

# Storable good market with intertemporal cost variations

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## Abstract

In a dynamic market for storable goods, we investigate a firm's pricing policy and the welfare effects associated with the firm's power to commit to future prices when production costs vary over time. We show that, when costs are expected to increase, the firm's lack of commitment generally lead to lower prices than full commitment if consumer storage costs are relatively small. This enhances consumer surplus and, under certain conditions, total welfare. However, for intermediate storage costs, the firm's full commitment tends to benefit consumers and, a fortiori, the whole economy. Our analysis provides potentially significant empirical and policy implications.

Keywords: commitment, consumer storage, monopoly, storable goods.

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# 1 Introduction

A critical issue for a firm operating in a storable good market is to deal with the consumers' strategic behavior in response to the firm's pricing policy. There exists systematic empirical evidence that purchasing patterns exhibit storage incentives in anticipation of higher future prices (e.g., Erdem et al. 2003; Hendel and Nevo 2004, 2006a, 2006b; Osborne forthcoming; Perrone 2017; Pesendorfer 2002; Pires 2016; Wang 2015). A natural reason that induces a firm to modify its price over time is a change in production costs.

The economic literature has extensively studied the firm's ability to commit to future prices in markets for durable goods. It is well established since Coase (1972) that the lack of commitment induces a monopolistic firm to engage in intertemporal price discrimination, which erodes the firm's monopoly power. The firm's commitment problem has recently received increasing attention in markets for storable goods, which are perishable in consumption but can be stored for later consumption (e.g., oil, coffee, groceries). Dudine et al. (2006) show that, in a storable good market where demand increases over time, the firm's inability to commit to future prices is definitively welfare detrimental.

In this paper, we investigate a firm's pricing policy and the welfare effects associated with the firm's ability to commit to future prices in a dynamic storable good market where forward-looking consumers are available to store in anticipation of higher future prices and production costs evolve over time. We characterize the conditions under which the firm's lack of commitment is beneficial for consumers and aggregate welfare. Our results provide potentially significant policy implications and shed new light on some empirical regularities about the firm's performance in environments with varying costs.

We consider a dynamic storable good model à la Dudine et al. (2006) where a monopolistic firm faces a continuum of consumers that are willing to store in anticipation of higher future prices. In this setting, we allow for intertemporal variations in the firm's production costs. This reflects a natural feature of many markets, where the prices for inputs and raw materials fluctuate over time. We find that, when production costs are expected to increase, the firm's lack of commitment generally leads to lower prices than full commitment as long as consumer storage costs are relatively small. Therefore, a myopic pricing policy increases consumer surplus and, despite the firm's loss, under certain circumstances it is even beneficial in terms of total welfare. Prima facie, this might seem surprising since one might believe that a myopic firm should increase its price in response to higher costs. Indeed, we show that an increase in production costs over time induces a firm with limited commitment powers to set lower prices than under full commitment. To understand the rationale for this result, it is helpful to start with the case of full commitment. When the increase in production costs exceeds the consumer storage cost, the firm prefers to stimulate consumer storage in order to avoid higher future production costs. In equilibrium, the firm commits to a price sequence that induces consumers to store the entire demand for future consumption. Under limited commitment, this outcome is no longer achievable, since the firm cannot refrain from reducing the price below the full commitment level and serving the future residual demand. Anticipating the firm's myopic behavior, the consumers are more reluctant to store. The only instrument to which the firm can resort to discourage sales in the second period is the price in the first period. Notably,

this affects the firm's problem in a non-trivial manner. A lower first period price stimulates consumer storage. Moreover, it leads to an increase in the second period demand gross of consumer storage, which is driven by a corresponding lower second period price. This is because the binding storability (no-arbitrage) constraint implies that in equilibrium the second period price reflects the first period price inflated by the consumer storage cost, which makes consumers indifferent to storing. When the increase in consumer storage associated with a lower first period price outweighs the increase in the second period gross demand, there is a decline in the second period demand net of consumer storage, which reduces production in the second period and therefore mitigates the firm's loss from the lack of commitment. This is the case if and only if the second period residual demand is upward sloping with respect to the first period price. As shown in Section 6, this condition holds as long as the demand is not significantly convex, which is satisfied under commonly used demand specifications. Since the storability constraint is binding irrespective of the firm's commitment powers, a lower price in the first period translates into a lower price in the second period as well, which definitively enhances consumer surplus. Notably, if the future residual demand is sufficiently small, the gain in consumer surplus more than compensates the firm's loss from the lack of commitment. Hence, a myopic pricing policy can even improve total welfare. These results provide potentially relevant policy recommendations, which are discussed in Section 9.

The firm's commitment issue identified in our paper is reminiscent of the classical Coase problem of a durable good monopolist that succumbs to the temptation to charge lower future prices in order to capture the consumers with lower valuations, which creates incentives to postpone purchases. However, the commitment issue we unveil exhibits novel features in various aspects, such as the mechanics behind the results, the properties of the equilibrium price sequence as well as the instruments to which the firm can resort in order to mitigate the commitment problem. Realizing these significant differences is important to better understand the issue at hand in our paper and the associated policy and empirical implications. In our setting, the storable, non-durable nature of the good implies that the consumers' intertemporal incentives are driven by demand anticipation and not by demand postponement. Since consumers are homogeneous in their valuations, the firm cannot engage in price discrimination among consumers, who are indifferent to storing in equilibrium. As previously explained, a firm with limited commitment powers manipulates the first period price in order to reduce the second period residual demand and alleviate the loss from limited commitment. While in the case of durable goods lower prices are driven by the consumers' demand postponement incentives, a firm that provides storable goods resorts to a lower price to stimulate the consumers' demand anticipation incentives, since higher consumer storage typically translates into lower future residual demand. The equilibrium price pattern also reveals significant differences from the case of durable goods. Since production costs increase over time and consumer storage is costly, the price sequence increases rather than decreases over time. The instruments that the firm can adopt in order to cope with the commitment problem reflect the different essence of the problem at hand. The traditional remedy for the Coase problem that consists in leasing (or renting) the good in each period is pointless with storable goods, since the quantity stored will be only consumed in the following period. Remarkably, while increasing costs mitigate

the durable good monopolist's incentives to cut prices in a Coasian fashion, the time inconsistency issue faced by the firm in our framework emerges exactly when costs are expected to increase.

For intermediate values of consumer storage costs, we find that the firm's full commitment tends to benefit consumers and, a fortiori, the whole economy. The lower capability to attract consumer storage induces a firm with limited commitment powers to forgo consumer storage when it is still profitable under full commitment. The firm charges higher prices at which consumer storage disappears. Alternatively, consumer storage is either allowed or removed ex ante suboptimally when it is too costly under full commitment, which again translates into higher prices. When consumer storage costs are large enough, the static monopoly solution always applies and the firm's commitment problem is welfare inconsequential.

As discussed in Section 6, our results naturally complement the findings of Dudine et al. (2006) that show the unambiguously welfare detrimental effect of the firm's lack of commitment in a storable good market where demand increases over time. The predictions of our model indicate that the welfare effects of the firm's commitment powers are significantly different and more intricate in markets with a larger degree of maturity, where demand tends to be stable while production costs vary over time.

Our results shed new light on some relevant empirical facts about the firm's performance. Since the firm's commitment problem emerges when production costs increase over time, we provide microfoundations for the well-established empirical evidence that the rate of inflation is negatively correlated with the average markup (e.g., Bénabou 1992; Banerjee et al. 2001; Banerjee and Russell 2001; Head et al. 2010). Our analysis is conducted in a fairly general setting without imposing any unduly restrictive assumptions on the functional forms. As explained in Section 8, the model is robust and the driving force of our results generalizes to various settings that involve uncertainty, larger time horizons or competition among firms.

