

BAYESIAN INFERENCE FOR CERTAIN TYPES OF NON-LINEAR ECOLOGICAL MODELS

Marios Kondakis and Nikolaos Demiris

First ecological problem

- Functional responses: Insects' developmental rate $y(T; \theta)$ depends on temperature T and model parameters θ . We focus on thermal thresholds T_{min} , T_{opt} , T_{max}
- Non-linear models used and the mean of the response $E\{y(T; \theta)\} = r(T)$ is related by nonlinear ecological functions to the predictor T .

$y(T; \theta) \sim F(\mu_F = r(T))$ which is:

- $N(r(T), \sigma^2)$
- $\text{Inv}\Gamma\left(\mu_F = r(T), \sigma^2 = \frac{r^2(T)}{\text{shape}-2}\right)$
- Zero inflated Inv.Gamma in case of excess zeros data. Some Ecological functions for $r(T)$ defined within thermal thresholds are:
- Bieri model: $r(T) = \alpha \cdot (T - T_{m1}) - \beta^{(T - T_{m2})}$
- Lactin model: $r(T) = \lambda + e^{\rho \cdot T} - e^{\rho \cdot T_m - \frac{(T_m - T)}{\Delta}}$

Second ecological problem

- Functional responses: Number of prey consumed y_t is related to prey and predator densities N_t and P_t , using ODE systems
- Holling II ODE: $\frac{dN}{dt} = -\frac{aNP}{1+aT_hN}$
- Beddington-DeAngelis ODE: $\frac{dN}{dt} = -\frac{aNP}{1+aT_hN+c(1-P)}$

- The Ornstein-Uhlenbeck (O-U) process can be thought of as the time-continuous AR(1) analog: $\kappa_t = \frac{p_t}{1-p_t}$, $d\kappa_s = \phi(\mu_t - \kappa_s)dt + \sigma dB_s$
- B_s : std Brownian motion, σ : instantaneous diffusion, ϕ : speed of reversion to the mean and $\mu_t = \text{logit} \frac{N_0 - N_t}{N_0}$
- Absorbs model mis-specification

Computation in Stan

HMC, ADVI-Meanfield, ADVI-Fullrank

Real-data applications

- Developmental rates using HMC and ADVI:

