

# NORMALITY CHECKS OF POSTERIOR PREDICTIVE DIAGNOSTICS

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## Objectives

- Review of posterior predictive diagnostics.
- The predictive posterior distribution in a closed form produced by a normal likelihood.
- Graphical checks.
- Study of certain statistic for checking the normality using posterior predictive diagnostics.

## Normal Model

- **Likelihood:**  $y_i \sim Normal(\mu, \sigma^2)$ .
- **Priors:**
  - $\mu/\sigma^2 \sim N(0, \sigma^2/n_0)$
  - $1/\sigma^2 \sim Gamma(u_0/2, u_0 S_0^2/2)$ .
- $\phi = 1/\sigma^2$ .
- $\phi|data \sim Gamma(u_n/2, u_n S_n^2/2)$  where  $u_n = n + u_0$ ,  $n_n = n_0 + n$  and  $S_n^2 = ((n - 1)S^2 + n_0 S_0^2 + \bar{y}^2 n n_0 / n_n) / u_n$
- $\mu|\phi, data \sim N(m_n, (\phi n_n)^{-1})$  where  $m_n = n\bar{y}/(n + n_0)$  and  $n_n = n + n_0$ .

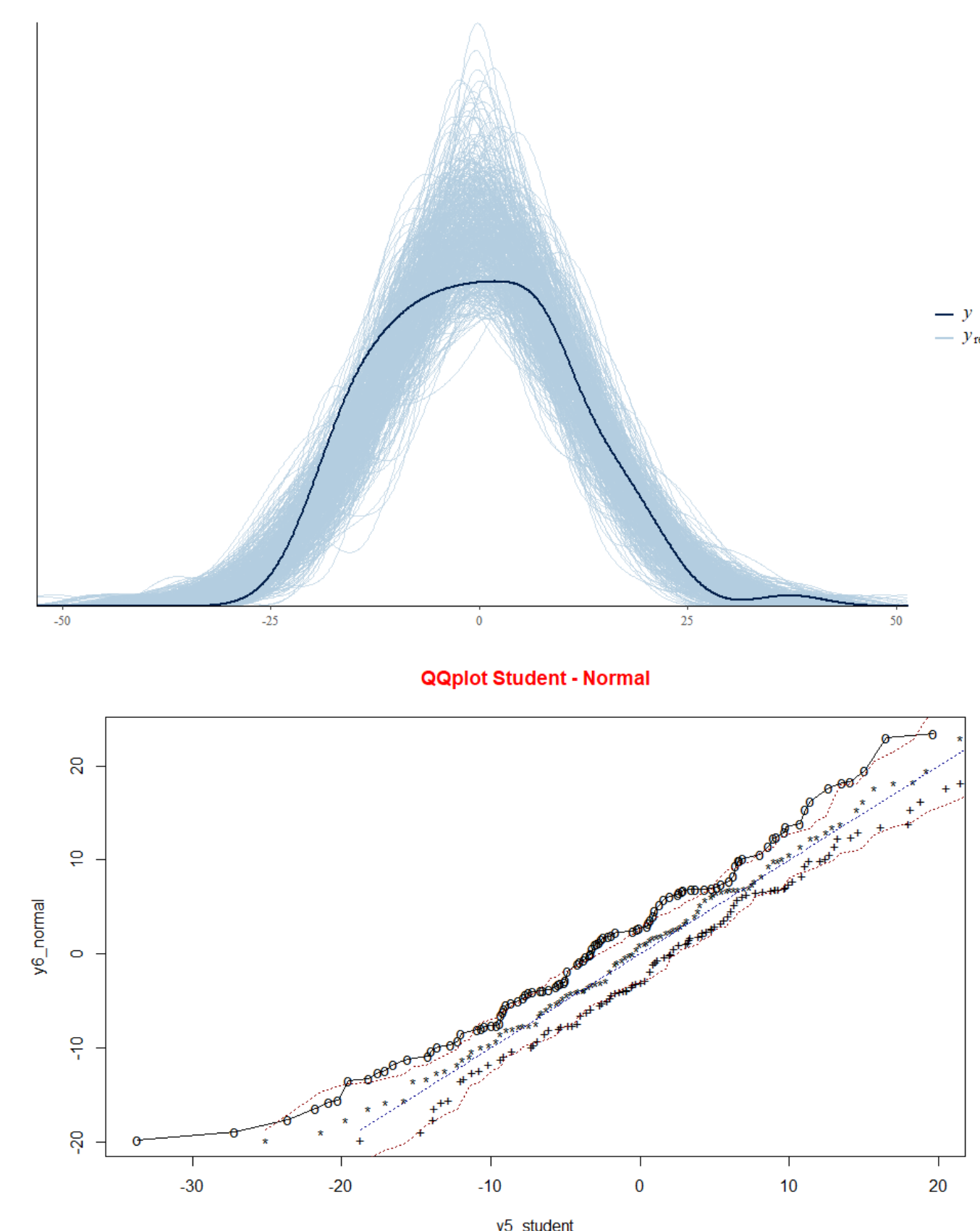
## The Posterior Predictive Distribution