**Related literature** Our paper naturally pertains to the extensive literature on storable goods. An early relevant contribution is Bénabou (1989), which characterizes the optimal pricing policy of a storable good monopolist that operates in an inflationary environment vis-à-vis a continuum of speculators available to store. In each period, the firm must decide whether to adjust its price to the rate of inflation by incurring a "menu cost" and the speculators engage in storage activities that are detrimental to the firm's profits. Differently from Bénabou (1989), in our setting the firm can freely change its price, which is not eroded by inflation, and may prefer to stimulate consumer storage in anticipation of higher production costs. Moreover, we investigate the impact of the firm's commitment powers on the equilibrium pricing policy and the associated welfare effects. Jeuland and Narasimhan (1985) find that price discrimination among consumers with different demand functions provides an explanation for temporary discounts in storable good markets. In a model where a share of consumers can store the good for future consumption, Hong et al. (2002) show that consumer storage leads to equilibrium price dispersion. As previously discussed, our study is closely related to the seminal paper of Dudine et al. (2006), which considers a storable good market where demand increases deterministically over time and a monopolistic firm faces a continuum of consumers that have

incentives to store in anticipation of higher future prices. In this setting, consumer storage harms the firm's profits, since it reduces future sales that occurs at higher prices. Hence, a firm with full commitment powers selects a price sequence that fully removes consumer storage. Under limited commitment, the firm succumbs to the temptation to increase the second period price to the static monopoly level in response to the absence of consumer storage. To mitigate wasteful storage driven by the consumers' anticipation of the firm's opportunistic behavior, the firm increases the price in the first period. As a result, the firm's lack of commitment reduces consumer surplus and firm's profits, which is definitely welfare detrimental. In our paper, we consider an alternative legitimate reason for time-varying prices, namely, intertemporal variations in the firm's production costs. As explained in Section 6, our significantly different results provide a complementary picture to Dudine et al. (2006) and can contribute to a fuller characterization of the dynamic interactions in storable good markets and of the associated welfare consequences.

Extending the analysis of Dudine et al. (2006) to non-linear pricing of storable goods, Hendel et al. (2014) find cyclical patterns in prices and sales. Su (2010) incorporates consumer storage into Su (2007)'s analysis of the optimal dynamic strategy of a seller vis-à-vis strategic buyers and shows that the seller may either charge a constant fixed price or offer periodic price promotions at predictable time intervals. Hendel and Nevo (2013) provide a theoretical and empirical investigation of the intertemporal price discrimination incentives of a firm that faces consumers with heterogeneous storage abilities. In equilibrium, the price pattern exhibits temporary reductions that allow the firm to discriminate among consumers. The effects of competition in markets for storable goods have been studied as well. In a Cournot duopoly framework, Anton and Das Varma (2005) show that firms compete for consumer storage. The equilibrium price sequence is increasing and prices are higher than when storage is unfeasible. In a differentiated good market with price competition, Guo and Villas-Boas (2007) find that preference heterogeneity leads to a differential consumer storage propensity, which exacerbates future price competition and may remove consumer storage in equilibrium.<sup>1</sup> Nava and Schiraldi (2014) investigate the impact of consumer storage on the firms' incentives to promote periodic price reductions in order to sustain collusion.

Our paper can also contribute to the voluminous literature on durable goods. The Coase conjecture, initially postulated by Coase (1972) and formally proved by Stockey (1981), Bulow (1982) and Gul et al. (1986), asserts that, when a durable good monopolist can change its price very quickly, the intertemporal profits tend to be dissipated. Since the firm cannot refrain from price-discriminating over time among consumers with different valuations of the good, consumers expect that prices will converge to the competitive level at any future instant and do not accept higher prices. The firm ends up charging prices close to the competitive level, consistently with the consumers' beliefs. We have previously emphasized the relation and the significant differences between the Coase problem that holds with durables and our mechanism that applies to storables. A recent relevant contribution by Ortner (2017) shows that stochastic costs introduce an option value of delaying trade, which restores the monopolist's power to extract some rents if the consumers' valuations are discrete. Since in our setting an

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<sup>1</sup>In Section 8.4 we discuss how our paper relates to Anton and Das Varma (2005) and Guo and Villas-Boas (2007).

increase in production costs undermines the firm's commitment ability, our results tend to go to the opposite direction to Ortner (2017). This provides further corroboration for the different nature of the problem at hand. Nava and Schiraldi (2016) find that a durable good monopolist can make positive profits when offering horizontally differentiated varieties. Recent important papers also include Board (2008), which solves the durable good monopolist's problem when incoming demand evolves over time, and Garrett (2016), which considers buyers arriving over time and whose valuations of the good vary stochastically.

**Structure of the paper** The rest of the paper unfolds as follows. Section 2 sets out the formal model. Section 3 considers the static solution to the firm's problem. Sections 4 and 5 characterize the firm's equilibrium pricing policy under full and limited commitment, respectively. Section 6 is devoted to price comparisons between the two commitment regimes. Section 7 provides a welfare analysis. Section 8 discusses the robustness of the results and examines various possible extensions. Section 9 concludes and illustrates the policy implications of our results. All formal proofs are collected in the Appendix. Additional formal results and associated proofs are relegated to the Supplementary Appendix.

## 2 The model

### 2.1 Setting

**Consumers** We consider a two-period market for a storable good where in each period  $\tau \in \{1, 2\}$  a monopolistic firm faces a (continuously differentiable) demand  $D(p_\tau)$ , which decreases with the price  $p_\tau$ , i.e.,  $D'(p_\tau) < 0$ . For the sake of simplicity and in line with Dudine et al. (2006), we assume no discounting on the second period. In Section 8.5 we discuss the role of the discount factor.

Consumers can store some units of the good in the first period for consumption in the second period at a unit cost  $s_c \geq 0$ . Competitive arbitrageurs can also engage in storage activities. Following Dudine et al. (2006), the consumer storage demand writes as

$$D_s(p_1) = \begin{cases} D(p_1 + s_c) & \text{if } p_1 + s_c < p_2 \\ [0, D(p_1 + s_c)] & \text{if } p_1 + s_c = p_2 \\ 0 & \text{if } p_1 + s_c > p_2 \end{cases} \quad (1)$$

For  $p_1 + s_c < p_2$ , the first period price augmented by the consumer storage cost is lower than the second period price, which implies that consumers prefer to store the entire quantity consumed in the second period. Conversely, for  $p_1 + s_c > p_2$ , consumers do not wish to store any quantity. For  $p_1 + s_c = p_2$ , consumers are indifferent between storing the good and waiting until the second period to purchase it, and therefore they are willing to store any quantity between zero and the consumption in the second period.

**Firm** In each period  $\tau \in \{1, 2\}$  the firm incurs a (constant) unit production cost  $c_\tau$ . The unit cost is  $c_1$  in the first period and  $c_2$  in the second period, where  $\Delta c \equiv c_2 - c_1$  denotes the in-

tertemporal cost variation. As it will be clear in the sequel, we are mainly interested in the case where production costs rise over time, i.e.,  $\Delta c > 0$ . For the sake of convenience, production costs change in a deterministic manner. This assumption is in the spirit of the framework of Dudine et al. (2006), where demand evolves deterministically over time. Moreover, this seems to be reasonable in storable good markets where cost shocks exhibit strong correlation over time (e.g., Deaton and Laroque 1996). In Section 8.1 we show that our qualitative results hold in a stochastic cost framework and discuss the implications of introducing cost uncertainty.

The firm's aggregate profits are  $\Pi \equiv \Pi_1 + \Pi_2$ , where

$$\Pi_1 = (p_1 - c_1) [D(p_1) + D_s(p_1)] \quad (2)$$

and

$$\Pi_2 = (p_2 - c_2) [D(p_2) - D_s(p_1)] \quad (3)$$

are the profits in the first and second period, respectively. Consumer storage inflates the demand in the first period but depresses it in the second period, since consumers resort to the quantity stored in the first period.

The firm's profits  $\Pi_\tau$  in each period  $\tau$  satisfy the following standard assumption.

**Assumption 1**  $\Pi''_\tau(p_\tau) < 0, \tau \in \{1, 2\}$ .

Assumption 1 states that the firm's profits in each period must be concave in the price, which ensures that the second order conditions for profit maximization are fulfilled.

## 2.2 Timing and equilibrium concept

Each period of the game includes the following two stages.

- (I) The firm determines the price for the good.
- (II) Consumers purchase a quantity of the good and decide on the amount to be stored.

Under full commitment, the firm is able to specify at the beginning of the game the pricing policy that maximizes the ex ante aggregate profits. Under limited commitment, the price in each period must be sequentially optimal and maximize the firm's continuation profits, namely, it arises as the subgame perfect Nash equilibrium of the game.