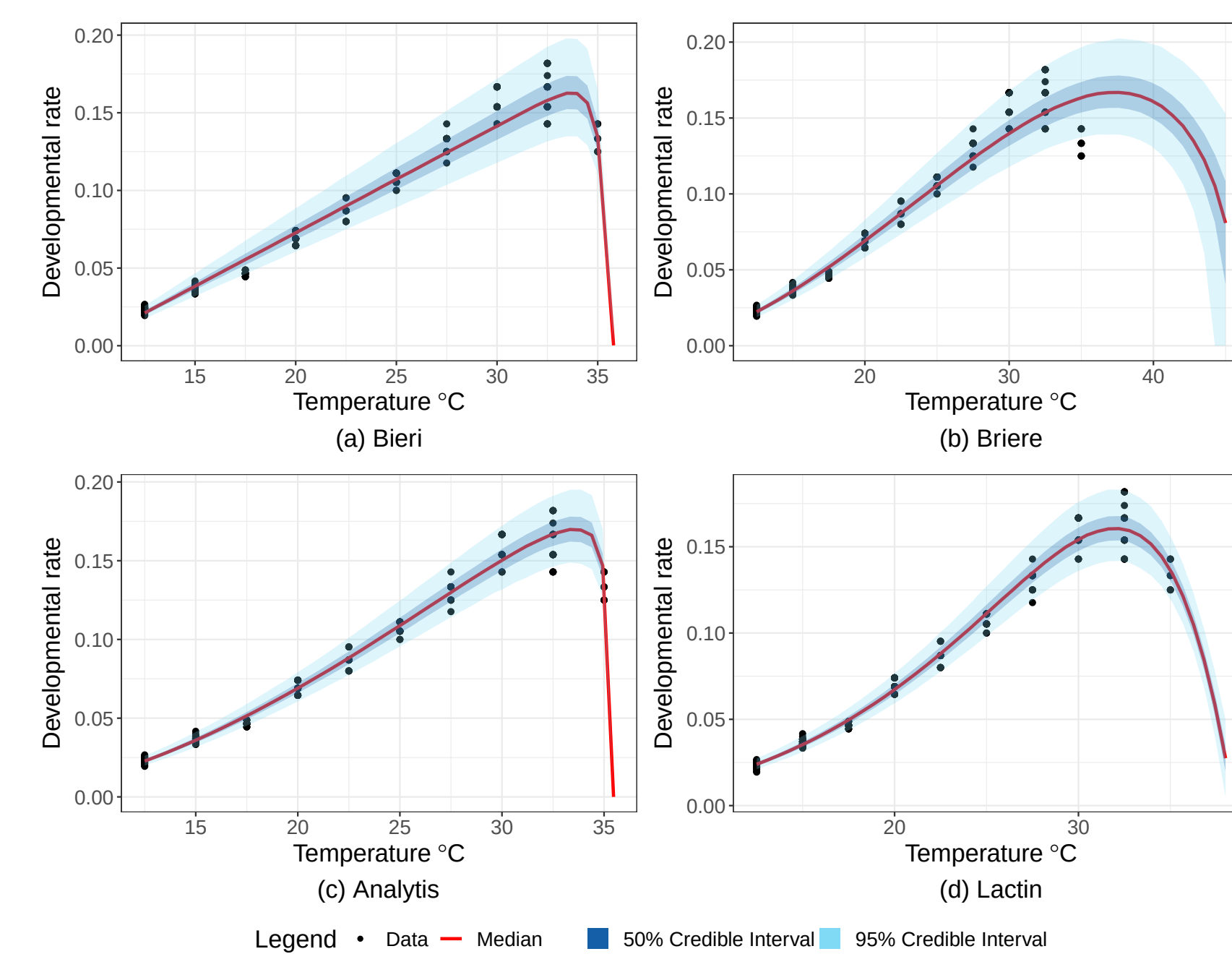


Fig. 1: Predicted posteriors vs *Tetranychus urticae* observed rates using Inv Γ models

	T_{min}		
	HMC	Meanfield	Fullrank
Analytis	4.2	7.6	6.0
Lactin	-18.7	7.8	-18.7
	T_{opt}		
	HMC	Meanfield	Fullrank
Analytis	33.6	99.9	79.9
Lactin	32.0	32.1	33.9
	T_{max}		
	HMC	Meanfield	Fullrank
Analytis	35.0	99.9	80.9
Lactin	38.4	42.6	38.6

Table 1: Estimates of T_{min} , T_{opt} and T_{max} at *Tetranychus urticae* observed rates using Inv Γ model

