Control Charts and Affine Term Structure Models

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Main Purpose of this work

- Application of statistical process monitoring (SPM) methods for the detection of changes in the yield curve and the parameters governing affine term structure models (ATSMs).

Affine Term Structure Model

- $\boldsymbol{X_{t+1}} = \mathbf{c} + \boldsymbol{\phi}\mathbf{X_t} + \mathbf{\Sigma u_{t+1}}$, $\boldsymbol{u_{t+1}} \sim N(0, \boldsymbol{I})$:State evolution process

$$\boldsymbol{X_{t+1}} = \boldsymbol{c^P} + \boldsymbol{\phi^P X_t} + \boldsymbol{\Sigma u^P_{t+1}}$$

$$\boldsymbol{X_{t+1}} = \boldsymbol{c^Q} + \boldsymbol{\phi^Q X_t} + \boldsymbol{\Sigma u^Q_{t+1}}$$

- $r_t = \delta_0 + \boldsymbol{\delta_1^{'} X_t}$: Short rate
- $M_{t,t+1} = exp(-r_t - \frac{1}{2}\boldsymbol{\lambda_t^{'}\lambda_t} - \boldsymbol{\lambda_t^{'} u_{t+1}})$: Pricing kernel
- $\lambda_t = \lambda_0 + \boldsymbol{\lambda_1 X_t}$: Market prices of risk
- $P_t^n = exp(\alpha_n + \boldsymbol{b_n^{'} X_t})$: Bond price
- $Y_t^n = -\frac{log(P_t^n)}{n} = A_n + \boldsymbol{B_n^{'} X_t}$: Yield of a n-period zero coupon bond
- $\boldsymbol{\theta} = (\mathbf{c}, \boldsymbol{\phi}, \mathbf{c}^Q, \boldsymbol{\phi}^Q, \boldsymbol{\Sigma}, \delta_0, \boldsymbol{\delta}_1)$: set of parameters for estimation

Data

- US bond yields of maturities 3-,24-,36-,48-,60-,120- months from April 1991 to December 2009.In-sample period: April,1991 to December,2000.
- Consumer Price Index (CPI). Industrial Production Index (IP) growth rate.
  Three latent factors.
- We estimate the model parameters using the MCSE approach

Sequential Monitoring

- Instead of monitoring directly the vector of parameters $\boldsymbol{\theta}$ we monitor the bond yield process $\mathbf{Y_t^n}$.
- Suppose $\boldsymbol{Y_t^{n,*}} = [Y_t^{1,*}, \ldots, Y_t^{N,*}]$ is the vector of observed zero-coupon bond yields at time t with maturity $n = 1, 2, \ldots, N$.
- Vector of residuals:

$$\boldsymbol{d_t} = \mathbf{Y_t}^{n,*} - E_0(\boldsymbol{Y_t^{n,*}} / \boldsymbol{Y_{t-1}}^{\boldsymbol{n,*}})$$

- Under the in-control condition:

$$E_0(\boldsymbol{Y_t^{n*}}) = E_0(\boldsymbol{Y_t^n})$$
$$= A_n + \boldsymbol{B_n^{'} X_{0,t}}$$