## 3 Static solution

When consumer storage is not feasible, the firm's problem reduces to the static monopoly problem in each period  $\tau \in \{1, 2\}$ , which is given by

$$\max_{p_\tau} (p_\tau - c_\tau) D(p_\tau). \quad (4)$$

It is helpful for our analysis to consider the following auxiliary function

$$\phi_\tau(p_\tau) \equiv D(p_\tau) + (p_\tau - c_\tau) D'(p_\tau), \quad (5)$$

which represents the left-hand side of the first order condition for the static monopoly problem in period  $\tau$ . The equilibrium static monopoly price is  $p_\tau^m = c_\tau - \frac{D(p_\tau^m)}{D'(p_\tau^m)}$ . We define  $\mu_\tau^m \equiv p_\tau^m - c_\tau = \frac{p_\tau^m}{\varepsilon_{p_\tau^m}}$  as the price-cost static monopoly markup (or profit margin) in period  $\tau$ , where  $\varepsilon_{p_\tau^m} \equiv -\frac{D'(p_\tau^m)p_\tau^m}{D(p_\tau^m)}$  is the demand elasticity evaluated at  $p_\tau^m$ . The static monopoly markup difference between the two periods is  $\Delta\mu^m \equiv \mu_2^m - \mu_1^m$ . It holds  $\Delta\mu^m \leq 0$  if and only if  $\frac{p_2^m}{p_1^m} \leq \frac{\varepsilon_{p_2^m}}{\varepsilon_{p_1^m}}$ . The sign of  $\Delta\mu^m$  crucially depends on the shape of the demand curvature. Since a cost increase ( $\Delta c > 0$ ) implies  $p_2^m > p_1^m$ , we find that  $\Delta\mu^m < 0$  as long as the increase in the demand elasticity is sufficiently large, which is the case when the demand is not too convex.<sup>2</sup> Standard computations show that with a linear demand specification  $D(p_\tau) = \alpha - \beta p_\tau$  it holds  $\Delta\mu^m = -\frac{\Delta c}{2} < 0$ , while an iso-elastic demand of the form  $D(p_\tau) = p_\tau^{-\eta}$  yields  $\Delta\mu^m = \frac{\Delta c}{\eta-1} > 0$  ( $\eta > 1$  stems from the second order condition).<sup>3</sup>

When storage is feasible, it follows from the consumer storage demand in (1) that the static monopoly solution is implementable if and only if  $p_1^m + s_c \geq p_2^m$ , i.e., the following static monopoly feasibility constraint is fulfilled

$$s_c \geq \Delta c + \Delta\mu^m. \quad (6)$$

The consumer storage cost  $s_c$  must be sufficiently large that no storage occurs at the static monopoly prices and therefore the static monopoly solution is implementable. The feasibility constraint (6) becomes weaker when  $\Delta\mu^m$  is smaller. As previously noted, if the demand is not too convex (which implies  $\Delta\mu^m < 0$ ), the second period static monopoly price is set in a region where the demand is more elastic, which mitigates the price increase and makes the feasibility constraint (6) more likely to be fulfilled.

## 4 Full commitment

A firm equipped with full commitment powers can credibly announce a price for each period and adhere to this pricing policy. Formally, the firm sets a price sequence that maximizes the aggregate profits given by the sum of the first period profits in (2) and the second period profits in (3). In principle, we can identify three pricing options that affect the consumer storage behavior. First, the firm may resort to a pricing policy such that the first period price augmented by the consumer storage cost is larger than the second period price, i.e.,  $p_1 + s_c > p_2$ . The consumer storage demand in (1) vanishes and the firm's problem reduces to a replica of the per period (unconstrained) maximization problem, which leads to the static monopoly prices

<sup>2</sup>Too see this, note that  $\Delta\mu^m < 0$  if and only if  $\frac{D(p_2^m)}{D(p_1^m)} < \frac{D'(p_2^m)}{D'(p_1^m)}$ . Since the left-hand side is lower than 1, a sufficient condition is that the demand is weakly concave. By continuity, this holds as long as the demand is not too convex.

<sup>3</sup>With an exponential demand  $D(p_\tau) = e^{-\gamma p_\tau}$  ( $\gamma > 0$ ), we obtain  $\Delta\mu^m = 0$ .

described in Section 3. This solution is implementable if and only if the feasibility constraint (6) is fulfilled. In other words, the consumer storage cost must be high enough to deter storage when the static monopoly prices are charged. The second option for the firm is to set a price sequence such that the first period price augmented by the consumer storage cost coincides with the second period price, i.e.,  $p_1 + s_c = p_2$ . In this case, the storability (no-arbitrage) constraint is binding and consumers are indifferent between storing the good for future consumption or waiting until the second period to purchase it. Note that this pricing policy may induce consumer storage, which can be profitable for the firm since production costs are higher in the second period ( $\Delta c > 0$ ). The third option consists in a price sequence such that the first period price inflated by the consumer storage cost is lower than the second period price, i.e.,  $p_1 + s_c < p_2$ . This ensures that consumers purchase and store in the first period the entire quantity that they are willing to consume in the second period. Since the full storage outcome can be replicated by setting  $p_1 + s_c = p_2$ , the third option is (at least weakly) dominated by the second option and is therefore irrelevant.

Intuitively, the firm faces the following trade-off. A lower price in the first period stimulates consumer storage, which is profitable in terms of cost savings. This benefit comes at the cost of a lower profit margin. Despite this basic trade-off, things are far from being trivial, since the equilibrium outcome depends on a range of factors, such as the consumer storage cost, the feasibility of the static monopoly solution and the shape of the demand function. The following proposition provides a full characterization of the equilibrium consumer storage behavior and of the equilibrium price sequence when the firm can commit to future prices.

**Proposition 1** *Under full commitment,*

- (i) if  $s_c < \min \{ \Delta c + \Delta \mu^m, \Delta c \}$ , consumer storage is  $D_s^{cs} = D(p_1^{cs} + s_c)$ , and prices are  $p_1^{cs} = c_1 - \frac{D(p_1^{cs} + s_c) + \phi_1(p_1^{cs})}{D'(p_1^{cs} + s_c)}$  and  $p_2^{cs} = p_1^{cs} + s_c$ ;
- (iia) if  $\Delta c + \Delta \mu^m \leq s_c \leq \Delta c$ , there exists a threshold  $\bar{s}_c^c \in (\Delta c + \Delta \mu^m, \Delta c)$  such that (1) for  $s_c < \bar{s}_c^c$  the outcome in (i) applies, (2) for  $s_c \geq \bar{s}_c^c$  consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m = c_1 + \mu_1^m$  and  $p_2^m = c_2 + \mu_2^m$ ;
- (iib) if, alternatively,  $\Delta c \leq s_c < \Delta c + \Delta \mu^m$ , consumer storage is  $D_s^{cn} = 0$ , and prices are  $p_1^{cn} = c_1 - \frac{D(p_1^{cn}) + \phi_2(p_1^{cn} + s_c)}{D'(p_1^{cn})}$  and  $p_2^{cn} = p_1^{cn} + s_c$ ;
- (iii) if  $s_c \geq \max \{ \Delta c + \Delta \mu^m, \Delta c \}$ , the outcome in (iia-2) applies.

To better appreciate the results in Proposition 1, we disentangle the analysis according to the sign of the static monopoly profit margin difference  $\Delta \mu^m$  between the two periods defined in Section 3. In Figure 1, panel (a) illustrates the case  $\Delta \mu^m \leq 0$  formalized in Corollary 1 and panel (b) illustrates the case  $\Delta \mu^m > 0$  formalized in Corollary 2.

We start with the case  $\Delta \mu^m \leq 0$ , which occurs when the demand is not too convex. This implies that the outcome in point (iia) of Proposition 1 is feasible instead of the outcome in point (iib).

