	(371.2209, 2.9371)	(372.9195, 2.951)	(372.8936, 2.9615)	(372.9375, 2.9712)	(372.6863, 2.9701)	(372.4575, 2.9716)	(372.1512, 2.9798)	(371.6123, 2.9848)
2.8	(0.03, 1.61)	(0.03, 1.6)	(0.03, 1.6)	(0.02, 1.3)	(0.02, 1.31)	(0.02, 1.3)	(0.02, 1.3)	(0.02, 1.31)
	(371.9642, 2.7514)	(371.1248, 2.751)	(371.937, 2.7288)	(371.9791, 2.773)	(371.1951, 2.8126)	(371.3207, 2.7771)	(371.2844, 2.806)	(371.262, 2.7933)
3	(0.01, 1)	(0.04, 1.87)	(0.04, 1.87)	(0.04, 1.87)	(0.04, 1.88)	(0.04, 1.87)	(0.04, 1.87)	(0.04, 1.87)
	(371.2209, 2.6306)	(371.0808, 2.6841)	(371.9815, 2.6828)	(371.9343, 2.6603)	(371.9077, 2.638)	(371.1989, 2.6435)	(371.7916, 2.6459)	(371.6123, 2.6424)

Table 9 - 7: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the Pareto distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of positive shifts k (m=150)

k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=122	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
-0.2	(0.72, 3.01)	(0.75, 4.25)	(0.75, 4.07)	(0.73, 3.75)	(0.73, 4.34)	(0.73, 3.54)	(0.73, 3.82)	(0.75, 4.99)
	(369.9802,	(369.6205,	(369.2357, 60.5044)	(369.8805, 60.2163)	(369.398, 60.938)	(369.4372,	(369.0546, 60.4008)	(369.0307,
-0.4	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.96, 2.54)	(0.98, 2.57)
	(372.975,	(378.0593,	(377.184, 15.8488)	(373.3507, 15.6444)	(373.5936,	(373.1078,	(373.7553, 15.7184)	(375.9362,
-0.6	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975,	(378.0593,	(377.184, 12.5359)	(373.3507, 12.4641)	(373.5936,	(373.1078,	(377.7787, 12.5095)	(375.9362,
-0.8	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 8.9376)	(378.0593, 8.9375)	(377.184, 8.9806)	(373.3507, 8.937)	(373.5936,	(373.1078, 8.9521)	(377.7787, 8.9754)	(375.9362,
-1	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 6.6441)	(378.0593, 6.6425)	(377.184, 6.6899)	(373.3507, 6.4623)	(373.5936,	(373.1078, 6.6457)	(377.7787, 6.6899)	(375.9362,
-1.2	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 5.4539)	(378.0593, 5.4557)	(377.184, 5.4845)	(373.3507, 5.484)	(373.5936,	(373.1078, 5.4868)	(377.7787, 5.4866)	(375.9362,
-1.4	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 4.3545)	(378.0593, 4.3575)	(377.184, 4.3709)	(373.3507, 4.3642)	(373.5936,	(373.1078, 4.3686)	(377.7787, 4.3735)	(375.9362,
-1.6	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 3.2712)	(378.0593, 3.2752)	(377.184, 3.2843)	(373.3507, 3.2797)	(373.5936,	(373.1078, 3.2816)	(377.7787, 3.2868)	(375.9362,
-1.8	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 3.2108)	(378.0593, 3.2126)	(377.184, 3.2212)	(373.3507, 3.218)	(373.5936,	(373.1078, 3.2195)	(377.7787, 3.2234)	(375.9362, 3.224)
-2	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 3.1639)	(378.0593, 3.1683)	(377.184, 3.1751)	(373.3507, 3.1718)	(373.5936,	(373.1078, 3.173)	(377.7787, 3.177)	(375.9362,
-2.2	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 2.1218)	(378.0593, 2.1238)	(377.184, 2.1282)	(373.3507, 2.1264)	(373.5936, 2.127)	(373.1078, 2.1275)	(377.7787, 2.1287)	(375.9362,
-2.4	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 2.1054)	(378.0593, 2.1071)	(377.184, 2.1206)	(373.3507, 2.1093)	(373.5936,	(373.1078, 2.1099)	(377.7787, 2.1217)	(375.9362, 2.122)
-2.6	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 2.0841)	(378.0593, 2.0862)	(377.184, 2.0889)	(373.3507, 2.0878)	(373.5936,	(373.1078, 2.0883)	(377.7787, 2.0897)	(375.9362,
-2.8	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 1.9689)	(378.0593, 1.9697)	(377.184, 1.9718)	(373.3507, 1.9709)	(373.5936,	(373.1078, 1.9712)	(377.7787, 1.9724)	(375.9362,
-3	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.54)	(0.98, 2.57)	(0.98, 2.57)
	(372.975, 1.9541)	(378.0593, 1.9546)	(377.184, 1.9572)	(373.3507, 1.9575)	(373.5936,	(373.1078, 1.9577)	(377.7787, 1.9593)	(375.9362,

Table 9 - 8: Optimal combinations (λ*, L*) (row above the dotted lines for each cell) for the individual EWMA control charts for the Pareto distribution and the corresponding in-control and out-of-control ARL values (ARL0, ARL1) (row below the dotted lines for each cell) for various values of negative shifts k (m=150)

9.8 Examples on the Individual Pareto Probability-Type, Shewhart-Type and EWMA Control Charts

This section is dedicated to the illustration of the proposed control charts by means of both simulated data generated from the distribution of concern and real data. The case of simulated data is presented in Subsection 9.9.1, while the real data case is discussed in Subsection 9.9.2.

9.9.1 Examples with Simulated Data from the Pareto Distribution

Once again, the R programming language version 4.0.2 (R Core Team (2020)) has been used for the simulation procedure, which is presented in the next lines: Suppose we take a sample of n = 30 observations from a Pareto process as follows. First, we take a sample of 15 observations from a Pareto process with in-control d value equal to 54 and in-control r value equal to 68. Now suppose that a shift of one standard deviation unit occurs in the process mean, and after that shift, we draw another set of 15 observations from the process. The resulting data set can be seen in Table 9-9. For this data set, we construct the individual probability-type Pareto control chart shown in Figure 9-1, using the most commonly used value for the significance level α = 0.27%, as mentioned in Section 9-2. As we can see in Figure 9-1, there is an increasing trend after the first 15 observations and the control chart detects an out-of-control point indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level.

For the same data set, we construct the individual Shewhart-type Pareto control chart shown in Figure 9-2, using L = 3.5493 standard deviations (which gives a desired value of in-control ARL close to 370). Figure 9-2 presents an increasing trend after the first 15 observations and the control chart detects two out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level. Comparing this chart to the previous one (Figure 9-1), we observe that the Shewhart-type chart detects the shift sooner than the probability-type control chart.

Data Set 1	69.36051536	68.96712115	68.12968429	70.77255643	70.92393351
	68.22971246	68.05579805	71.25504484	68.23515216	70.02474861
	68.46571824	68.44581946	69.65809923	68.12931190	68.52044245
	73.72568547	70.14012411	72.21420331	69.90486487	70.37417125
	73.57554491	74.10423378	70.96487403	76.15594664	69.81805034
	70.50767778	70.21280465	77.98746171	72.12578008	74.32275503

Table 9 - 9: Data from a Pareto process with in control d=54, in-control r=68 and a shift of one standard deviation unit in the process mean due to an increasing shift after the first 15 observations (gray shading)

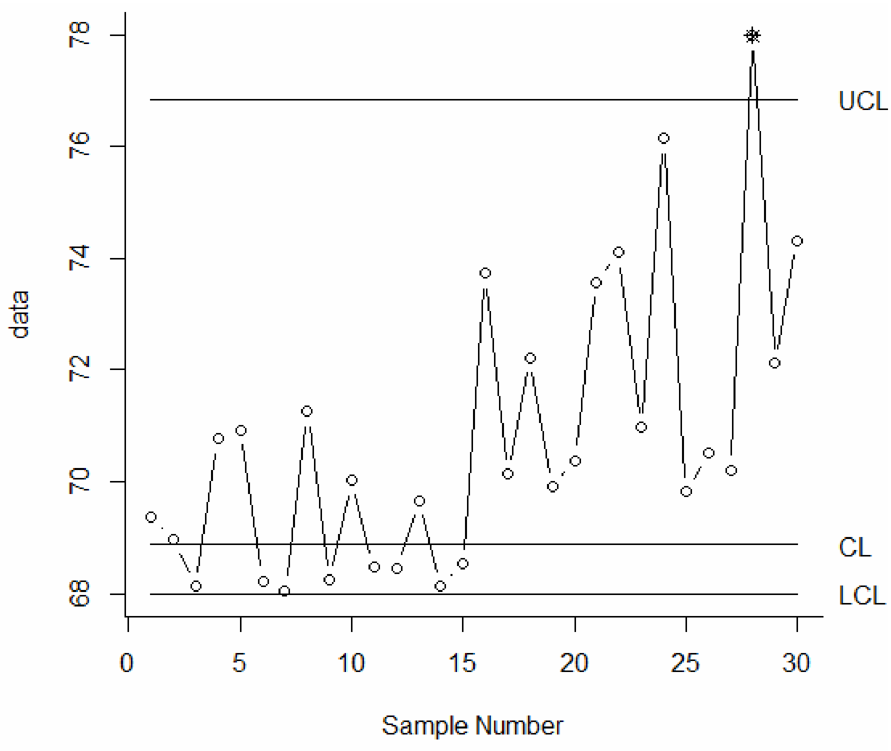

Figure 9 - 1: Individual probability type Pareto control chart for the data set in Table 9-9 with a shift of one standard deviation unit in the process mean

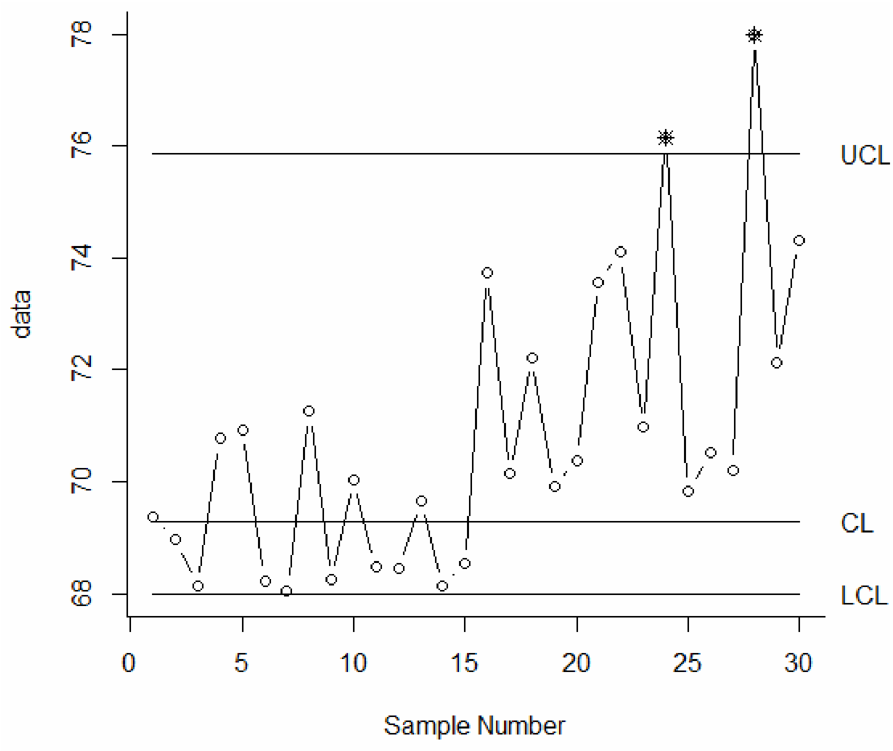

Figure 9 - 2: Individual Shewhart type Pareto control chart for the data set in Table 9-9 with a shift of one standard deviation unit in the process mean

Using the data set in Table 9-9 for the case of a shift of one standard deviation unit, we now construct the individual EWMA Pareto control chart shown in Figure 9-2, using λ=0.05 and L = 2.1812 standard deviations (which gives a desired value of in-control ARL close to 370). As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 21st observation which, compared to Figure 9-1, is sooner than the Shewhart-type control chart, as expected.