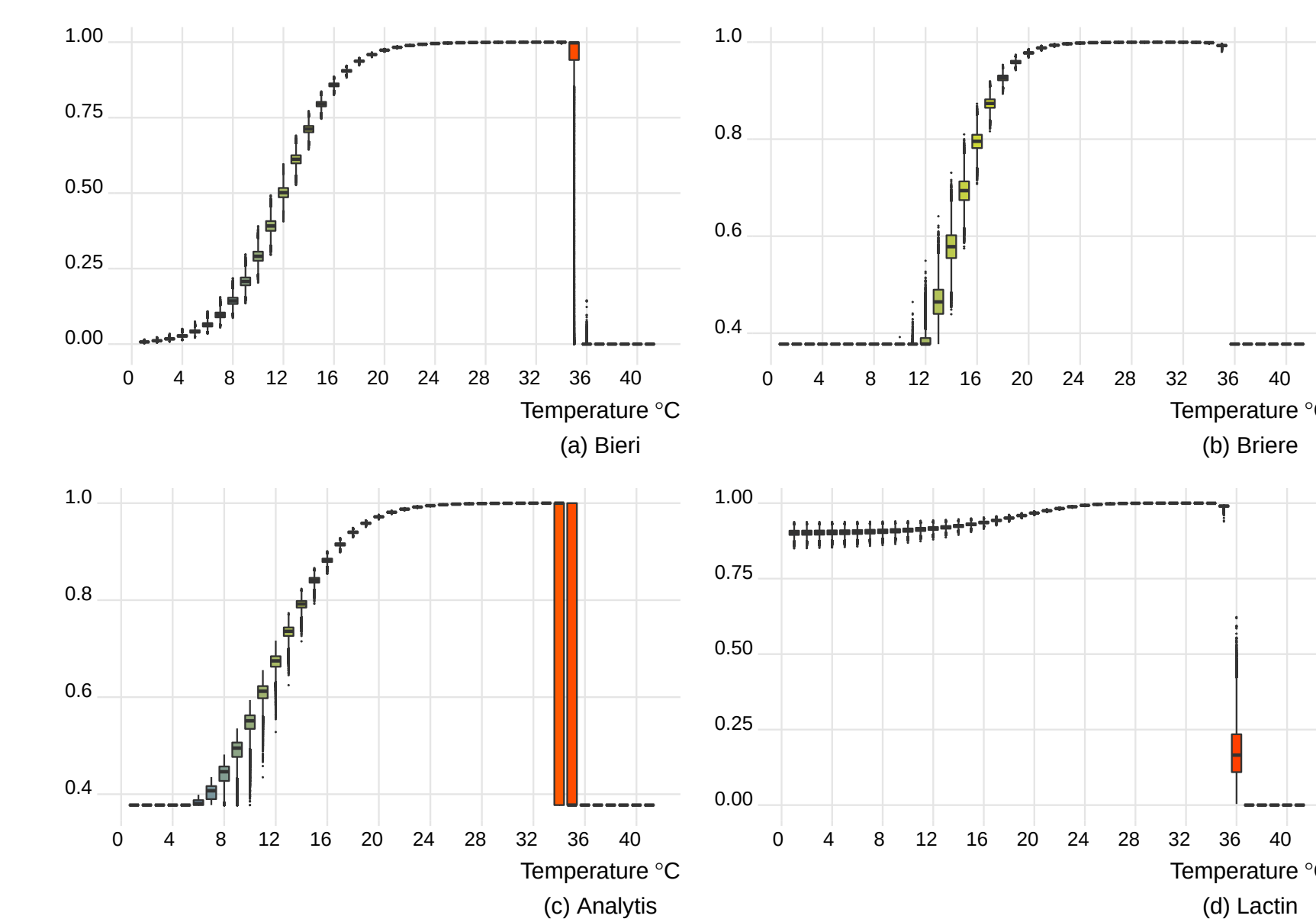


Fig. 2: Boxplots of Posteriors of the probability of non-zero development in *Propylea Coccinellidae* data using Zero Inflation Inv Γ distribution

- Prey and predator system using HMC:

	DETERMINISTIC CASE				STOCHASTIC CASE			
	Holling II	Beddington	Crowley	Hassell	Holling II	Beddington	Crowley	Hassell
AIC	359,98	362,04	361,94	361,96	209,73	213,22	212,91	213,13
BIC	365,72	369,68	369,59	369,61	219,29	224,69	224,39	224,6
DIC	356,04	356,17	355,89	355,94	231,88	232,87	232,31	232,64
LooC	362,56	361,38	361,09	361,18	257,85	259,89	257,71	257,49
WAIC	362,44	361,34	361,04	361,14	232,89	234,31	233,62	234,11

Table 2: Information criteria among different ODE systems for deterministic and stochastic case