**Corollary 1** *Suppose  $\Delta \mu^m \leq 0$ . Then, under full commitment,*

- (i) if  $s_c < \bar{s}_c^c$ , consumer storage is  $D_s^{cs} = D(p_1^{cs} + s_c)$ , and prices are  $p_1^{cs}$  and  $p_2^{cs} = p_1^{cs} + s_c$ , where  $\frac{\partial p_1^{cs}}{\partial s_c} < 0$  for  $D''(\cdot) < \tilde{D}''$  and  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ ;

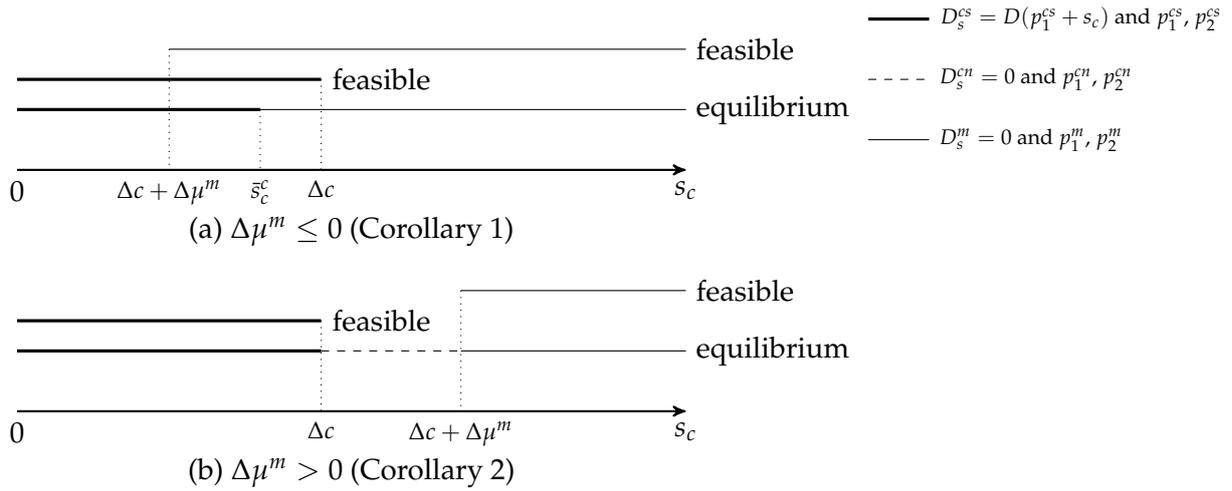


Figure 1: Full commitment (Proposition 1)

- (ii) if  $s_c \geq \bar{s}_c^c$ , consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m$  and  $p_2^m$ .  
It holds  $p_1^m \geq p_1^{cs}$  for  $D''(\cdot) < \tilde{D}''$ , where the equality follows if and only if  $s_c = 0$ .

As point (i) of Corollary 1 indicates, when the consumer storage cost is relatively low, i.e.,  $s_c < \bar{s}_c^c$ , the firm finds it optimal to commit to a price sequence that induces consumers to store the entire quantity for the second period. Since the storability constraint is binding, i.e.,  $p_2^{cs} = p_1^{cs} + s_c$ , consumers are indeed indifferent to storing. However, full consumer storage occurs in equilibrium, since the firm could slightly reduce the first period price and induce full storage, which yields a discontinuous increase in profits. Note from panel (a) of Figure 1 that for  $\Delta c + \Delta\mu^m \leq s_c \leq \Delta c$  the firm can choose between allowing consumer storage and enjoying the static monopoly profits (the feasibility constraint (6) holds). Since the firm's profits in the presence of consumer storage decrease with storage costs (as consumers are more reluctant to store) while the static monopoly profits do not change, there exists a threshold  $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$  below which the firm prefers to allow consumer storage.<sup>4</sup> The threshold  $\bar{s}_c^c$  increases with the second period cost  $c_2$ , i.e.,  $\frac{\partial \bar{s}_c^c}{\partial \Delta c} > 0$  (for a given  $c_1$ ), because the static monopoly profits decline but the profits in the presence of consumer storage are unaffected. Hence, consumer storage is more likely to be promoted in response to a more significant cost increase that makes production more convenient in the first period. For  $s_c \geq \bar{s}_c^c$ , the firm sets the static monopoly prices and consumers abstain from storing, as point (ii) of Corollary 1 establishes.<sup>5</sup>

The equilibrium consumer storage and price sequence as a function of  $s_c$  are illustrated in panel (a) of Figure 2 for the example of linear demand. Note that they exhibit a discontinuity at  $\bar{s}_c^c$ , where the firm is indifferent between allowing consumer storage and implementing the static monopoly solution. Clearly, consumer storage decreases with  $s_c$ , since consumers are more reluctant to store. When consumer storage is costless, i.e.,  $s_c = 0$ , the storing price is the same in the two periods, i.e.,  $p_1^{cs} = p_2^{cs}$ , and coincides with the first period static monopoly

<sup>4</sup>We refer to the proof of Proposition 1 in the Appendix for a formal demonstration of the existence and uniqueness of the threshold  $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$ .

<sup>5</sup>When production costs decrease over time ( $\Delta c < 0$ ), the static monopoly solution trivially applies irrespective of the magnitude of consumer storage costs, since both the firm and the consumers prefer to postpone the consumers' purchases ( $\Delta c < 0$  implies  $p_1^m > p_2^m$ , which makes the feasibility constraint (6) irrelevant).

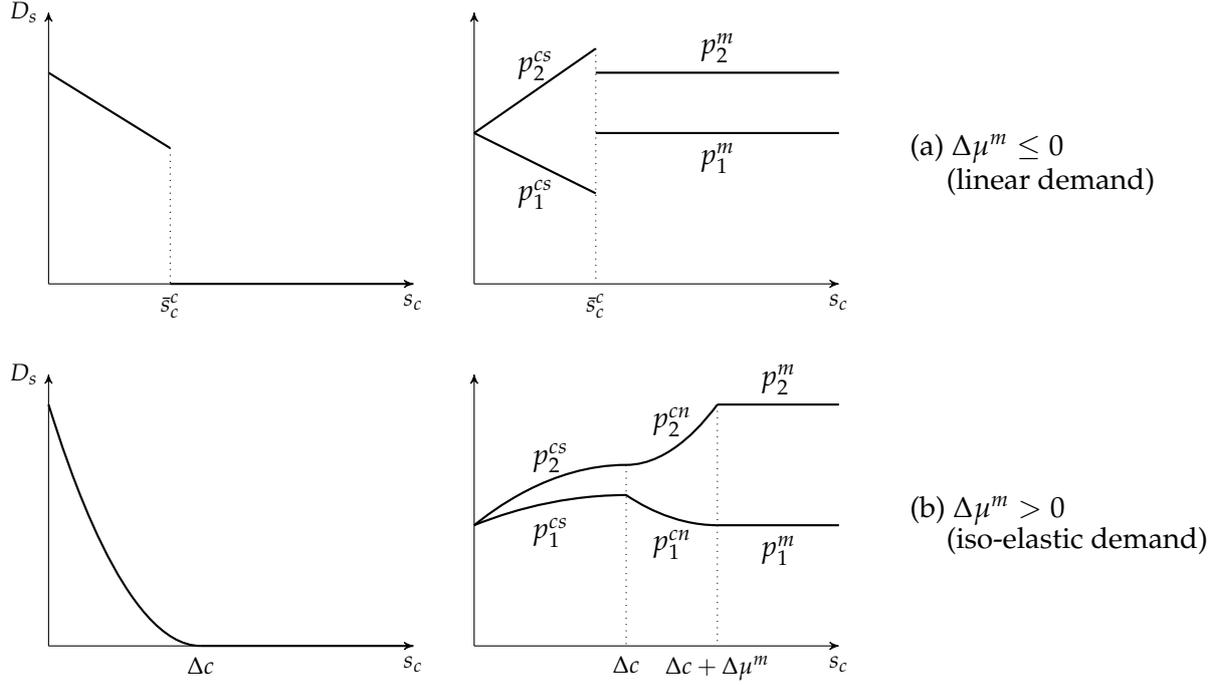


Figure 2: Equilibrium consumer storage and price patterns under full commitment

price  $p_1^m$ . This is because consumers are so eager to store that the firm cannot discriminate between the two periods and faces twice the same demand in the first period. However, when  $s_c$  is higher, the storing price  $p_1^{cs}$  declines to incentivize consumer storage.<sup>6</sup> The price sequence is non-monotonic with respect to  $s_c$  in the first period and possibly in the second period as well, especially when the second period cost is not too large.

We now turn to the case  $\Delta\mu^m \geq 0$ , which occurs when the demand is sufficiently convex. This is formalized in the following corollary and illustrated in panel (b) of Figure 1. The outcome in point (iib) of Proposition 1 is feasible instead of the outcome in point (iia).