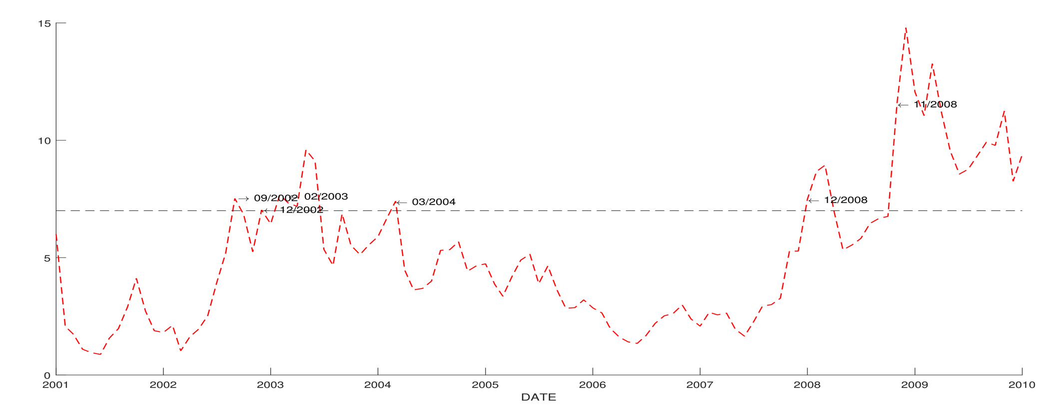
Control chart procedures

- Modified EWMA charts based on
  - Mahalanobis distance (ModMah)
  - MEWMA Statistic (ModMEWMA)
- Residual based control charts
  - EWMA chart based on the Mahalanobis distance (ResMah)
  - Control chart based on the MEWMA Statistic (ResMEWMA)
- MCUSUM chart
- MMOEWMA chart.

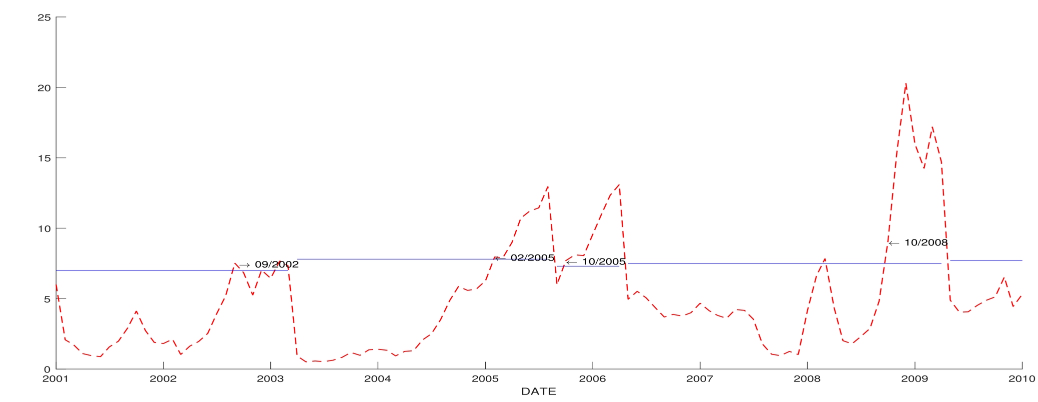
- The EWMA statistic based on the Mahalanobis distance $T_{t,n}$ is given by
  $Z_{t,n} = (1-\lambda)Z_{t-1,n} + \lambda T_{t,n}, \ t \geq 1$
  with $Z_{0,n} = N$ and $\lambda \in (0, 1]$.
- A signal is given if $Z_{t,n} > h$,$h > 0$.
- The Mahalanobis distance is :
  $\mathrm{T}_{t,n} = (\boldsymbol{Y_t^{n,*}} - \boldsymbol{\mu}_t)^{'} \boldsymbol{Cov_0}(\boldsymbol{Y_t^{n,*}})^{-1} (\boldsymbol{Y_t^{n,*}} - \boldsymbol{\mu}_t), t \geq 1$
- The in-control mean $\boldsymbol{\mu_t}$ is
  $E_0(\boldsymbol{Y_t^{n,*}}) = A_n + \boldsymbol{B_n}(\boldsymbol{\mu^Q} + \boldsymbol{\phi^Q} E_0(\boldsymbol{X_{t+1}}))$
- The in-control covariance matrix is
  $\mathrm{Cov}_0(\boldsymbol{Y_t^{n,*}}) = \boldsymbol{B_n}(\boldsymbol{\phi^Q} Var_0(\boldsymbol{X_t}) \boldsymbol{\phi^{Q'}} + \boldsymbol{\Sigma}) \boldsymbol{B_n} + \boldsymbol{U}$
  where $\boldsymbol{U} = E(\boldsymbol{u_t u_t^{'}})$

Empirical Example

- EWMA control chart based on the Mahalanobis distance for $\lambda = 0.9$.



- We perform the two sample T-test for the equality of the means with unequal variances and if the results of the test confirm the change we estimate the ATSM for an estimation window equal to 25 months that contains the period of six months after the signal.



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