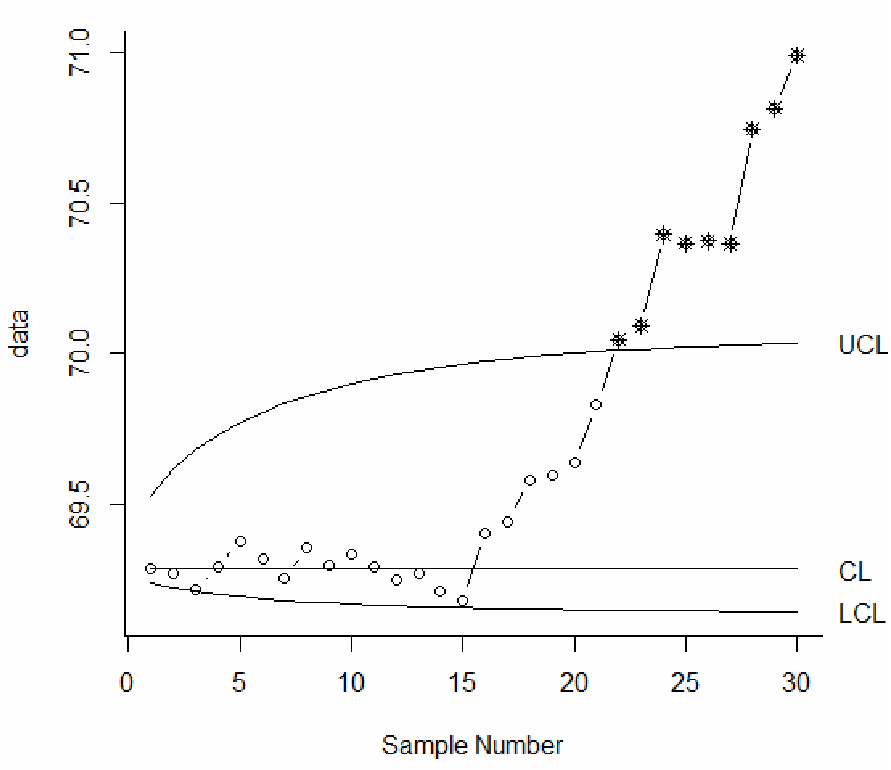

Figure 9 - 3: Individual EWMA type Pareto control chart for the data set in Table 9-9 with a shift of one standard deviation unit in the process mean

9.8.2 Application of the Individual Pareto Probability-Type, Shewhart-Type and EWMA Control Charts to Real Data

This section presents the usefulness of the proposed control charts using two real datasets. The first dataset comes from Goegebeur et al. (2005), used among others by Vandewalle et al. (2007), representing the calcium content in soil in the Condroz region in Belgium. This data set, however, is very large (1428 observations) and it is too right-skewed and long-tailed to be fitted by a Pareto distribution in its whole. Smaller samples of this data set, however, are great for fitting this distribution. Therefore, a sample of 30 consecutive observations from this data set has been chosen randomly after its 10^{th} observation, and this sample is presented here (with its observations in random order) in Table 9-10. Another scenario will be analyzed immediately afterwards (Table 9-11).

First of all, when dealing with any dataset, the normality assumption should be checked. Both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test give a p-value < 0.01 which is a very clear indication that normality assumption does not hold for our data. For the case of the Pareto distribution, on the other hand, the Kolmogorov-Smirnov test gives an approximate p-value=0.586 with the presence of ties in our data and a p-value=0.7912 without them. In both cases p-value is large. Therefore, we do not reject the null hypothesis that our data may be coming from the assumed distribution and this is an indication that the Pareto distribution fits our data well.

The values of the parameters of our assumed Pareto distribution being equal to 3.9728 and 242.9444 for d and r, respectively, are going to be used for the construction of the individual probability-type control chart (along with the significance level value α=0.27%) and for the Shewhart-type control chart for our data, in conjunction with the value of L=3.6376 standard deviations (for which in-control ARL is close to 370). The resulting control charts can be seen in Figure 9-4 and Figure 9-5 for the probability-type and Shewhart-type control chart, respectively, which show all the observations being inside the control limits, which is an indication that the calcium content in soil is within the expected ranges. The Shewhart-type control chart, however, presents a point close to the upper control limit which needs attention.

For the construction of the individual EWMA control chart for our data, using the same parameter values of the assumed Pareto distribution from the data in conjunction with the values of λ=0.05 and L=2.0836 standard deviations (for which in-control ARL is close to 370), the resulting control chart can be seen in Figure 9-6, which shows all the observations being inside the control limits, which, once again, is an indication that the calcium content in soil is within the expected ranges.

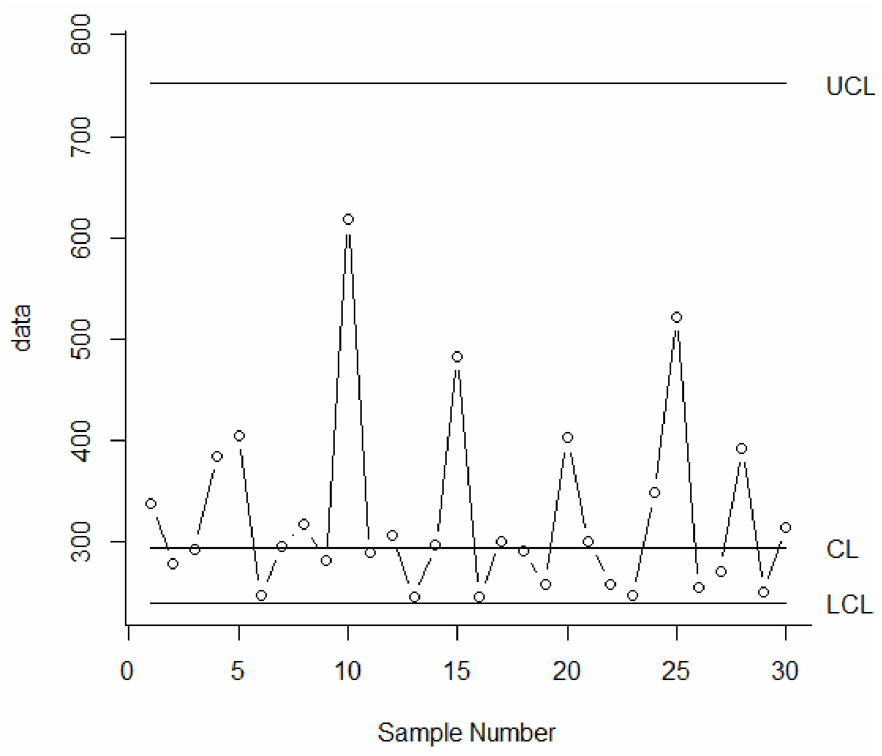

Figure 9 - 4: Individual probability type Pareto control chart for the Condroz calcium data set in Table 9-10

337	278	293	385	405	248	296	317	281	618
289	307	245	297	483	246	301	291	259	403
300	259	247	348	522	255	271	393	251	315

Table 9 - 10: First dataset of calcium content in soil in Condroz region in Belgium

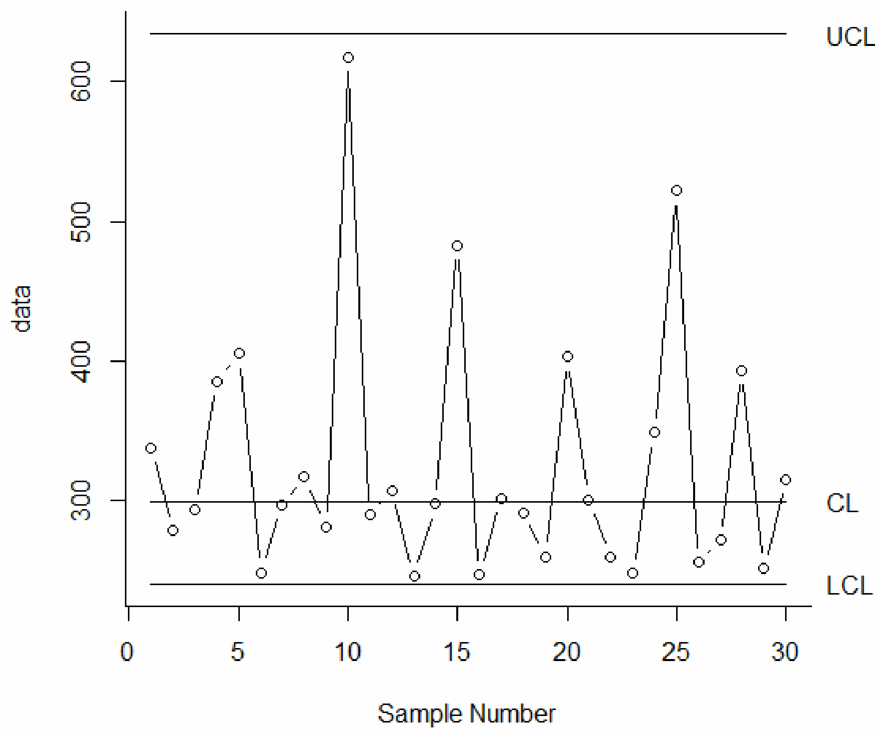

Figure 9 - 5: Individual Shewhart type Pareto control chart for the Condroz calcium data set in Table 9-10