**Corollary 2** *Suppose  $\Delta\mu^m > 0$ . Then, under full commitment,*

(i) *if  $s_c < \Delta c$ , consumer storage is  $D_s^{cs} = D(p_1^{cs} + s_c)$ , and prices are  $p_1^{cs}$  and  $p_2^{cs} = p_1^{cs} + s_c$ , where  $\frac{\partial p_1^{cs}}{\partial s_c} > 0$  for  $D''(\cdot) > \tilde{D}''$  and  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ ;*

(ii) *if  $\Delta c \leq s_c < \Delta c + \Delta\mu^m$ , consumer storage is  $D_s^{cn} = 0$ , and prices are  $p_1^{cn}$  and  $p_2^{cn} = p_1^{cn} + s_c$ , where  $\frac{\partial p_1^{cn}}{\partial s_c} < 0$  and  $\frac{\partial p_2^{cn}}{\partial s_c} > 0$ ;*

(iii) *if  $s_c \geq \Delta c + \Delta\mu^m$ , consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m$  and  $p_2^m$ .*

*It holds  $p_1^{cs} \geq p_1^m$  for  $D''(\cdot) > \tilde{D}''$ , where the equality follows if and only if  $s_c = 0$ , and  $p_1^{cn} > p_1^m$ . Moreover,  $p_2^m > p_2^{cn} > p_2^{cs}$ .*

Point (i) of Corollary 2 shows that, as in point (i) of Corollary 1, when the consumer storage cost is low enough, i.e.,  $s_c < \Delta c$ , the firm commits to a price sequence that induces consumers to store the entire quantity for the second period. A comparison between panels (a) and (b) in Figure 1 reveals that for  $\Delta\mu^m > 0$  consumer storage is promoted as long as the additional cost of producing in the second period exceeds the additional price that consumers are available

<sup>6</sup>According to Corollary 1, this holds if the demand is not too convex. Recall from the discussion in Section 3 that this is consistent with the case  $\Delta\mu^m \leq 0$ .

to pay ( $s_c < \Delta c$ ). Moreover, there exists an interval for  $s_c$ , i.e.,  $\Delta c \leq s_c < \Delta c + \Delta\mu^m$ , where consumer storage is too costly ( $\Delta c \leq s_c$ ) but the static monopoly solution is not implementable (the feasibility constraint (6) fails to hold). As point (ii) of Corollary 2 indicates, the firm must resort to prices distorted from the static monopoly level in order to remove consumer storage. Note that, since the storability constraint is binding, i.e.,  $p_2^{cn} = p_1^{cn} + s_c$ , consumers are indeed indifferent to storing. Contrary to the outcome in point (i), consumer storage does not take place in equilibrium, since the firm could slightly increase the first period price and fully remove consumer storage, which yields a discontinuous increase in profits. Clearly, the firm selects the static monopoly prices as soon as they are available, i.e., for  $s_c \geq \Delta c + \Delta\mu^m$ , as point (iii) of Corollary 2 indicates.

The equilibrium consumer storage and price sequence as a function of  $s_c$  are illustrated in panel (b) of Figure 2 for the example of iso-elastic demand. Note that they are now continuous functions. A comparison between panels (a) and (b) of Figure 2 shows that for  $\Delta\mu^m > 0$  the first period storing price  $p_1^{cs}$  is distorted above (rather than below) the static monopoly level and increases (rather than decreases) with  $s_c$ .<sup>7</sup> To understand the rationale for this result, note that with convex demand an increase in  $s_c$  for a given  $p_1$  not only reduces the consumer storage demand  $D_s(p_1) = D(p_1 + s_c)$ , which pushes  $p_1$  downwards to promote storage (as with concave demand), but it also makes  $D_s(\cdot)$  flatter, which creates the opposite incentive to move  $p_1$  upwards. When the demand is sufficiently convex, the latter effect dominates the former effect and the first period storing price  $p_1^{cs}$  increases with  $s_c$ .

The no-storing prices  $p_1^{cn}$  and  $p_2^{cn}$  lie in between the static monopoly prices  $p_1^m$  and  $p_2^m$ . To deter consumer storage, the firm distorts the price upwards in the first period and downwards in the second period relative to the static monopoly level. Contrary to  $p_1^{cs}$ , the no-storing price  $p_1^{cn}$  decreases with  $s_c$ . When storage becomes more costly for consumers, the firm can alleviate the price distortion from the static monopoly level to prevent storage. While the first period pricing policy is non-monotonic, the second period pricing policy monotonically increases with  $s_c$  (the storability constraint is binding) and reaches the maximum when the static monopoly price is implemented.

## 5 Limited commitment

We now investigate the situation where the firm is unable to commit to a long term pricing policy. We know from point (i) of Proposition 1 that, when consumer storage costs are low enough, a firm with full commitment powers finds it optimal to announce a price sequence that induces consumers to store their entire future demand. Moreover, as point (iib) of Proposition 1 indicates, for intermediate storage costs, the firm prefers to commit to a price sequence such that the first period price is distorted above while the second period price is distorted below the static monopoly level, which fully removes consumer storage (see Corollary 2). These pricing policies cannot be implemented when the firm lacks the ability to commit to future prices. This is because, after the second period commences, the firm has an incentive to revise its price. In

<sup>7</sup>According to Corollary 2, this holds if the demand is sufficiently convex. It follows from the discussion in Section 3 that this is consistent with the case  $\Delta\mu^m > 0$ .

other terms, the firm's pricing policy is not sequentially optimal. Specifically, in the first case, after consumers stored their entire second period demand, the firm has an incentive to reduce the price in the second period below the pre-announced level in order to stimulate its sales. In the second case, if consumers did not store in the first period, the firm's best response is to increase the price in the second period to the static monopoly level. Anticipating the firm's myopic behavior, consumers modify their storage strategies and tend to store to a smaller or larger extent relative to full commitment.

As in the case of full commitment, the firm can resort to three pricing options. First, the firm may select a pricing policy such that consumer storage does not occur, i.e.,  $p_1 + s_c > p_2$ . This leads to the static monopoly prices, provided that the feasibility constraint (6) is satisfied. The second pricing option is to make consumers indifferent between storing in the first period or purchasing in the second period, i.e.,  $p_1 + s_c = p_2$ . Differently from full commitment, the firm cannot freely manipulate the equilibrium storing level, which is dictated by the constraint of sequential optimality described above. The third option for the firm is to set  $p_1 + s_c < p_2$ , which induces full consumer storage. Yet, this pricing policy is not implementable, since the firm succumbs to the temptation to reduce  $p_2$  and gain positive profits in the second period.<sup>8</sup> To preserve the relevance of the firm's limited commitment problem, we focus on the sensible case where the cost increase is not so pronounced as to make production ex post unprofitable in the second period. When the storing price under full commitment is below the costs in the second period, the full commitment storing solution can be trivially replicated under limited commitment.

The following proposition provides a full characterization of the equilibrium consumer storage behavior and of the equilibrium price sequence when the firm cannot commit to future prices.

**Proposition 2** *Under limited commitment,*

(i) if  $s_c < \min \{ \Delta c + \Delta \mu^m, \tilde{s}_c^* \}$ , consumer storage is  $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$ , and prices are  $p_1^{*s} = c_1 - \frac{D(p_1^{*s}) + \phi_2(p_1^{*s} + s_c) + (\Delta c - s_c)\phi_2'(p_1^{*s} + s_c)}{D'(p_1^{*s})}$  and  $p_2^{*s} = p_1^{*s} + s_c$ ;

(ii) if  $\Delta c + \Delta \mu^m \leq s_c \leq \tilde{s}_c^*$ , there exists a threshold  $\bar{s}_c^* \in (\Delta c + \Delta \mu^m, \tilde{s}_c^*)$  such that (1) for  $s_c < \bar{s}_c^*$  the outcome in (i) applies, (2) for  $s_c \geq \bar{s}_c^*$  consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m = c_1 + \mu_1^m$  and  $p_2^m = c_2 + \mu_2^m$ ;

(iii) if, alternatively,  $\tilde{s}_c^* \leq s_c < \Delta c + \Delta \mu^m$ , consumer storage is  $D_s^{*n} = 0$ , and prices are  $p_1^{*n} = p_2^{*n} - s_c$  and  $p_2^{*n} = p_2^m$ ;

(iii) if  $s_c \geq \max \{ \Delta c + \Delta \mu^m, \tilde{s}_c^* \}$ , the outcome in (ii-2) applies.