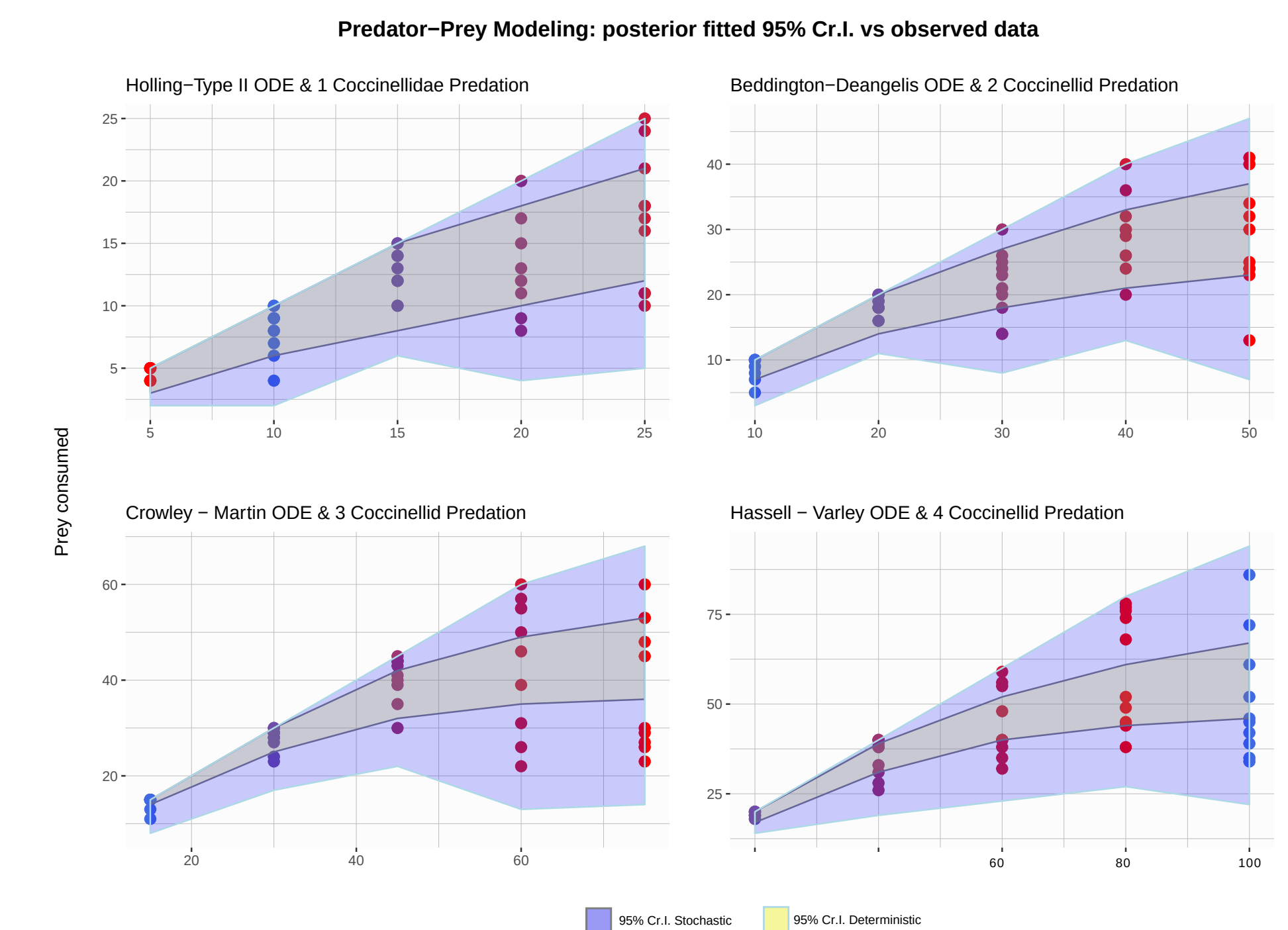


Fig. 3: Prediction results based on HMC sampler

Remarks

In developmental rate case:

- Inv Γ models are variance adaptive
- Zero inflation model deals with zero developmental rates
- ADVI methods do not always give robust estimates for T_{opt} and T_{max}
- In predator-prey systems:
- Stochasticity absorbs noise and allows for more accurate parameter estimation
- The attack rate decreases in case of the presence of at least two predators (competition)