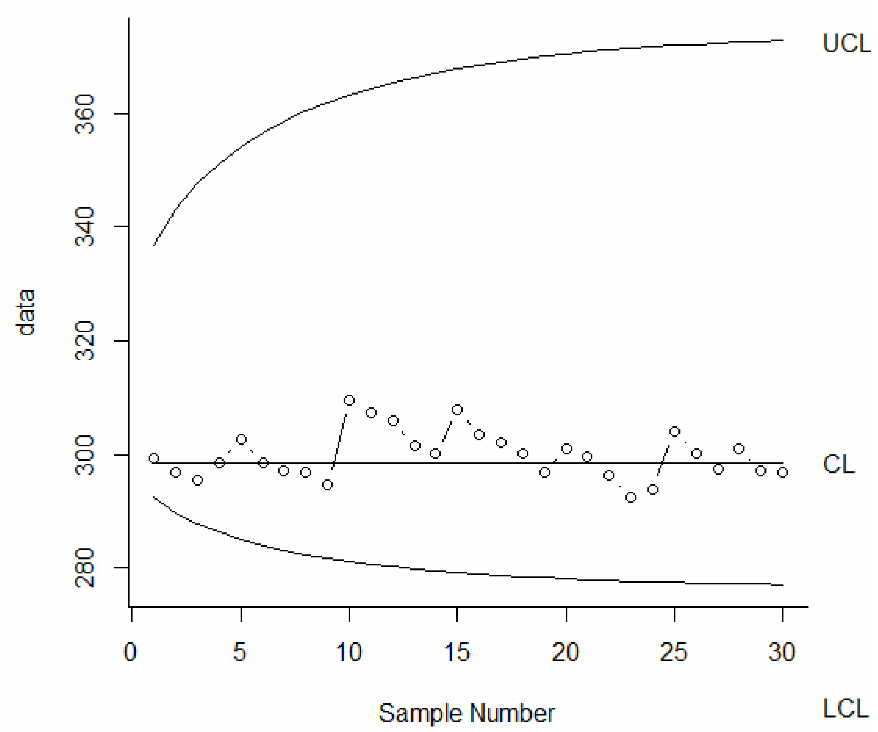

Figure 9 - 6: Individual EWMA Pareto control chart for the Condroz calcium data set in Table 9-10

The second dataset represents breaking angles of chocolate cakes found by fixing one half of a slab of cake and then pivoting the other half about the middle until breakage occurs. The chosen data set is a subset of the data in Cohran and Cox (1959) and consists of two subsets of data regarding the last two of the three recipes considered there for the specific temperature of 205^{o}C and is presented for convenience in Table 9-11. We are going to see whether the choice of recipe is significant, in other words whether the observations in the second data subset are significantly different or still in-control relatively to the observations from the first subset of our dataset. For the first subset of this dataset (first row of Table 9-11), the Shapiro-Wilk normality test gives a p-value equal to 0.008101 and and the Kolmogorov-Smirnov test gives an approximate p-value=0.02806 with the presence of ties in our data and a p-value=0.0336 without them. The results of both tests are a very clear indication that normality assumption does not hold for our data. For the case of the Pareto distribution the Kolmogorov-Smirnov test gives very large p-values (an approximate p-value=0.9251 with the presence of ties in our data and a p-value=0.9895 without them) which are evidence that we cannot reject the null hypothesis that our data may be coming from the assumed Pareto distribution. For the case of the second subset of our dataset (second row of Table 9-11), the Kolmogorov-Smirnov test for a Pareto distribution gives an approximate p-value=0.3752 with the presence of ties in our data and a p-value=0.2115 without them, which is an indication that we do not have enough evidence to reject the null hypothesis that our data may be coming from the assumed Pareto distribution. The two processes corresponding to the two recipes for chocolate cakes are different. Let's see if our control charts can detect that difference.

The values of the parameters of our assumed Pareto distribution (for the case of recipe 3) being equal to 4.2621 and 23.6246 for d and r, respectively, are going to be used for the construction of the individual probability-type control chart (along with the significance level value $\alpha = 0.27\%$) and for the Shewhart-type control chart for our data, in conjunction with the value of L=5.4457 standard deviations (for which in-control ARL is close to 370). The resulting control charts can be seen in Figure 9-7 and Figure 9-8 for the probability-type and Shewhart-type control chart, respectively. Both charts present an out-of-control point below the lower control limit. This is an indication of a process improvement due to the fact that the breaking angle decreased with recipe 2 compared to using recipe 3.

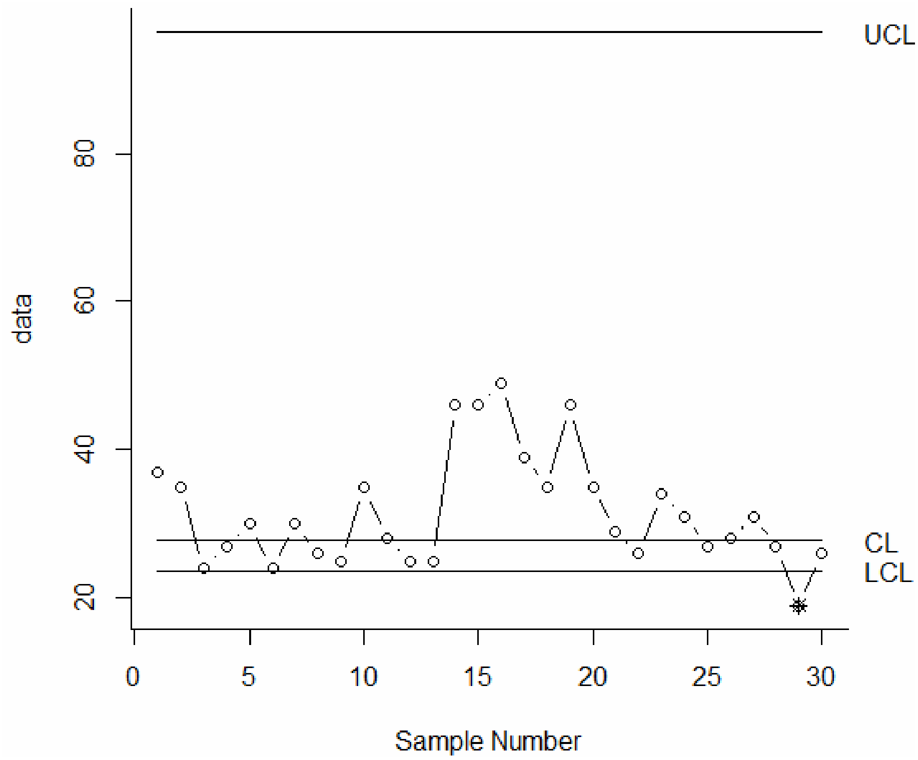

Figure 9 - 7: Individual probability type Pareto control chart for the breaking angles data set of Table 9-11

Recipe 3	37	35	24	27	30	24	30	26	25	35	28	25	25	46	46
Recipe 2	49	39	35	46	35	29	26	34	31	27	28	31	27	19	26

Table 9 - 11: Breaking angles of chocolate cakes

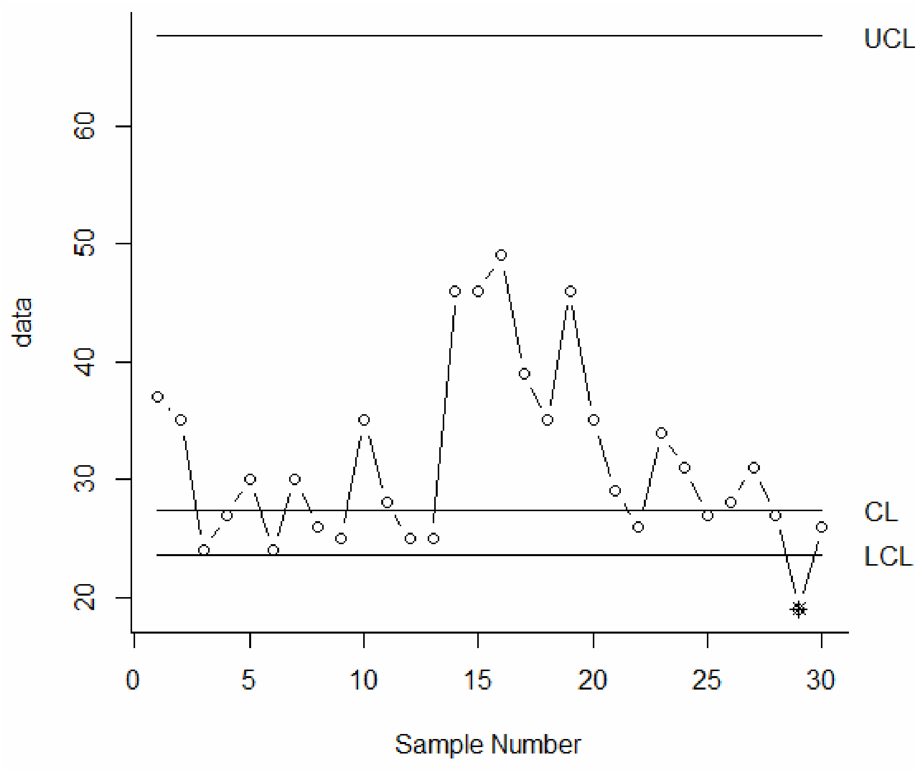

Figure 9 - 8: Individual Shewhart type Pareto control chart for the breaking angles data set in Table 9-11

For the construction of the individual EWMA control chart for our data, using the same parameter values of the assumed Pareto distribution from the data in conjunction with the values of λ=0.05 and L=3.5495 standard deviations (for which in-control ARL is close to 370), the resulting control chart can be seen in Figure 9-9, which, contrarily to the previous two charts, does not detect the out-of-control state of the process. This is probably due to the small λ value which gives bigger weight to the past much bigger values.

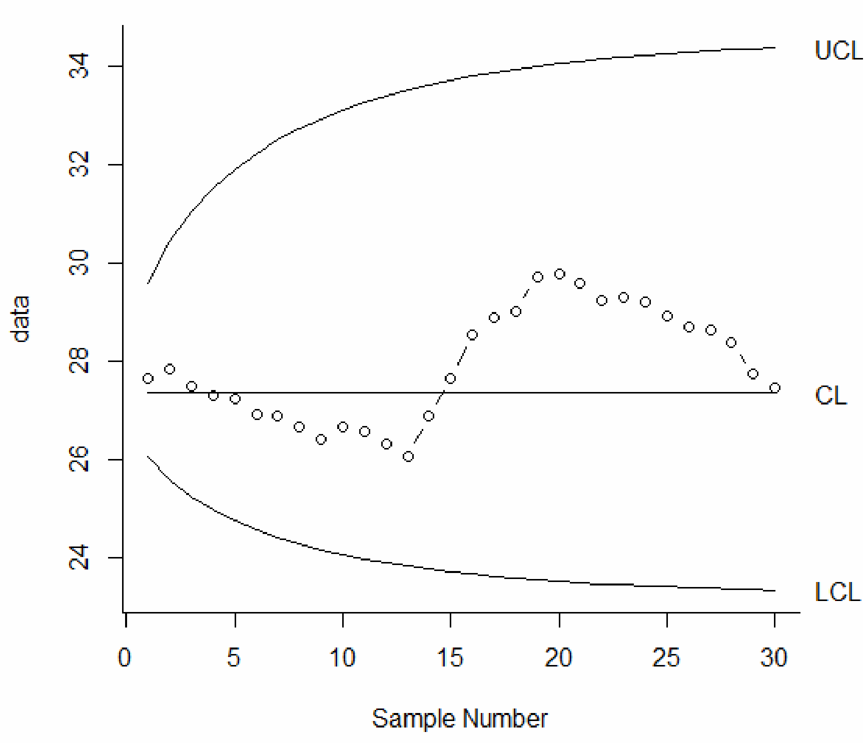

Figure 9 - 9: Individual EWMA Pareto control chart for the breaking angles data set of Table 9-11

9.9 Control Charts for Individual Observations from the Pareto Distribution with the Scaled Weighted Variance Method

The control charts for the Pareto distribution presented so far were based on the skewness correction method proposed by Chan and Cui (2003). It would be worth also investigating some other method for taking into account the distribution's skewness, such as the scaled weighted variance method proposed by Castagliola (2000). This method is going to be used hereafter for the construction and investigation of the performance of the control charts for individual observations from the Pareto distribution and the comparison with the control charts of the preceding sections of this chapter.