$$t(u_n, m_n, S_n^2(1 + 1/n_n))$$

## Example with Normal data and some graphical checks

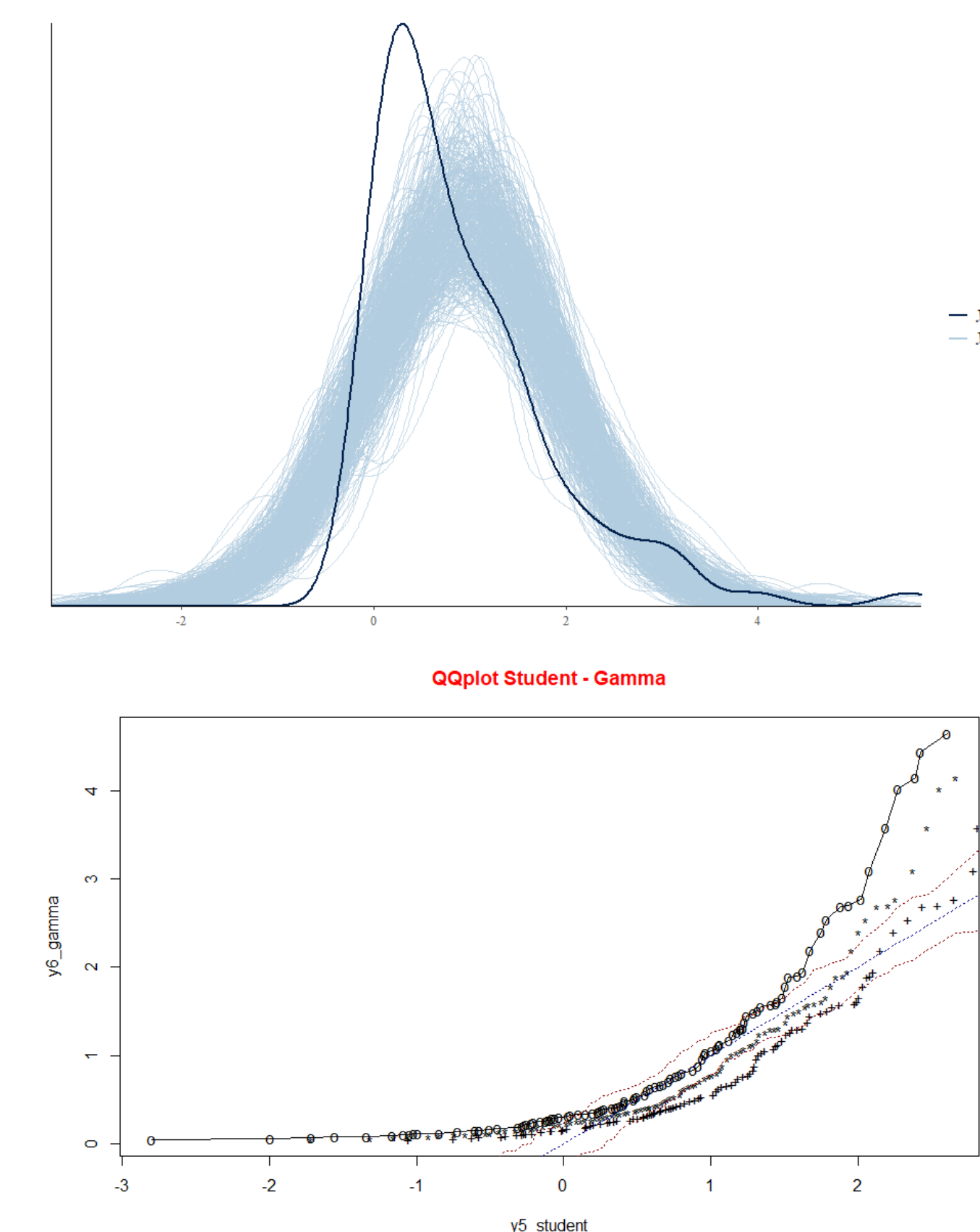
$y_i \sim Normal(0, 100)$ .  
and  $n = 100, n_0 = 1, u_0 = 2, s_0^2 = 1$   
predictive posterior distribution:  
 $t(102, 0.8, 99.1)$

