AN EWMA-TYPE CHART BASED ON SIGNED RANKS WITH EXACT RUN LENGTH PROPERTIES

T. Perdikis[a] , S. Psarakis[a] P. Castagliola[b] , G. Celano[c]

[a] Athens University of Economics and Business, Athens, Greece; [b]Université de Nantes, Nantes, France; [c]Università di Catania, Catania, Italy

Goal of this work

We propose [1] an EWMA-type chart based on the Wilcoxon signed rank (WSR) statistic where:

- The *general* distribution of WSR is derived without any approximation
- *Exact* in- and out-of-control performances are provided

The WSR statistic

- The WSR statistic SR_t is equal to

$$\mathrm{SR}_t = \sum_{j=1}^{n} \mathrm{sign}(X_{t,j} - \theta_0) L_{t,j}$$

- Additionally: $\mathrm{SR}_t = 2\mathrm{SR}_t^+ - \frac{n(n+1)}{2}$,
- The p.g.f., $G_{\mathrm{SR}_t^+}(\omega)$ is ([2]):

$$G_{\mathrm{SR}_t^+}(\omega) = \prod_{i=1}^{n} (p\omega^i + (1-p))$$

- The p.m.f. $f_{\mathrm{SR}_t^+}(s|n,p)$ is:

$$f_{\mathrm{SR}_t^+}(s|n,p) = \frac{1}{s!} G^{(s)}_{\mathrm{SR}_t^+}(\omega)\Big|_{\omega=0}$$

where

$$\frac{1}{s!} G^{(s)}_{\mathrm{SR}_t^+}(\omega) = \sum_{j=0}^{\frac{n(n+1)}{2}-s} c_{s,j} w^j$$

Phase II implementation

X a quality characteristic from an continuous distribution and θ the location parameter of interest.

- Process is in-control $\mathrm{P}(X_{t,j} > \theta_0 | \theta = \theta_0) = \mathrm{P}(X_{t,j} < \theta_0 | \theta = \theta_0) = p_0$
- Otherwise $p_1 = \mathrm{P}(X_{t,j} > \theta_0 | \theta = \theta_1) = 1 - F_X(\theta_0|\theta_1)$

Proposed chart

- Similarly with [3], the plotting statistic, Y_t, is obtained through:

$$(\gamma_x + \gamma_y) Y_t + R_t = \gamma_x \mathrm{SR}_t + \underbrace{\gamma_y Y_{t-1} + R_{t-1}}_{B_{t-1}}$$

- $(\gamma_x, \gamma_y) \in \mathbb{N}^2$
- $R_t = \gamma_x \mathrm{SR}_t + B_{t-1} - (\gamma_x + \gamma_y) Y_t$
- $Y_t = \left\lfloor \frac{\gamma_x \mathrm{SR}_t + B_{t-1}}{\gamma_x + \gamma_y} \right\rceil, Y_0 = R_0 = 0$
- Signals if $Y_t < -K$ or $Y_t > K$

Run Length properties

- $\mathrm{ARL} = \mathbf{q}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}$

$$\mathbf{Q} = (q)_{i,j} = f_{\mathrm{SR}^+}\left(\frac{\mathrm{SR}_t + \frac{n(n+1)}{2}}{2} \Big| n, p_1\right)$$

Optimization

For $\mathrm{ARL}_0 = 370.4, p_0 = 0.5$, the optimal vector of design parameters $(\mathbf{v}^* = K^*, \gamma_x^*, \gamma_y^*)$ is obtained as:

$$\mathbf{v}^* = \underset{\mathbf{v}}{\mathrm{argmin}} \mathrm{ARL}(\mathbf{v}, n, p_1) \quad (1)$$