9.9.1. Construction of Shewhart-type Control Charts for Individual Observations from a Process Following the Pareto Distribution Using the Scaled Weighted Variance Method

The method proposed by Castagliola (2000) is as follows: the central line is placed at the mean of the Pareto distribution, which is computed using equation (5-3), while the control limits are placed around the mean at two different multiples of the standard deviation of the Pareto distribution, which is computed using equation (5-4). These multiples are functions of appropriate values of the quantiles of the standardized Normal distribution, the probability of type I error or false alarm rate, α, and the cumulative distribution function of the Pareto distribution, which is computed using equation (5-2). More specifically, the lower control limit is defined as $LCL = \mu - \sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)\sigma$, while the upper control limit is defined as $UCL = \mu + \sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)\sigma$.

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Pareto control chart are as follows.

$$UCL = dr(d-1)^{-1} + \sqrt{\frac{1-\left(\frac{r}{x}\right)^d}{\left(\frac{r}{x}\right)^d}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(\frac{r}{x}\right)^d}\right)\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}$$

$$CL = dr(d-1)^{-1} \qquad ,d>3$$

$$LCL = dr(d-1)^{-1} - \sqrt{\frac{\left(\frac{r}{x}\right)^d}{1-\left(\frac{r}{x}\right)^d}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-\left(\frac{r}{x}\right)^d\right]}\right)\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}$$

(9-11)

9.9.2. Performance Investigation for the Individual Pareto Control Charts Constructed With the Scaled Weighted Variance Method

For the investigation of the performance of the control chart constructed above, we will use the ARL (ARL_0 and ARL_1) values obtained by equations (9-4) and (9-5) where $F_{in}\left(x\right)$ is the cumulative distribution function of the two-parameter Lindley distribution in equation (5-2) with in-control parameters, $F_{out}\left(x\right)$ is the cumulative distribution function for the distribution of concern with out-of-control parameters given by $d_{new}=1+\frac{\sqrt{\sigma^2+\left(\mu_0+k\sigma\right)^2}}{\sigma}$ and $r_{new}=\left(\mu_0+k\sigma\right)\frac{\sqrt{\sigma^2+\left(\mu_0+k\sigma\right)^2}}{\sigma+\sqrt{\sigma^2+\left(\mu_0+k\sigma\right)^2}}$, as earlier, and control limits computed with equation (9-11) in both cases. Using the above formulas we obtain Table 9-13 which shows the in-control and out-of-control ARL values for the individual Pareto control chart for various values of the two parameters d and r of the distribution of concern and for various values of k which, as mentioned before, shows the shift we want to detect in the process mean in terms of the process standard deviation. For the significance level the most commonly used value of 0.27% has been chosen which corresponds to 0.27% probability of falsely rejecting the null hypothesis that our process is in control.

Comparing Tables 9-13 and 9-2 we observe that the performance of the chart improves significantly when using the scaled weighted variance method instead of the skewness corrected limits. The difference in ARL values between those two control charts is greater than 5% for all shift sizes greater than k=±1 while for smaller shift sizes the difference is slightly less than 5% for larger values of the Pareto distribution parameters. Comparison of the ARL values for positive and negative shifts shows that, although the control charts can detect both positive and negative shifts well, there are some slight differences with all values being a little smaller for the negative shifts than for the corresponding positive ones. The only differences that are above 5% concern shift sizes of k equal to 0.2 or between 1.2 and 1.6.

k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=112	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
-3	2.2500	2.2503	2.2505	2.2507	2.2509	2.2510	2.2510	2.2512
-2.8	3.0615	3.0620	3.0622	3.0625	3.0627	3.0628	3.0628	3.0631
-2.6	3.5750	3.5754	3.5757	3.5773	3.5775	3.5777	3.5778	3.5781
-2.4	4.0932	4.0937	4.0953	4.0959	4.0962	4.0964	4.0964	4.0970
-2.2	7.1200	7.1203	7.1206	7.1210	7.1273	7.1284	7.1286	7.1288
-2	8.1481	8.1484	8.1489	8.1500	8.1505	8.1510	8.1512	8.1519
-1.8	9.1848	9.1875	9.1882	9.1896	9.1903	9.1909	9.1912	9.1930
-1.6	17.2373	17.2393	17.2403	17.2420	17.2428	17.2436	17.2441	17.2452
-1.4	27.3068	27.3095	27.3107	27.3120	27.3141	27.3151	27.3157	27.3170
-1.2	40.3739	40.3973	40.4089	40.4124	40.4128	40.4146	40.4154	40.4172
-1	60.5405	60.5428	60.5453	60.5489	60.5519	60.5557	60.5578	60.5593
-0.8	92.0541	92.0704	92.0731	92.0784	92.0812	92.0843	92.0848	92.0880
-0.6	110.1273	110.1463	110.1502	110.1578	110.1617	110.1648	110.1680	110.1716
-0.4	164.8934	164.9068	164.9128	164.9312	164.9350	164.9363	164.9369	164.9396
-0.2	224.1212	224.1608	224.1735	224.1987	224.2127	224.2221	224.2290	224.2441
0	373.6122	373.4845	373.0184	372.2896	371.7508	372.5373	373.2699	373.9157
0.2	224.6368	224.6182	224.6099	224.5972	224.5715	224.5509	224.5355	224.4897
0.4	166.2370	166.2287	166.2246	166.2182	166.2103	166.1937	166.1872	166.1693
0.6	112.2070	112.2007	112.1979	112.1936	112.1884	112.1779	112.1727	112.1606
0.8	93.9993	93.9972	93.9939	93.9899	93.9821	93.9781	93.9691	93.0041
1	60.9702	60.9646	60.9643	60.9620	60.9578	60.9525	60.9373	60.9321
1.2	40.8199	40.8168	40.8153	40.8122	40.8105	40.8053	40.8027	40.7968
1.4	28.3171	28.3145	28.3124	28.3123	28.3093	28.3048	28.3026	28.2875
1.6	18.2437	18.2414	18.2404	18.2378	18.2369	18.2331	18.2312	18.2269
1.8	10.1897	10.1877	10.1868	10.1848	10.1844	10.1805	10.1789	10.1752
2	9.1484	9.1480	9.1464	9.1453	9.1439	9.1412	9.1287	9.1264
2.2	7.2179	7.2164	7.2157	7.2148	7.2124	7.2120	7.2097	7.2070
2.4	4.1937	4.1934	4.1918	4.1909	4.1898	4.1877	4.1864	4.1842
2.6	3.5937	3.5937	3.5934	3.5932	3.5914	3.5796	3.5787	3.5754
2.8	3.0800	3.0790	3.0784	3.0778	3.0770	3.0754	3.0736	3.0728
3	2.2484	2.2482	2.2469	2.2463	2.2455	2.2441	2.2435	2.2419

Table 9 - 12: ARL values for individual Pareto control charts with scaled weighted variance, with $\alpha = 0.0027$.

9.9.3. Construction of the EWMA Control Charts for Individual Observations from the Pareto Distribution Using the Scaled Weighted Variance Method

Here we will use the scaled weighted variance method for constructing EWMA control charts, too. As will be exhibited in the next subsection, this method will improve the performance of the chart. The procedure for deriving the control limits of the chart with this method is the following: in equation (2-3) we will replace L by $\sqrt{\frac{1-F_X(\mu)}{F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4F_X(\mu)}\right)$ for the lower control limit and $\sqrt{\frac{F_X(\mu)}{1-F_X(\mu)}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-F_X(\mu)\right]}\right)$ for the upper control limit, where μ is the mean of the Pareto distribution, which is computed with equation (5-3), and $F_X(x)$ is its cumulative distribution function given by equation (5-2). For the construction of the EWMA control charts we will also need the standard deviation of the Pareto distribution computed from equation (5-4).

As a result, the central line (CL) and the upper and lower control limits (UCL and LCL, respectively) of the Pareto EWMA control chart are as follows.

$$UCL = dr\left(d-1\right)^{-1} + \sqrt{\frac{1-\left(\frac{r}{x}\right)^{d}}{\left(\frac{r}{x}\right)^{d}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(\frac{r}{x}\right)^{d}}\right)\sqrt{dr^{2}\left(d-1\right)^{-2}\left(d-2\right)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-\left(1-\lambda\right)^{2i}\right]}$$

$$CL = dr\left(d-1\right)^{-1} \qquad ,d>3$$

$$LCL = dr\left(d-1\right)^{-1} - \sqrt{\frac{\left(\frac{r}{x}\right)^{d}}{1-\left(\frac{r}{x}\right)^{d}}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-\left(\frac{r}{x}\right)^{d}\right]}\right)\sqrt{dr^{2}\left(d-1\right)^{-2}\left(d-2\right)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}\left[1-\left(1-\lambda\right)^{2i}\right]}$$

(9-12)

9.9.4. Performance Investigation for the Individual EWMA Pareto Control Charts Constructed With the Scaled Weighted Variance Method

The performance of the control chart proposed in the previous subsection is going to be investigated here using the ARL as obtained in equation (9-9). For the transient probabilities in (9-8) the cumulative distribution function for the Pareto distribution, i.e. equation (5-2), is going to be used with either in-control parameters for the case of computing the in-control ARL value or the out-of-control parameters for the case of the out-of-control ARL, with the asymptotic control limits as computed with equation (9-12) for $i \to \infty$. This means that the control limits that will be used for the computation of ARL will be of the form

$$UCL = dr(d-1)^{-1} + \sqrt{\frac{1-\left(\frac{r}{x}\right)^d}{\left(\frac{r}{x}\right)^d}}\Phi^{-1}\left(1-\frac{\alpha}{4\left(\frac{r}{x}\right)^d}\right)\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = dr(d-1)^{-1} - \sqrt{\frac{\left(\frac{r}{x}\right)^d}{1-\left(\frac{r}{x}\right)^d}}\Phi^{-1}\left(1-\frac{\alpha}{4\left[1-\left(\frac{r}{x}\right)^d\right]}\right)\sqrt{dr^2(d-1)^{-2}(d-2)^{-1}}\sqrt{\frac{\lambda}{2-\lambda}}$$

(9-13)

For the out-of-control case we assume that the shift of the process mean is in terms of the process standard deviation. In other words, the new mean is assumed to be of the form $\mu_1 = \mu_0 + k\sigma$. Using this relationship, the new parameters of the distribution with the shifted mean will be computed by solving equations (5-3) and (5-4) in terms of its two parameters, as for the Shewhart-type control chart.