## Graph of $y$ vs $y_{rep}$ and QQplot



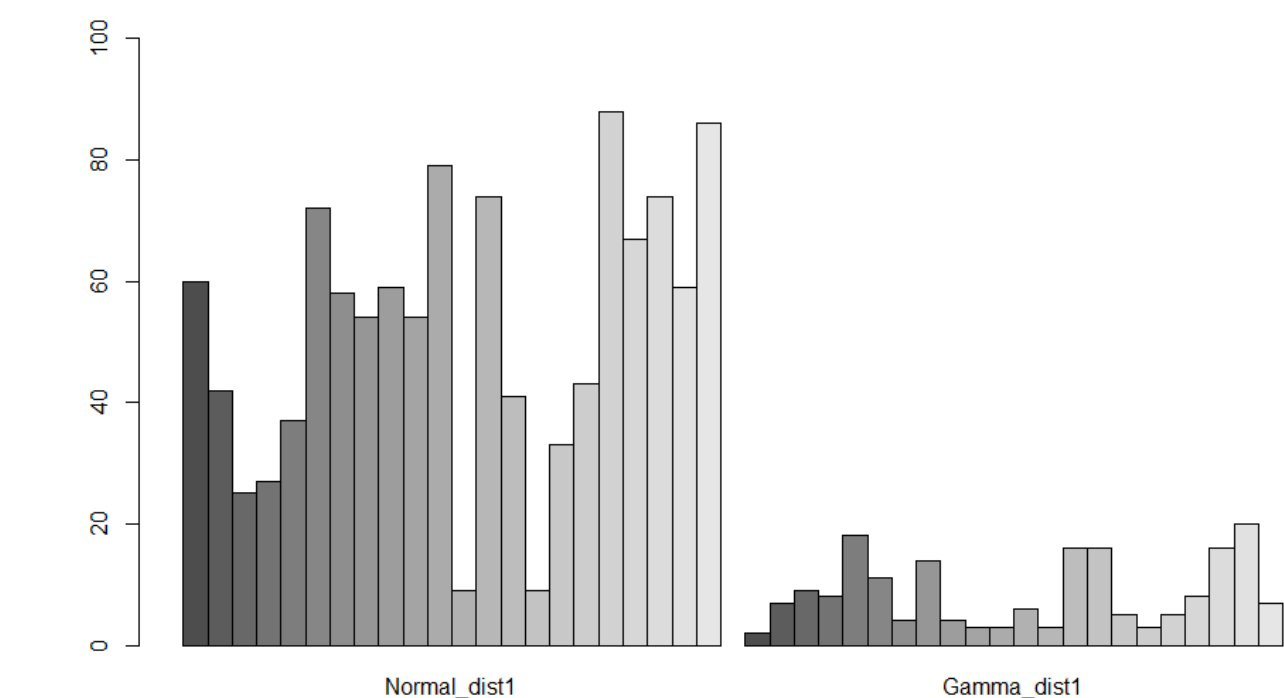
## Example with data from a gamma distribution

Student predictive posterior vs.  $gamma(1,1)$ .



## Study of certain statistic for checking the normality using posterior predictive diagnostics

Distance calculation of sample data with those from the predictive posterior at quantiles' level. Then comparison of those distances and +1, if the quantile's distance of the first sample is higher than the second or -1 for the opposite.



## Future Work

- Finding a function which will propose a specific distribution for every using sample.
- Finding a normality check model process.

## Bibliography

- Ioannis Ntzoufras (2009) Bayesian Modeling Using Winbugs (John Wiley Sons)
- Andrew Gelman (2013) Bayesian Data Analysis (Chapman & Hall)
- F Javier Rubio and Jairo Fuquene (2017) Posterior QQ envelopes: normality test