$$\mathrm{ARL}(\mathbf{v}^*, n, p_0) = \mathrm{ARL}_0 \quad (2)$$

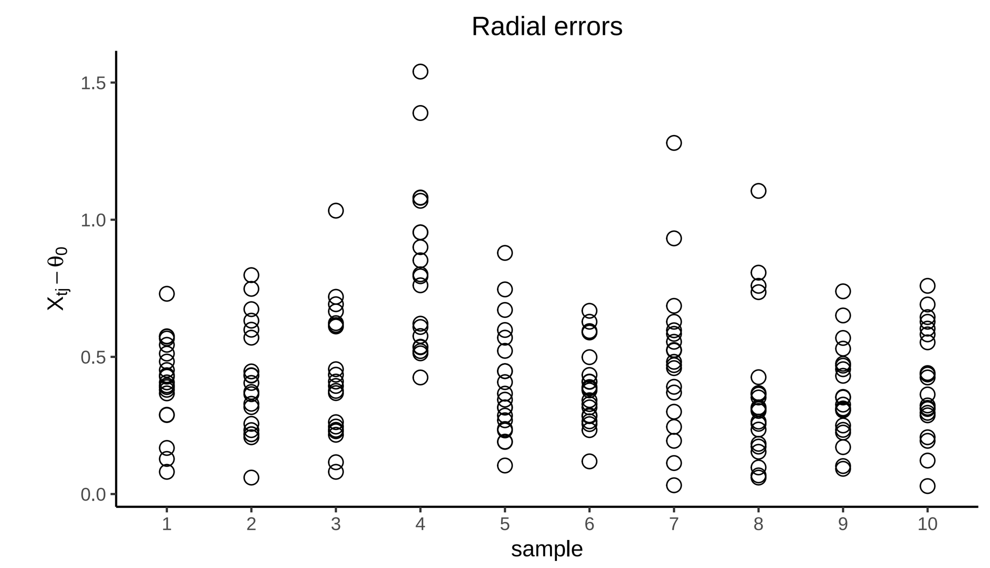
and

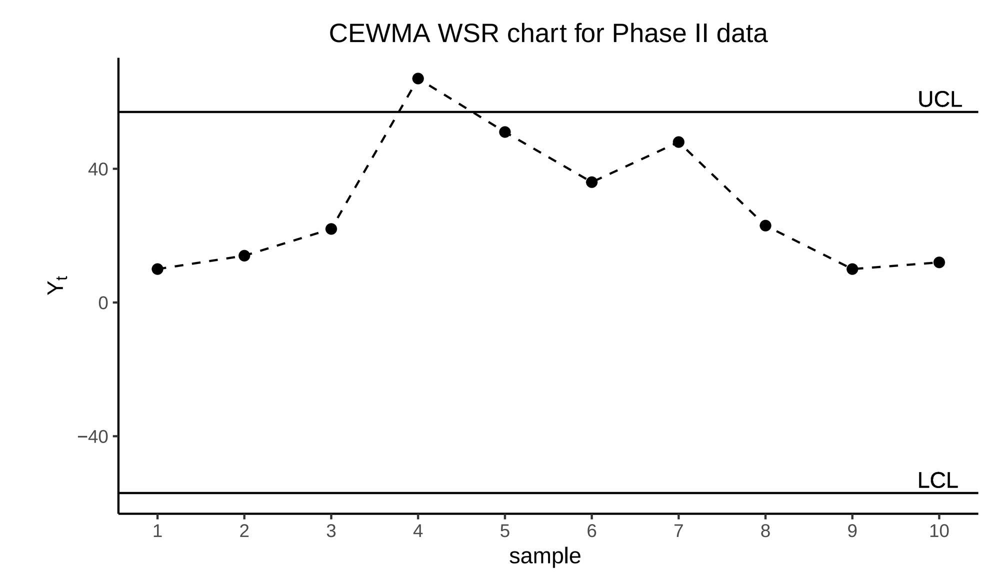
$$\frac{|\mathrm{ARL}(\mathbf{v}^*, n, p_0) - \mathrm{ARL}_0|}{\mathrm{ARL}_0} \leq \delta \quad (3)$$

Performance Comparisons

p	CEWMA WSR	CEWMA SN	EWMA	A-EWMA	CUSUM
0.50	370.6	370.2	370.0	367.6	373.7
0.45	84.7	37.3	31.0	**30.9**	40.3
0.40	20.2	**11.4**	12.3	12.2	11.7
0.30	4.9	**4.5**	5.6	5.4	4.5
0.25	**3.3**	3.5	4.4	4.2	3.5
0.20	**2.6**	2.9	3.7	3.5	2.9
0.15	**2.1**	2.4	3.2	2.9	2.4
0.10	**1.8**	2.1	2.9	2.5	2.1
0.05	**1.5**	2.0	2.6	2.1	2.0
γ_X	3	3	-	-	-
γ_Y	5	16	-	-	-
K	75	4	-	-	-
λ	-	-	0.05	0.05	-
W	-	-	2.487	2.487	-
k	-	-	-	-	1.0
h	-	-	-	-	11.62

Illustrative example





References

[1] T. Perdikis, S. Psarakis, P. Castagliola, and G. Celano. An ewma-type chart based on signed ranks with exact run length properties. *Journal of Statistical Computation and Simulation*, 0(0):1–20, 2020. doi: 10.1080/00949655.2020.1828415.

[2] B.M. Bennett. On the Non-Null Distribution of wilcoxon's Signed Rank Test. *Metrika*, 19(1):36–38, 1972.

[3] P. Castagliola, K.P. Tran, G. Celano, A.C. Rakitzis, and P.E. Maravelakis. An Ewma-Type Sign Shart with Exact Run Length Properties. *Journal of Quality Technology*, 51(1):51–63, 2019.