Using those formulae we get Tables 9-14, 9-15 and 9-16 which show the in-control and out-of-control ARL values for the individual EWMA control chart for the Pareto distribution for various values of the two parameters d and *r* of the distribution of concern and for various values of *k* which shows the shift of the process mean in terms of the process standard deviation. More specifically, Table 9-14 contains the ARL values for λ=0.3 for various values of the m for the subintervals into which the region between the upper and lower control limits is divided, as mentioned earlier. From this table we see that when keeping λ the same, the ARL value increases as the number m of subintervals increases and the rate of this increase is high until the value of about m=150, above which ARL increases very slightly. As a result, the suggested value of m for the computation of ARL in the formulae above is m=150. Therefore, Tables 9-15 and 9-16 show the ARL values for m=150 for various values of λ for positive and negative shifts, respectively.

m	k	d=25 r=37	d=42 r=68	d=57 r=93	d=86 r=112	d=105 r=154	d=128 r=184	d=210 r=250	d=300 r=310
50	0	371.5758	371.0168	371.0081	370.5865	370.8686	370.5524	371.5331	371.1008
	0.2	59.8295	59.3618	59.1681	58.8446	58.7309	58.4468	58.3312	56.7517
	0.5	14.9145	14.6919	14.5936	14.0530	12.8128	12.6895	12.5925	12.2326
	1	6.6781	6.6459	6.6152	6.4216	6.3107	6.1885	5.9867	5.8076
	1.5	4.2220	4.1529	4.0620	4.0502	4.0341	4.0073	4.0056	3.9923
	2	4.1054	4.0700	3.9726	3.9657	3.9238	3.8090	3.7523	3.7067
	2.5	2.9120	2.8976	2.8690	2.8590	2.8501	2.8433	2.8377	2.8277
	3	2.8254	2.8057	2.7974	2.7969	2.7939	2.7914	2.7869	2.7847
70	0	378.1232	377.0832	377.9046	375.8071	379.1504	378.2037	376.6732	376.0175
	0.2	65.9246	65.3759	65.0816	64.9809	64.2198	64.1701	64.1229	63.3873
	0.5	20.5240	19.8470	19.3273	19.2157	18.9803	18.4633	18.0838	17.7680
	1	8.8012	8.7410	8.5506	8.3101	8.1405	8.1208	8.0947	6.7965
	1.5	4.7353	4.5982	4.5001	4.4534	4.4037	4.3954	4.3254	4.2694
	2	4.3004	4.2774	4.2444	4.2389	4.2290	4.2171	4.0807	4.0178
	2.5	2.9122	2.8986	2.8696	2.8677	2.8625	2.8564	2.8462	2.8412
	3	2.8485	2.8334	2.8246	2.8235	2.8203	2.8177	2.8129	2.8106
90	0	383.6351	382.8061	383.9329	385.5677	383.8574	382.5915	386.4353	385.2196
	0.2	73.4373	72.7980	72.6190	71.3762	71.2834	70.3124	70.2994	70.1964
	0.5	25.2232	24.6547	24.5592	23.7238	23.6078	23.0156	22.7657	21.2519
	1	10.3129	9.8446	9.7098	9.6393	9.4467	9.4179	9.2276	9.1891
	1.5	5.0484	4.9864	4.7861	4.7627	4.7496	4.7086	4.7064	4.6430
	2	4.2931	4.2632	4.1993	4.1939	4.1851	4.1720	4.1697	4.1543
	2.5	3.0496	3.0163	2.9936	2.9920	2.9840	2.9829	2.9774	2.9762
	3	2.8653	2.8496	2.8457	2.8364	2.8333	2.8312	2.8306	2.8257
120	0	393.2872	392.3872	391.7430	391.5994	396.4727	394.1259	399.8895	397.7120
	0.2	81.8707	81.6355	81.3067	80.0159	78.1693	77.9230	77.8775	75.2450
	0.5	30.9563	30.2487	30.2325	30.1241	29.9998	28.8472	28.7415	27.8087
	1	12.7264	12.4725	12.3551	12.3042	12.2954	12.0755	10.9243	10.6524
	1.5	5.2592	5.1812	5.1274	5.0670	4.9777	4.9327	4.9044	4.8202
	2	4.4681	4.4474	4.4314	4.4107	4.3936	4.3633	4.3434	4.2779
	2.5	3.0643	3.0556	3.0486	3.0425	3.0359	3.0191	2.9778	2.9707
	3	2.9198	2.9140	2.9128	2.9092	2.9072	2.8798	2.8656	2.8562
150	0	402.0264	401.9673	406.9197	404.1559	400.2907	407.4877	410.0490	407.7949
	0.2	91.1765	89.6207	89.2094	88.1732	87.9252	86.6706	86.3972	83.6450
	0.5	34.3881	34.3457	34.3434	34.1554	33.2012	32.9812	32.9312	32.1297
	1	12.9677	12.6738	12.5380	12.4915	12.3512	12.2122	12.1224	12.0026
	1.5	5.7494	5.4120	5.3148	5.3144	5.2398	5.2308	5.2100	5.1061
	2	4.5298	4.5178	4.4971	4.4724	4.4542	4.4505	4.4364	4.4230
	2.5	3.0857	3.0385	3.0272	3.0160	3.0122	3.0077	2.9989	2.9931
	3	2.9290	2.9224	2.9156	2.9128	2.9087	2.9037	2.9030	2.8924
180	0	412.6440	409.3255	412.6762	419.3127	413.2346	419.3093	410.9173	417.3880
	0.2	98.3295	97.2621	96.4632	96.2564	95.3351	92.4808	92.1970	89.4834
	0.5	39.0394	38.2058	38.0012	37.2606	37.0861	36.7748	36.1860	36.1254
	1	14.3122	12.8481	12.7187	12.7064	12.5420	12.3579	12.2489	12.0301
	1.5	5.7396	5.5853	5.5159	5.4919	5.4724	5.4455	5.4357	5.3787
	2	4.5832	4.5425	4.5409	4.5041	4.4901	4.4782	4.4676	4.4521
	2.5	3.0903	3.0756	3.0612	3.0572	3.0526	3.0455	3.0426	3.0344
	3	2.9452	2.9270	2.9194	2.9090	2.9088	2.9057	2.9007	2.9001
210	0	417.6446	416.8991	417.5348	420.9717	425.0539	419.2086	419.9477	426.4312
	0.2	105.4988	103.9125	102.6883	101.7706	101.4749	100.5252	100.5173	97.6418
	0.5	42.1220	40.5233	40.4837	40.4640	40.0442	39.4334	38.8501	38.7326
	1	14.4321	14.3072	14.1999	14.1799	14.1673	14.1535	12.8564	12.8049
	1.5	5.9409	5.7518	5.6308	5.5904	5.5798	5.5291	5.4790	5.3838
	2	4.5877	4.5858	4.5503	4.5098	4.5074	4.4884	4.4798	4.4618
	2.5	3.0997	3.0860	3.0786	3.0739	3.0676	3.0581	3.0426	3.0275
	3	2.9421	2.9231	2.9150	2.9076	2.9064	2.9035	2.8979	2.8978
240	0	422.0406	423.9249	433.2742	433.6987	424.9526	429.4557	428.7146	435.0043
	0.2	110.9449	108.5876	107.5777	106.4571	105.1487	103.7477	102.9301	101.8321
	0.5	45.3584	44.4856	44.4084	43.8785	43.3433	43.2354	41.7069	41.2649
	1	14.8357	14.8219	14.7731	14.5459	14.5147	14.4522	14.2442	14.0989
	1.5	5.9525	5.9120	5.7641	5.7183	5.6493	5.6146	5.6141	5.5457
	2	4.6078	4.5873	4.5569	4.5358	4.5121	4.5092	4.5000	4.4817
	2.5	3.1240	3.0870	3.0772	3.0610	3.0525	3.0523	3.0468	3.0407
	3	2.9401	2.9224	2.9144	2.9064	2.9026	2.9017	2.8961	2.8960

Table 9 - 13: ARL values for individual EWMA control charts for the Pareto distribution (λ=0.3) with scaled weighted variance, with α = 0.0027, for various values of m.