In line with the analysis of full commitment in Section 4, we identify two main cases. In Figure 3, panel (a) illustrates the case  $\Delta c + \Delta \mu^m \leq \tilde{s}_c^*$  formalized in Corollary 3 and panel (b) illustrates the case  $\Delta c + \Delta \mu^m > \tilde{s}_c^*$  formalized in Corollary 4. As shown in the proof of Proposition 2 in the Appendix, the threshold  $\tilde{s}_c^*$  represents the highest value for  $s_c$  such that consumer storage is feasible.

<sup>8</sup>Alternatively, the competition between arbitrageurs — that buy the good in the first period at the unit price  $p_1$  and, after incurring the storage cost  $s_c$ , resell it in the second period — will push the second period price to  $p_2 = p_1 + s_c$ .

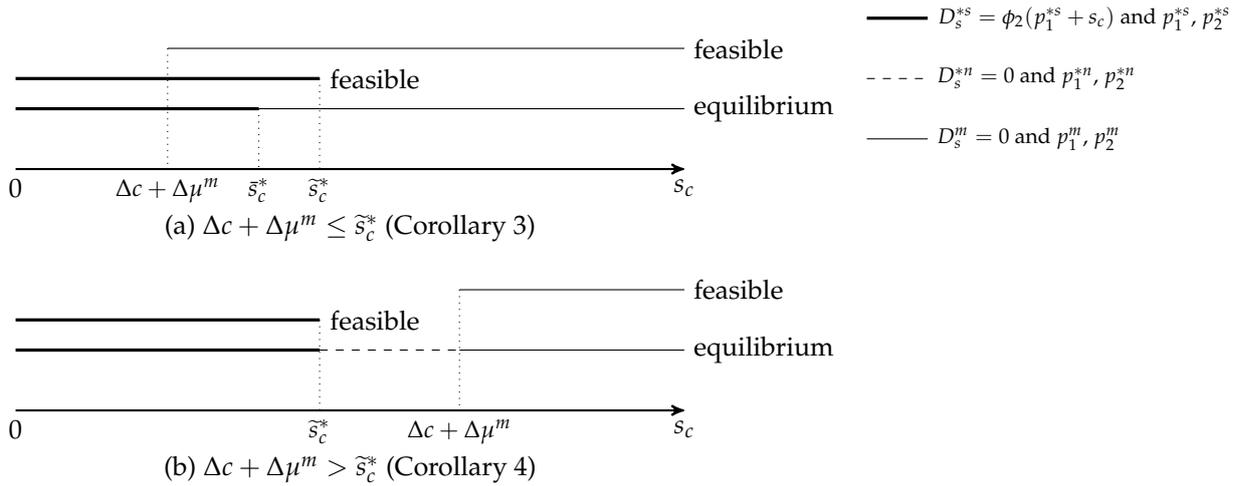


Figure 3: Limited commitment (Proposition 2)

We start with the case  $\Delta c + \Delta \mu^m \leq \bar{s}_c^*$ , which implies that the outcome in point (iia) of Proposition 2 is feasible instead of the outcome in point (iib). Note that a (necessary) condition for this case to apply is that  $\Delta \mu^m \leq 0$ , which occurs as long as the demand is not too convex.<sup>9</sup>

**Corollary 3** Suppose  $\Delta c + \Delta \mu^m \leq \bar{s}_c^*$ . Then, under limited commitment,

(i) if  $s_c < \bar{s}_c^*$ , consumer storage is  $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$ , and prices are  $p_1^{*s}$  and  $p_2^{*s} = p_1^{*s} + s_c$ , where  $\frac{\partial p_1^{*s}}{\partial s_c} = 0$  for  $D''(\cdot) = 0$  and  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ ;

(ii) if  $s_c \geq \bar{s}_c^*$ , consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m$  and  $p_2^m$ .

It holds  $p_1^m > p_1^{*s}$  for  $D''(\cdot) = 0$ , and  $p_2^m > p_2^{*s}$ .

Point (i) of Corollary 3 shows that, if the consumer storage cost is sufficiently low, i.e.,  $s_c < \bar{s}_c^*$ , consumers partially store in the first period the quantity demanded in the second period. Since the storability constraint is binding, i.e.,  $p_2^{*s} = p_1^{*s} + s_c$ , consumers are indeed indifferent to storing. Contrary to the case of full commitment, the equilibrium storing level is now established by the sequential optimality constraint. Under limited commitment, the firm can only resort to the first period price to promote consumer storage in anticipation of higher future costs. Consumers realize that, if they stored the entire quantity for the second period, the firm will succumb to the temptation to set a relatively low price in the second period in order to encourage further purchases. This mitigates the consumer storage incentives and the firm can only induce partial storing of future demand. As illustrated in panel (a) of Figure 3, for  $\Delta c + \Delta \mu^m \leq s_c \leq \bar{s}_c^*$  the firm can choose between allowing consumer storage and implementing the static monopoly solution (the feasibility constraint (6) is satisfied). Since the firm's profits in the presence of consumer storage decrease with storage costs while the static monopoly profits are unaffected, the storing option is profit superior as long as the storage cost is below a certain threshold, i.e.,  $s_c < \bar{s}_c^*$ , where  $\bar{s}_c^* \in (\Delta c + \Delta \mu^m, \bar{s}_c^*)$ .<sup>10</sup> For  $s_c \geq \bar{s}_c^*$ , the firm sets the static monopoly prices and consumers abstain from storing, as point (ii) of Corollary 3 indicates.

<sup>9</sup>We refer to the proof of Proposition 2 in the Appendix for technical details.

<sup>10</sup>The proof of Proposition 2 in the Appendix provides a formal characterization of the existence and uniqueness of the threshold  $\bar{s}_c^* \in (\Delta c + \Delta \mu^m, \bar{s}_c^*)$ .

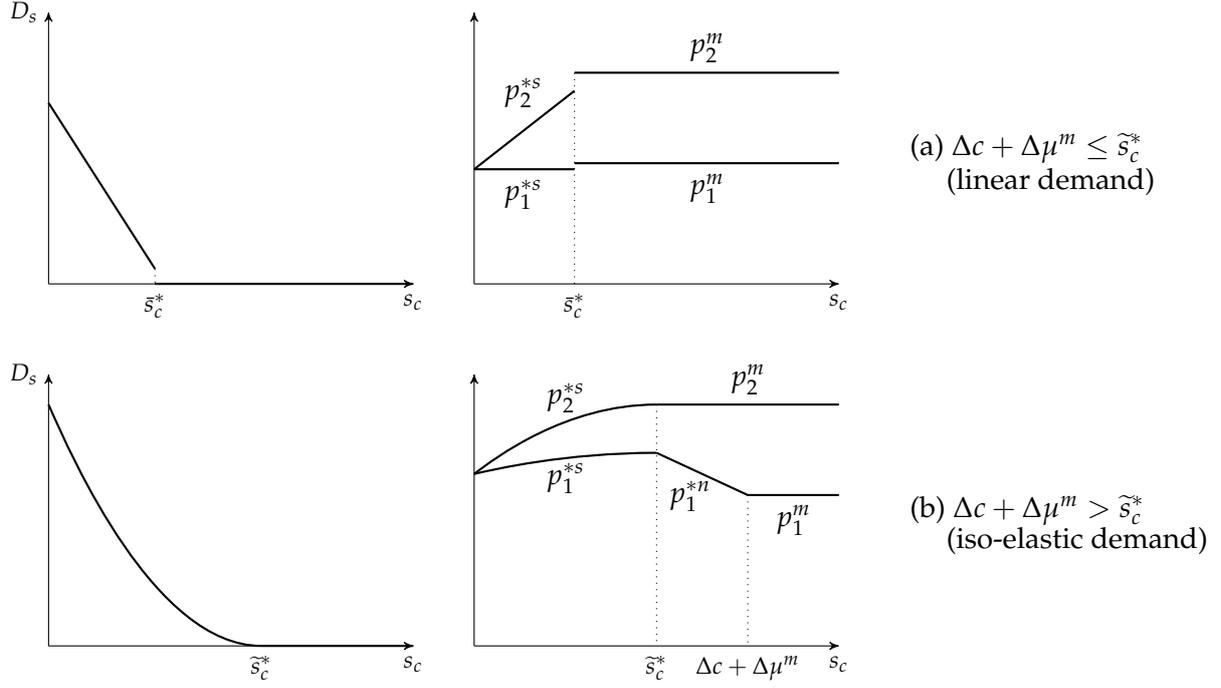


Figure 4: Equilibrium consumer storage and price patterns under limited commitment

The equilibrium consumer storage and price sequence as a function of  $s_c$  are illustrated in panel (a) of Figure 4 for the example of linear demand. Similarly to the case of full commitment, they are discontinuous at  $\tilde{s}_c^*$ . The first period storing price  $p_1^{*s}$  is independent of  $s_c$ , which is, however, an artifact of the linear demand specification. To attract consumer storage,  $p_1^{*s}$  is distorted below  $p_1^m$ . A more general result is that the second period storing price  $p_2^{*s}$  is now unambiguously lower than  $p_2^m$ . The firm's lack of commitment removes the possibility of a second period price above the static monopoly level, since the firm would have an incentive to reduce this price irrespective of the consumer storage behavior.