λ	k	d=25 r=37	d=42 r=68	d=57 r=93	d=86 r=112	d=105 r=154	d=128 r=184	d=210 r=250	d=300 r=310
λ=0.05	0	376.3072	376.1827	375.7370	375.5070	375.3984	375.2816	375.8410	375.7201
	0.2	57.9523	57.7348	57.7324	57.4843	57.4579	57.2881	57.1935	57.0363
	0.4	14.1373	14.0148	13.7170	12.9887	12.9124	12.8706	12.7725	12.5054
	0.6	10.5434	10.3734	9.8122	9.6977	9.6848	9.5445	9.4422	8.4363
	0.8	8.1931	8.1893	8.1443	8.0790	7.9887	7.9607	7.4127	6.5684
	1	4.1069	4.0848	4.0317	3.9750	3.9016	3.8893	3.8489	3.0361
	1.5	3.2160	3.1995	3.0484	3.0412	3.0351	3.0244	3.0193	2.7978
	2	3.0906	3.0691	2.9864	2.9753	2.8487	2.7512	2.7127	2.6886
	2.5	2.3730	2.2523	2.1548	2.1416	2.1032	2.0724	2.0181	1.9931
	3	2.0060	1.9998	1.9905	1.9841	1.9818	1.9791	1.9732	1.9718
λ=0.08	0	377.2600	377.2270	376.8937	376.7364	376.8281	376.6396	376.5155	376.1784
	0.2	59.0557	57.7712	57.7352	57.6073	57.6057	57.4591	57.2003	57.1518
	0.4	14.3575	14.3048	14.2812	13.4215	13.2281	12.9793	12.8022	12.5754
	0.6	10.5193	10.3254	9.3612	9.2616	9.1254	8.9070	8.8820	8.3439
	0.8	8.3455	8.2328	8.1481	8.1044	8.0504	7.9608	7.9069	7.0934
	1	4.9353	4.8412	4.7819	4.6430	4.6145	4.5714	4.5437	3.8448
	1.5	3.2481	3.2307	3.2231	3.2169	3.2059	3.2008	3.1973	3.1773
	2	3.2173	3.2015	3.1757	3.1554	2.9862	2.9841	2.9773	2.9737
	2.5	3.0270	3.0126	3.0007	2.9901	2.9548	2.9373	2.9318	2.8157
	3	2.4730	2.4039	2.3803	2.3757	2.3644	2.3614	2.3428	2.2842
λ=0.10	0	379.8407	379.6124	378.8632	378.0722	379.3710	379.0972	378.6395	378.4375
	0.2	60.6984	60.2625	59.8030	59.6964	59.4875	59.1254	59.0789	57.8617
	0.4	14.4957	14.3484	14.3302	14.1308	14.0580	13.1914	12.8484	12.6284
	0.6	11.3284	11.1039	10.9370	10.6301	10.0323	9.8197	9.6430	9.5733
	0.8	8.4848	8.3730	8.3631	8.2372	8.0971	8.0418	8.0317	7.8797
	1	5.9369	5.8481	4.8287	4.6486	4.6484	4.5973	4.5488	4.3445
	1.5	4.0281	3.9844	3.9757	3.7264	3.6824	3.6805	3.6422	3.6420
	2	3.3548	3.3391	3.3284	3.3219	3.3169	3.3127	3.3107	3.3072
	2.5	3.1436	3.1375	3.1262	3.1171	3.1048	3.1034	3.0891	3.0842
	3	2.7578	2.7575	2.6784	2.6637	2.6548	2.6450	2.6220	2.4312
λ=0.12	0	380.6445	380.5486	380.2864	379.8122	379.8401	379.5955	379.5373	379.4244
	0.2	61.6393	61.4393	61.1887	60.9932	60.8489	60.6373	60.5197	59.7759
	0.4	14.5209	14.5073	14.3786	14.1578	14.1275	14.1037	14.0912	12.8193
	0.6	12.3720	11.4802	11.4551	10.6890	10.6253	10.2289	10.1884	10.0537
	0.8	8.5084	8.4377	8.4346	8.4306	8.4048	8.3288	8.2002	7.9182
	1	6.1712	6.1484	5.8464	5.6882	5.2846	5.1615	4.8480	4.4284
	1.5	4.3515	4.3200	4.3152	3.9622	3.9370	3.8937	3.8930	3.6448
	2	3.4573	3.4418	3.4275	3.4237	3.4223	3.4180	3.4104	3.4068
	2.5	3.3978	3.3635	3.3575	3.3573	3.3487	3.3434	3.3323	3.3271
	3	2.6880	2.6698	2.6424	2.6410	2.6343	2.6284	2.6127	2.4984
λ=0.15	0	381.2848	380.7848	380.7575	380.4370	380.3726	380.2548	379.7188	379.8971
	0.2	61.7828	61.5579	61.2068	61.0372	60.9700	60.8482	60.6410	60.2864
	0.4	14.5270	14.5259	14.4848	14.2337	14.1436	14.1273	14.1015	13.9828
	0.6	12.5937	12.4812	12.3788	12.1993	10.7373	10.4870	10.3533	10.1701
	0.8	8.8641	8.7843	8.7004	8.6575	8.5910	8.4934	8.4346	8.1482
	1	6.4030	6.2706	6.0628	5.8181	5.7120	5.5575	5.0548	4.4405
	1.5	4.4287	4.3733	4.3484	4.2884	4.2364	4.2162	4.2128	3.8626
	2	3.9334	3.9054	3.8998	3.8893	3.8809	3.8757	3.8648	3.8484
	2.5	3.5481	3.5407	3.5079	3.5004	3.4893	3.4893	3.4842	3.4840
	3	3.0336	3.0128	2.7489	2.7446	2.7353	2.7287	2.7281	2.7018
λ=0.20	0	382.6054	381.0557	383.2480	381.6848	381.1812	380.8028	380.1736	380.1733
	0.2	63.1889	61.7125	61.3248	61.1277	61.0240	60.9126	60.8181	60.3393
	0.4	14.5912	14.5772	14.5007	14.3754	14.3028	14.1757	14.1736	14.0164
	0.6	12.8754	12.5319	12.3793	12.2817	12.0734	12.0428	10.8880	10.8484
	0.8	8.9887	8.8328	8.7870	8.6917	8.6289	8.5778	8.5151	8.4754
	1	6.7510	6.4882	6.4812	6.3004	5.9640	5.8121	5.5173	5.4848
	1.5	4.5143	4.4484	4.4228	4.4007	4.2871	4.2612	4.2321	4.1818
	2	4.1917	4.1781	4.1542	4.1537	4.1488	4.1462	4.1425	4.1289
	2.5	3.9643	3.9375	3.9343	3.9120	3.9015	3.8848	3.8812	3.8717
	3	3.2861	3.2590	3.2575	3.2528	3.2528	3.2395	3.2371	3.2355

Table 9 - 14: ARL values for individual EWMA control charts for the Pareto distribution (m=150), with scaled weighted variance, with $\alpha = 0.0027$, for various positive shifts

λ	k	d=25, r=37	d=42, r=68	d=57, r=93	d=86, r=112	d=105, r=154	d=128, r=185	d=210, r=250	d=300, r=310
λ=0.05	0	376.3072	376.1827	375.7370	375.5070	375.3984	375.2816	375.8410	375.7201
	-0.2	57.0973	57.1406	57.2796	57.3773	57.4070	57.5340	57.7091	57.7324
	-0.4	12.0375	12.0600	12.2518	12.3617	12.4805	12.5017	12.8442	12.9579
	-0.6	7.7309	7.8812	8.0362	8.1048	8.1482	8.1982	8.2548	8.3450
	-0.8	6.3271	6.3401	6.3687	6.3732	6.4059	6.4648	6.4864	6.5359
	-1	3.2484	3.3501	3.3512	3.3537	3.3550	3.3552	3.3625	3.3645
	-1.5	2.8816	2.8843	2.8878	2.8912	2.8934	2.8981	2.9100	2.9300
	-2	2.5557	2.6379	2.6390	2.6415	2.6428	2.6448	2.6448	2.6823
	-2.5	2.4122	2.5054	2.5345	2.5573	2.6024	2.6202	2.6318	2.6486
	-3	2.1955	2.2017	2.2061	2.2157	2.2184	2.2198	2.2214	2.2284
λ=0.08	0	377.2600	377.2270	376.8937	376.7364	376.8281	376.6396	376.5155	376.1784
	-0.2	57.1015	57.2864	57.2889	57.5196	57.5259	57.7516	57.7990	57.8198
	-0.4	13.2224	13.4726	13.5357	13.6093	14.2512	14.8318	15.1275	15.2003
	-0.6	7.8419	8.2860	8.3243	8.3644	8.4680	8.4682	8.5464	8.6159
	-0.8	6.6448	6.6868	6.6870	6.6936	6.7041	6.7127	6.7128	6.7308
	-1	3.6206	3.6222	3.6248	3.6254	3.6287	3.6370	3.6444	3.6445
	-1.5	3.0548	3.2808	3.2814	3.2845	3.2860	3.3025	3.3099	3.3255
	-2	2.7272	2.7282	2.7301	2.7315	2.7319	2.7331	2.7364	2.7544
	-2.5	2.4846	2.5179	2.5439	2.6063	2.6198	2.6218	2.6480	2.6848
	-3	2.4040	2.4069	2.4077	2.4162	2.4170	2.4201	2.4246	2.4393
λ=0.10	0	379.8407	379.6124	378.8632	378.0722	379.3710	379.0972	378.6395	378.4375
	-0.2	59.0789	59.8030	59.9693	60.3175	60.4072	60.5195	60.8182	60.9772
	-0.4	13.3684	13.7899	13.8068	13.8572	14.3577	14.9790	15.3798	15.7257
	-0.6	8.1546	8.6884	9.1226	9.1712	9.1841	9.2035	9.3177	9.3648
	-0.8	6.6455	6.6881	6.7248	6.7324	6.7519	6.7848	6.7978	6.8844
	-1	3.8101	3.9812	3.9846	3.9900	3.9934	3.9936	4.0101	4.0272
	-1.5	3.4099	3.4354	3.4812	3.4848	3.4882	3.4889	3.4896	3.5462
	-2	2.8975	2.9041	2.9312	3.0426	3.0446	3.0489	3.0504	3.0536
	-2.5	2.8846	2.8846	2.8937	2.8991	2.9045	2.9064	2.9373	2.9544
	-3	2.5172	2.5573	2.6054	2.6177	2.6448	2.6890	2.7084	2.7506
λ=0.12	0	380.6445	380.5486	380.2864	379.8122	379.8401	379.5955	379.5373	379.4244
	-0.2	60.641	60.9126	60.97	60.9932	61.0484	61.4393	61.7201	62.0533
	-0.4	13.6014	14.0689	14.0864	14.1573	14.6888	15.125	15.1689	15.2737
	-0.6	8.4088	9.121	9.1937	9.3908	9.6844	9.8288	9.8723	9.9328
	-0.8	6.937	6.9373	6.9397	6.9553	6.9726	6.9932	6.9937	7.0284
	-1	4.0101	4.0124	4.0148	4.0161	4.0204	4.0375	4.0441	4.0703
	-1.5	3.4643	3.4691	3.759	3.7719	3.7784	3.7878	3.7933	3.8004
	-2	3.0936	3.2509	3.255	3.2595	3.2606	3.2641	3.268	3.2842
	-2.5	3.0015	3.012	3.0127	3.0337	3.0464	3.0627	3.1904	3.2716
	-3	2.5482	2.5575	2.6091	2.644	2.6484	2.7868	2.8632	2.905
λ=0.15	0	381.2848	380.7848	380.7575	380.437	380.3726	380.2548	379.7188	379.8971
	-0.2	61.6037	61.6048	61.8482	61.8489	62.2068	62.3044	62.5579	63.1248
	-0.4	13.8689	14.1402	14.1775	14.2624	14.8253	15.2573	15.3825	15.7005
	-0.6	9.0484	9.1248	9.2481	9.5439	9.7515	9.8407	9.9307	10.0023
	-0.8	7.5148	7.5577	7.5591	7.5796	7.6148	7.6248	7.6393	7.6984
	-1	4.1206	4.1243	4.1243	4.1255	4.1284	4.1481	4.1637	4.1848
	-1.5	3.8221	3.8284	3.84	3.8412	3.8412	3.8733	3.8848	3.9073
	-2	3.3321	3.5488	3.571	3.5723	3.5731	3.5757	3.5939	3.6048
	-2.5	3.3068	3.3324	3.3421	3.345	3.3572	3.3617	3.3702	3.4006
	-3	2.9069	2.9112	2.9395	2.958	2.9737	2.9837	2.9988	3.003
λ=0.20	0	382.6054	381.0557	383.248	381.6848	381.1812	380.8028	380.1736	380.1733
	-0.2	61.9782	61.9802	62.7771	63.0536	63.0612	63.2737	63.6828	63.9578
	-0.4	14.0737	14.4378	14.6525	14.7082	15.3959	16.2164	16.4641	16.773
	-0.6	9.1021	9.1406	9.6425	9.7357	9.8163	9.9875	10.0548	10.2034
	-0.8	7.5281	7.5702	7.5784	7.6051	7.6173	7.6484	7.6932	7.7369
	-1	4.5788	4.5791	4.597	4.6084	4.6435	5.127	5.1284	5.1593
	-1.5	4.4084	4.422	4.4284	4.4307	4.4323	4.4391	4.4433	4.4886
	-2	3.8104	3.8155	3.8188	3.8264	3.8424	3.8431	3.8486	3.8843
	-2.5	3.4823	3.5173	3.5377	3.539	3.5414	3.5432	3.806	3.8454
	-3	2.9542	2.9698	2.979	3.0298	3.0788	3.1045	3.1845	3.2457

Table 9 - 15: ARL values for individual EWMA control charts for the Pareto distribution (m=150), with scaled weighted variance, with α = 0.0027, for various negative shifts

Comparing those two tables, we observe that the proposed control chart can detect both positive and negative shifts well, but there are some differences in ARL values between those two tables, with most of the differences being in favour of the ARL values for positive shifts. The only cases for which the ARL values for negative shifts are bigger are for values of k less than 0.4 for large values of the distribution's parameters and for all values of the distribution's parameters for shifts of magnitude k equal to or greater than 2.5 when lambda is very small (up to 0.05).

When comparing Table 9-15 with Table 9-4 and Table 9-16 with Table 9-5 the improvement of the performance of the control chart when using the scaled weighted variance instead of the skewness correction method is revealed. The in-control ARL values for the case of using the scaled weighted variance are greater than the corresponding ones for the case of using the skewness correction method, while the out-of-control ARL values are smaller for the scaled weighted variance than for the skewness correction method. The differences between the ARL values are almost all higher than 5% for either positive or negative shifts.

9.9.5 Example on the Pareto individual Shewhart-type and EWMA control charts with scaled weighted variance using simulated data

This section is dedicated to the illustration of the proposed control charts by means of simulated data generated from the Pareto distribution. The case of real data will be covered in section 9.9.6. For the same dataset as in Table 9-9 we construct the individual Shewhart-type Pareto control charts with scaled weighted variance presented in Figure 9-10, using the most commonly used value for the significance level α = 0.27%, as mentioned earlier. As we can see in Figure 9-10, there is an increasing trend after the first 15 observations and the control chart detects out-of-control points indicating that an assignable cause has occurred in the process causing its mean to shift to an out-of-control level sooner than the corresponding control chart with skewness correction in Figure 9-2.

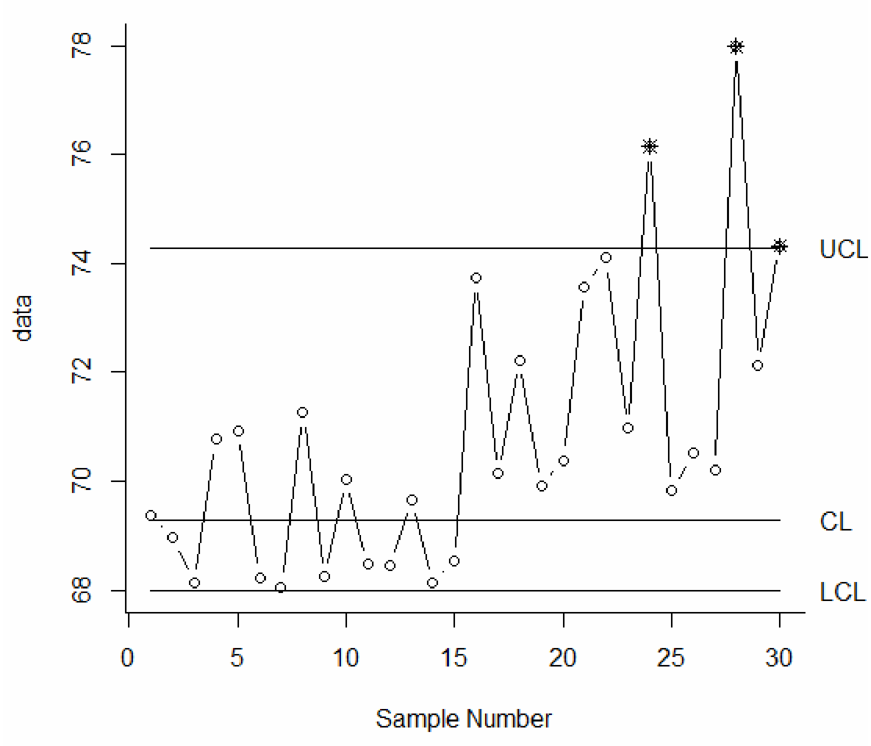

Figure 9 - 10: Individual Pareto control chart with scaled weighted variance for the data set in Table 9-9 with a shift of one standard deviation unit in the process mean

Using the data set in Table 9-9 for the case of a shift of one standard deviation unit, we now construct the individual EWMA Pareto control chart with scaled weighted variance shown in Figure 9-11, using $\lambda=0.05$. As we can see, there is an increasing trend after the first 15 observations and the control chart gives an out-of-control signal after the 20^{th} observation which, compared to Figure 9-10, is sooner than the individual control chart with scaled weighted variance, as expected, and compared to Figure 9-3 it is also sooner than it was detected by the EWMA control chart with the skewness correction method.

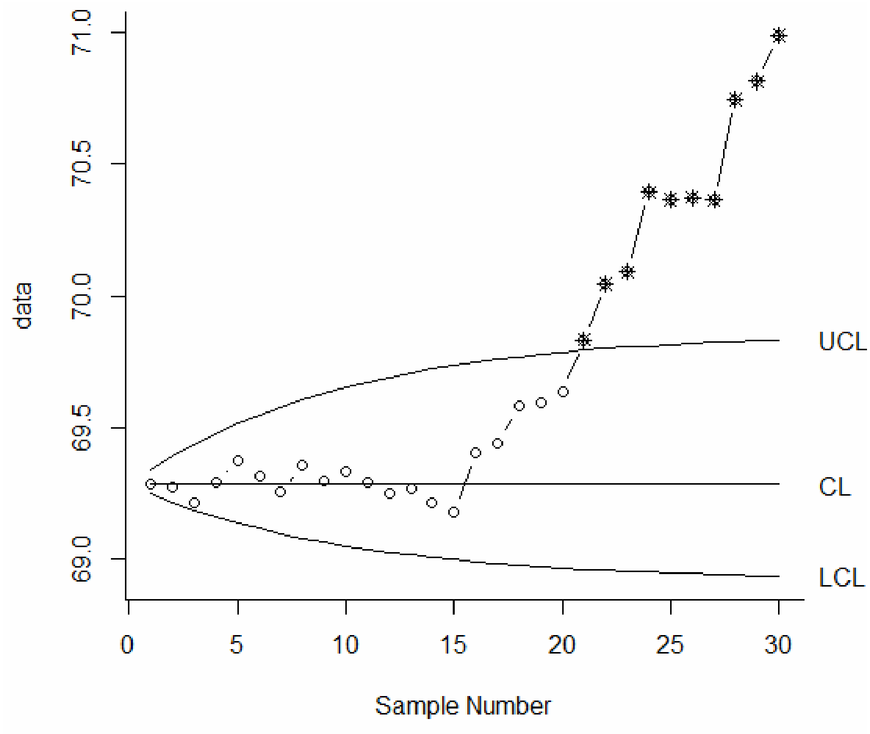

Figure 9 - 11: Individual EWMA Pareto control chart with scaled weighted variance for the data set in Table 9-9 with a shift of one standard deviation unit in the process mean

9.9.6 Application of the Pareto individual Shewhart-type and EWMA control charts with scaled weighted variance to real data

This section discusses the illustration of the proposed control charts through application to the same real datasets as earlier (Tables 9-10 and 9-11) and for the same values of the parameters of our assumed Pareto distribution. For the first dataset (Table 9-10) the distribution's parameters are once again equal to 3.9728 and 242.9444 for d and r, respectively. For the construction of the control charts, the significance level value α = 0.27% has been chosen. The resulting control chart for the first dataset can be seen in Figure 9-12 which presents an out-of-control point which was not detected by the corresponding control chart with skewness correction.

For the construction of the individual EWMA control chart for our data, using the same parameter values of the assumed Pareto distribution from the data in conjunction with λ=0.05, the resulting control chart can be seen in Figure 9-13, which does not detect any out-of-control observations. This is probably due to the inertia effect we mentioned in Section 2-14, because the small λ value gives bigger weight to the past smaller values and reacts slower to the shift in the opposite direction. The large value is then followed by a few small values and therefore the chart does not detect the shift.

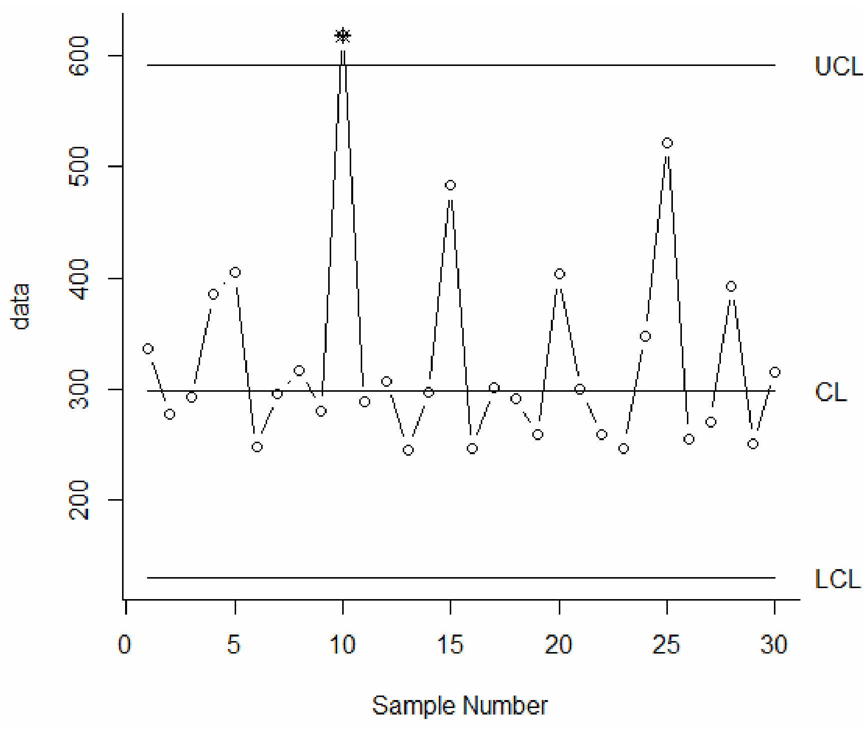

Figure 9 - 12: Individual Pareto control chart with scaled weighted variance for the Condroz calcium data set in Table 9-10

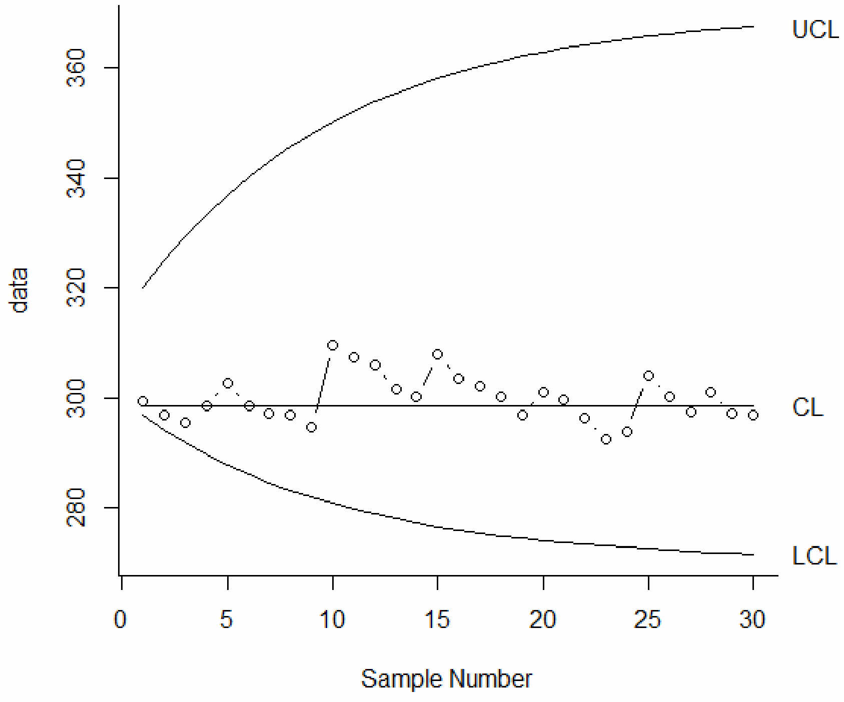

Figure 9 - 13: Individual EWMA Pareto control chart with scaled weighted variance for the Condroz calcium data set in Table 9-10

Now, let's deal with the second data set which was presented earlier in Table 9-11. The significance level is chosen to be equal to the value α=0.27%. The resulting individual Pareto control chart with scaled weighted variance can be seen in Figure 9-14 which also detects the out-of-control point.