We now turn to the case  $\Delta c + \Delta \mu^m > \tilde{s}_c^*$ , which is formalized in the following corollary and illustrated in panel (b) of Figure 3. The outcome in point (iib) of Proposition 2 is feasible instead of the outcome in point (iia). This case applies if  $\Delta \mu^m > 0$ , which occurs when the demand is sufficiently convex.

**Corollary 4** Suppose  $\Delta c + \Delta \mu^m > \tilde{s}_c^*$ . Then, under limited commitment,

(i) if  $s_c < \tilde{s}_c^*$ , consumer storage is  $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$ , and prices are  $p_1^{*s}$  and  $p_2^{*s} = p_1^{*s} + s_c$ , where  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ ;

(ii) if  $\tilde{s}_c^* \leq s_c < \Delta c + \Delta \mu^m$ , consumer storage is  $D_s^{*n} = 0$ , and prices are  $p_1^{*n} = p_2^{*n} - s_c$  and  $p_2^{*n} = p_2^m$ ;

(iii) if  $s_c \geq \Delta c + \Delta \mu^m$ , consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m$  and  $p_2^m$ .

It holds  $p_1^{*n} > p_1^m$  and  $p_2^m > p_2^{*s}$ .

We can see from point (i) of Corollary 4 that, as in point (i) of Corollary 3, consumer storage occurs in equilibrium for sufficiently low storage costs. A comparison between panels (a) and (b) of Figure 3 shows that the firm now prefers to induce consumer storage as long as it is feasible, i.e., if  $s_c < \tilde{s}_c^*$ . For intermediate consumer storage costs, i.e.,  $\tilde{s}_c^* \leq s_c < \Delta c + \Delta \mu^m$ ,

consumer storage is unfeasible but the firm cannot implement the static monopoly solution (the feasibility constraint (6) is violated). The outcome in point (ii) of Corollary 4 reveals that the firm sets a price sequence such that consumers are indifferent to storing (the storability constraint is binding) but consumer storage vanishes in equilibrium. This resembles the full commitment outcome in point (ii) of Corollary 2 illustrated in panel (b) of Figure 1. However, under limited commitment the first period no-storing price  $p_1^{*n}$  is distorted above the static monopoly level while second period no-storing price  $p_2^{*n}$  coincides with the static monopoly level, which constitutes the firm's best response to the absence of consumer storage. As point (iii) of Corollary 4 indicates, the static monopoly solution is implemented as soon as it is available, i.e., for  $s_c \geq \Delta c + \Delta \mu^m$ .

The equilibrium consumer storage and price sequence as a function of  $s_c$  are illustrated in panel (b) of Figure 4 for the example of iso-elastic demand. As under full commitment in panel (b) of Figure 2, they are continuous functions. The depicted pattern of first period storing price  $p_1^{*s}$  does not hold generally, because  $p_1^{*s}$  varies with  $s_c$  according to the shape of the demand curvature. The first period no-storing price  $p_1^{*n} = p_2^m - s_c$  lies above the static monopoly level and decreases linearly with  $s_c$  (due to the binding storability constraint). Hence, the first period pricing policy is non-monotonic. The second period pricing policy monotonically increases with  $s_c$  and coincides with the static monopoly level when consumer storage is no longer feasible.

It is worth noting that the consumer storage demand  $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$  in point (i) of Corollaries 3 and 4 (with  $\phi_2(\cdot)$  defined by (5)) is positive if and only if  $\frac{p_2^{*s} - c_2}{p_2^{*s}} < \frac{1}{\varepsilon_{p_2^{*s}}}$ , where  $\frac{p_2^{*s} - c_2}{p_2^{*s}}$  is the relative markup (or Lerner index) and  $\varepsilon_{p_2^{*s}}$  is the demand elasticity evaluated at the second period storing price  $p_2^{*s}$ . We know from Corollaries 3 and 4 that consumer storage can be sustained in equilibrium only if the second period price is lower than the static monopoly level. This downward price distortion implies that the relative markup falls below the inverse of the demand elasticity. In order to relax this constraint, the firm charges the second period price in a relatively more inelastic demand region.

## 6 Price comparisons

Equipped with the results in the previous sections, we are now in a position to compare the equilibrium prices under the two regimes of full and limited commitment. For the sake of convenience, we define  $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1)$  as the second period demand net of consumer storage.

**Proposition 3** *Suppose  $s_c < s_c^l$ , where  $s_c^l$  is defined by (20) in the Appendix. Then, in each period the price under limited commitment is lower than the price under full commitment, i.e.,  $p_\tau^{*s} < p_\tau^{cs}$ ,  $\tau \in \{1, 2\}$ , if and only if  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$ .*

Proposition 3 shows that, under certain circumstances, the firm's lack of commitment leads to lower prices in each period. Since production costs rise in the second period and a myopic firm only cares about its current profits after the second period starts, one might be tempted to believe that the firm ends up charging excessively high prices. Indeed, we show that an

increase in production costs can lead to unambiguously lower prices under limited commitment. To appreciate the rationale for this result as substantiated in the introduction, recall from Proposition 1 that the full commitment storing price sequence is such that consumers store the entire second period demand and therefore the market shuts down in the second period. However, as discussed in Section 5, this pricing policy is not sequentially optimal, since the firm succumbs to the temptation to lower its price and serve the market in the second period. As stated in Proposition 3, suppose that the consumer storage cost is sufficiently low, i.e.,  $s_c < s_c^l$ , where  $s_c^l$  defined by (20) in the Appendix is the value for  $s_c$  below which consumer storage occurs irrespective of the firm's commitment powers, which implies that the storability constraint is always binding. Since  $s_c^l \leq \Delta c$ , the additional cost  $\Delta c$  of producing in the second period outweighs the additional price  $s_c$  that consumers are available to pay. Hence, the firm has an ex ante incentive to discourage purchases in the second period. To this aim, the only instrument that the firm can adopt under limited commitment is the price in the first period. Notably, a manipulation of the first period price affects the firm's problem in a non-trivial manner.

Consider the case in which the second period demand net of consumer storage is upward sloping, i.e.,  $\frac{\partial D_2^N}{\partial p_1} > 0$ . A lower first period price translates into a lower second period net demand, since the increase in consumer storage  $D_s(p_1)$  associated with a lower first period price  $p_1$  more than compensates the increase in the second period gross demand  $D(p_1 + s_c)$  driven by a corresponding lower second period price  $p_1 + s_c$  (the storability constraint is binding). As formally shown in the Appendix, this occurs as long as the demand is not significantly convex, which is satisfied under commonly used demand specifications, such as linear, exponential, and iso-elastic demand (if the degree of elasticity is not too high). In this case, a reduction in the first period price is profitable, because the stimulation of consumer storage leads to a lower second period net demand, which alleviates the firm's loss from the lack of commitment. Given that the storability constraint is binding, the second period price declines as well. As a result, a firm with limited commitment powers charges unambiguously lower prices than under full commitment. Since for  $s_c < s_c^l$  the storing prices  $p_\tau^{cs}$  and  $p_\tau^{*s}$  are charged under full and limited commitment, we find that  $p_\tau^{*s} < p_\tau^{cs}$ ,  $\tau \in \{1, 2\}$ .