For the construction of the individual EWMA control chart for our data, using the same parameter values of the assumed Pareto distribution from the data in conjunction with the value of λ=0.05, the resulting control chart can be seen in Figure 9-15, which, once again, does not detect the out-of-control state of the process, probably due to the small λ value which gives bigger weight to the past much bigger values.

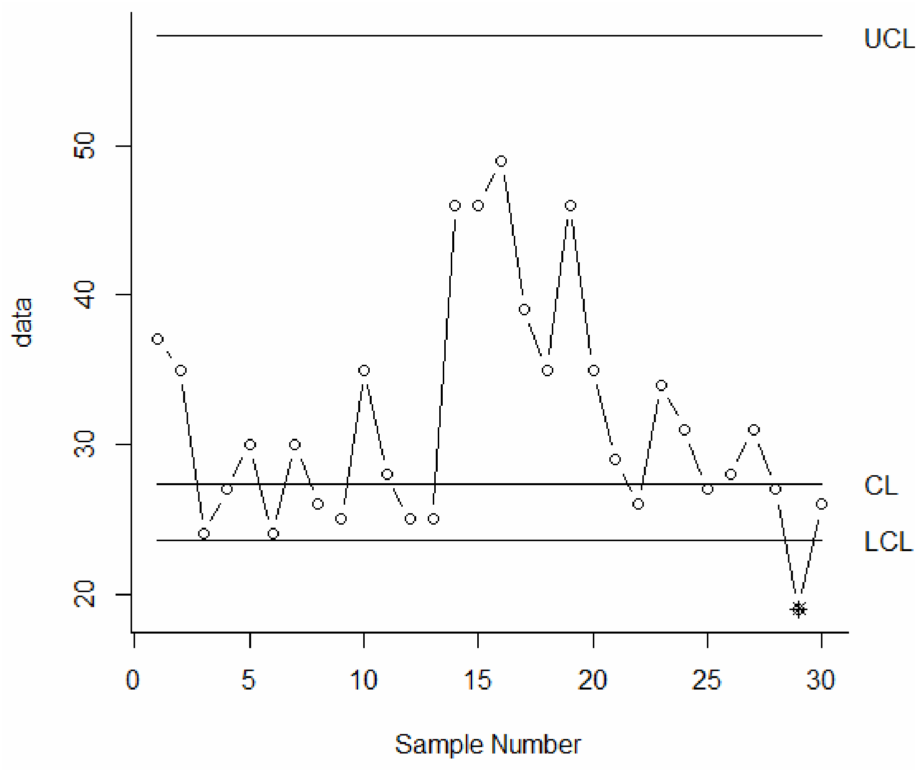

Figure 9 - 14: Individual Pareto control chart with scaled weighted variance for the breaking angles data set of Table 9-11

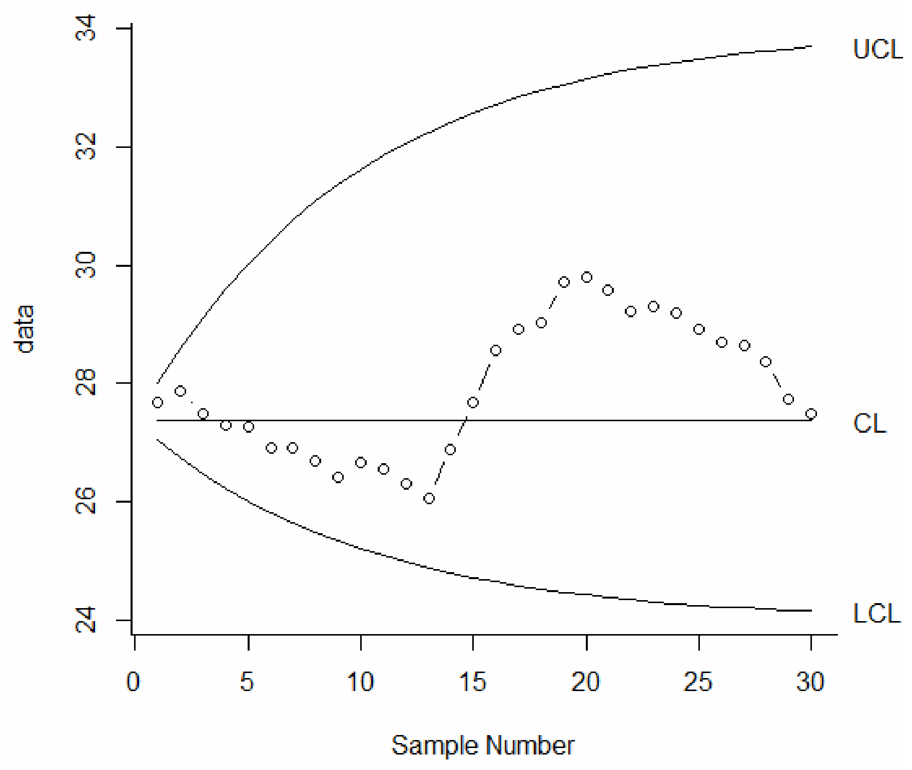

Figure 9 - 15: Individual EWMA Pareto control chart with scaled weighted variance for the breaking angles data set of Table 9-11

9.10 Conclusions and Further Research

In this chapter probability-type, Shewhart-type and EWMA control charts have been constructed for monitoring individual observations from a process which is assumed to follow the Pareto distribution for the theoretical scenario of known distributions' parameters. Two different methods for taking into account the distribution's skewness have been considered. The performance of the proposed control charts has been investigated for the cases of all the proposed control charts (probability-type, Shewhart-type and EWMA control charts with both skewness correction methods). Optimal design for the EWMA control chart has also been presented. The five types of proposed control charts have been illustrated with both simulated and real data.

The proposed control charts take into account the skewness of the distribution and this leads to a significant improvement of their performance as has been demonstrated along this chapter. The performance of the control charts seems to improve more when the scaled weighted variance method by Castagliola (2000) is used instead of the skewness correction method proposed by Chan and Cui (2003).

This study can also be applied to other Lindley-related distributions (generalizations, mixtures, transformations, etc.). Furthermore, for future research, the whole analysis can be extended to include supplementary runs rules for the detection of small shifts. For this purpose it would also be useful to construct CUSUM control charts for the Pareto distribution, as well.

CHAPTER 10

CONCLUSIONS AND FURTHER RESEARCH

The concept of quality is essential in every aspect of our everyday lives and it is of major need to keep it at the best possible level. This purpose is accomplished through Statistical Process Control and control charts play the most crucial role in this effort. Therefore, an overview of the literature on statistical process control charts was presented in the present essay covering the various types of control charts proposed over the years beginning from the original Shewhart control charts and proceeding to their modifications and alternatives (such as the CUSUM and EWMA charts). The basic assumptions considered when those control charts were originally proposed were covered in the present study. Special emphasis was given on control charts for individual observations as well as the assumption of Normality which is usually violated in real life situations. Control charts have been proposed in the literature for various non-Normal distributions, but there are still some distributions with many applications in real life which were not covered as far as SPC is concerned. This gap was filled with the present thesis. Examples of those distributions include the Lindley and Lindley-related distributions and the Logarithmic distribution. Pareto distribution is also a distribution which presents an increasing interest recently in the field of SPC, but there are still a lot of possibilities for new work. These were the motivations for the present thesis. The first part of the current study was completed with an overview of the research for the aforementioned distributions in order to reveal what has already been done for them and the lack of efforts regarding control charts for these distributions.

Individual observations are very common in our everyday lives, as was presented in the introduction to the second part of this thesis. Therefore, control charts were constructed herein for individual observations from the one-parameter and two-parameter Lindley distributions, as well as the

Logarithmic and Pareto distributions. First of all probability-type control charts were constructed. Then Shewhart-type and EWMA control charts were proposed using the skewness correction method proposed by Chan and Cui (2003) in order to improve the performance of the charts without it. The corrected Shewhart-type charts with this method were proved to perform better than the probability-type ones and the corrected EWMA charts were proved to have better performance than the corrected Shewhart-type charts. Optimal design of the corrected EWMA charts for all the distributions was also discussed. The performance of all the charts was investigated and illustrated with both simulated and real data. Then another method for taking into account each distribution's skewness was considered. This was the scaled weighted variance method proposed by Castagliola (2000). Shewhart-type and EWMA charts were constructed using this method, too, and their performance was compared with the corresponding charts with the other skewness correction method. These comparisons along with the illustrations of the proposed charts with the same simulated and real data revealed the superiority of the scaled weighted variance method.

This dissertation contributes to SPC literature in several ways. First of all, control charts are created for distributions with many applications in real life for which control charts had not been addressed (Logarithmic and Lindley-related distributions) and new methods for constructing control charts have been proposed for the case of the Pareto distribution. Moreover, comparison of two different methods for taking into account each distribution's skewness has been considered herein, which had not been conducted earlier in literature for discrete distributions, since the scaled weighted variance method by Castagliola (2000) was applied only to continuous distributions. Furthermore, the first part of chapter 7 regarding probability-type charts for the two-parameter Lindley distribution and Shewhart-type and EWMA charts with the skewness correction method by Chan and Cui (2003) has already been published [Demertzi and Psarakis (2024)] contributing to the existing literature on control charts for skewed distributions.

In addition, this thesis creates the need for further research. Firstly, this study was concentrated on the theoretical case of known distributions' parameters. This, however, is not usually the case in real life situations. Therefore, the proposed control charts should also be studied for the case of estimated parameters and the effect of parameter estimation on the charts' performance should be investigated.

Furthermore, the present study can also be applied to other distributions related to the ones chosen here (generalizations, mixtures, transformations, etc.). Moreover, for future research, the whole analysis can be extended to include supplementary runs rules (but not with individual data due to risk of high false alarm rate) for the detection of small shifts. For this purpose it would also be useful to construct CUSUM control charts for the distributions of concern. Shewhart-EWMA and Shewhart-CUSUM charts might also be interesting to be developed for the specific distributions, since they have been proved in the literature to be effective in overall detection of small and large shifts.

Additionally, the design of control charts presented so far was purely statistical, meaning that the construction of the control charts was based on the underlying distribution of our data. In practice, however, it is often needed to design control charts considering the economic point of view, too, so as to minimize some function of the costs of sampling and testing, producing items which are not conforming to the specifications, repairing, false alarms and assignable causes' detection and elimination. Therefore, an economic-statistical design for the proposed control charts might also be interesting to be developed. Last but not least, as it was mentioned from the very beginning, this essay was focused on the univariate case, so the whole study can be further extended to cover the case of more dimensions, too.

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