When the net demand in the second period is downward sloping, i.e.,  $\frac{\partial D_2^N}{\partial p_1} < 0$ , the firm must increase the first period price in order to reduce the second period net demand. This occurs only when the demand is significantly convex. In this case, the reduction in the second period gross demand associated with a higher price more than compensates the reduction in consumer storage, which implies that the second period residual demand declines. Consequently, the firm charges higher prices than under full commitment in order to reduce the second period residual demand and alleviate the loss from the lack of commitment. Despite higher prices, consumer storage still occurs in equilibrium. Since the storability constraint is binding, consumers are indeed indifferent to storing, and the equilibrium storing level is dictated by the sequential optimality constraint.

As discussed in the introduction, our paper is closely related to Dudine et al. (2006), which shows that, in a storable good market where demand increases over time, the firm's inability to commit to future prices leads to unambiguously higher prices and lower welfare. We find

that allowing for intertemporal cost variations in a storable good market significantly alters the effects of the firm's lack of commitment on pricing and welfare. It follows from Proposition 3 that, for relatively low consumer storage costs, limited commitment generally leads to lower prices. We have seen that this result is driven in a non-trivial manner by the firm's incentive to manipulate its price strategy to reduce the second period residual demand. As shown in the sequel, the price and welfare effects critically change with the magnitude of consumer storage costs. Therefore, our results complement the findings of Dudine et al. (2006) and indicate that the impact of the firm's commitment powers becomes more heterogeneous in relatively mature markets where demand tends to be stable and production costs vary over time.

The following proposition completes the characterization of the price comparisons between full and limited commitment for higher values of consumer storage costs.

**Proposition 4** *A. Suppose  $s_c^l \leq s_c < s_c^h$ , where  $s_c^l$  and  $s_c^h$  are defined by (20) and (21) in the Appendix. Then, in the first period the price under limited commitment is higher than the price under full commitment. If  $\Delta\mu^m > 0$ , the price under limited commitment is also higher in the second period.*

*B. Suppose  $s_c \geq s_c^h$ . Then, in each period the price under limited commitment coincides with the price under full commitment and corresponds to the static monopoly price.*

Proposition 4A indicates that, for intermediate storage costs, i.e.,  $s_c^l \leq s_c < s_c^h$ , the price comparisons change substantially. The price in the first period is now higher under limited commitment. Although the equilibrium price differs within each regime according to the parameter constellations, a common rationale for this result can be identified. The lower ability to stimulate consumer storage implies that a firm with limited commitment powers forgoes consumer storage when it is still profitable under full commitment. Alternatively, consumer storage is either allowed or deterred ex ante suboptimally when it is unprofitable under full commitment. In the case where consumer storage is removed, the firm charges a first period price higher than under full commitment at which consumer storage vanishes. In the case where consumer storage is allowed, differently from Proposition 3, limited commitment leads to higher prices, since consumer storage is ex ante inefficient.<sup>11</sup> As Proposition 4 indicates, when the demand is sufficiently convex ( $\Delta\mu^m > 0$ ), the second period price is also unambiguously higher under limited commitment, because the storability constraint is binding under both commitment regimes. However, this may not hold if the demand is not too convex ( $\Delta\mu^m \leq 0$ ). The comparison between the second period prices becomes problematic when the static monopoly prices  $p_1^m$  and  $p_2^m$  are set under limited commitment, while the storing prices  $p_1^{cs}$  and  $p_2^{cs}$  are chosen under full commitment. Since the storability constraint is slack under limited commitment ( $p_1^m + s_c \geq p_2^m$  by the feasibility constraint (6)) but it is binding under full commitment ( $p_1^{cs} + s_c = p_2^{cs}$  by Proposition 1), a higher first period price under limited commitment does not necessarily imply a higher price in the second period as well. Indeed, since  $p_2^{cs}$  increases with  $s_c$  (see Corollaries 1 and 2) but  $p_2^m$  is unaffected, there might exist a threshold for  $s_c$  above which the second period price is lower under limited commitment.

<sup>11</sup>This case occurs when the consumer storage cost  $s_c$  is larger than the increase in production costs  $\Delta c$  and the storability constraint is binding under both commitment regimes. The firm's problem is equivalent to charging the same price  $p_1$  in each period and facing an additional cost in the first period equal to  $s_c - \Delta c > 0$ . Hence, consumer storage leads to higher prices under limited commitment. We refer to the proof of Proposition 4 in the Appendix for technical details.

Proposition 4B indicates that the firm opts for the static monopoly prices irrespective of its commitment powers when consumer storage costs are sufficiently high, i.e.,  $s_c \geq s_c^h$ , where  $s_c^h$  defined by (21) in the Appendix is such that  $s_c^h \geq \Delta c + \Delta \mu^m$ . Consumer storage is so costly that it renders the static monopoly solution not only implementable (the feasibility constraint (6) holds) but also optimal under both commitment regimes.

It is worth exploring the impact of a change in the second period costs on the equilibrium storing prices under full and limited commitment. This is formalized in the following corollary.

**Corollary 5** *For a given  $c_1$ , it holds*

- (i)  $\frac{\partial p_\tau^{cs}}{\partial \Delta c} = 0, \tau \in \{1, 2\}$ ;
- (ii)  $\frac{\partial p_\tau^{*s}}{\partial \Delta c} < 0$  if and only if  $\frac{\partial}{\partial \Delta c} \left[ (\Delta c - s_c) \frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \right] > 0, \tau \in \{1, 2\}$ .

The full commitment storing price  $p_\tau^{cs}$  is clearly independent of the magnitude of the cost increase  $\Delta c$ , since production only takes place in the first period. The relation between the cost increase  $\Delta c$  and the limited commitment storing price  $p_\tau^{*s}$  reflects the impact of  $\Delta c$  on the firm's loss  $(\Delta c - s_c)$  from serving the market in the second period weighted by the slope of the second period net demand function  $\frac{\partial D_2^N}{\partial p_1}$ . This is the result of the trade-off between two effects. To gain some intuition, note that  $\frac{\partial}{\partial \Delta c} \left[ (\Delta c - s_c) \frac{\partial D_2^N}{\partial p_1} \right] = \frac{\partial D_2^N}{\partial p_1} + (\Delta c - s_c) \frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c}$ . The first term captures the direct effect of  $\Delta c$  on the firm's loss, which corresponds to the price impact on the second period net demand, i.e.,  $\frac{\partial D_2^N}{\partial p_1}$ . The second term measures the indirect effect of  $\Delta c$  on the firm's loss through the price channel, which corresponds to the responsiveness of  $\frac{\partial D_2^N}{\partial p_1}$  to  $\Delta c$ , i.e.,  $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c}$ , weighted by the loss  $(\Delta c - s_c)$ . Since  $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c)$ ,<sup>12</sup> the trade-off between the two effects crucially depends on the demand curvature. First, suppose that the demand is concave, i.e.,  $D''(\cdot) < 0$ . We know from the discussion after Proposition 3 that the second period net demand is upward sloping, i.e.,  $\frac{\partial D_2^N}{\partial p_1} > 0$ . Hence, the first effect is positive and pushes toward a reduction in the first period price in order to dampen the second period net demand. The reason is that a higher  $\Delta c$  induces the firm to implement a more aggressive pricing policy to save production costs in the second period. However, the second effect is negative, i.e.,  $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c) < 0$ . This is because a higher  $\Delta c$  mitigates the negative slope of the consumer storage function, which becomes flatter ( $\frac{\partial^2 D_s}{\partial p_1 \partial \Delta c} = -\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} > 0$ ). A more rigid storage demand creates an incentive to increase the first period price. As long as  $\Delta c$  is small enough, the first effect dominates the second effect since the firm's loss  $(\Delta c - s_c)$  is relatively low, and therefore the limited commitment price decreases with  $\Delta c$ . However, for relatively high values of  $\Delta c$ , the second effect tends to prevail, and the limited commitment price increases in response to a rise in  $\Delta c$ .

Suppose now that the demand is (weakly) convex, i.e.,  $D''(\cdot) \geq 0$ . It follows from the discussion after Proposition 3 that, as long as the demand is not significantly convex, the first effect is still positive, i.e.,  $\frac{\partial D_2^N}{\partial p_1} > 0$ . The second effect is now (weakly) positive as well, i.e.,  $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c) \geq 0$ . The idea is that a higher  $\Delta c$  steepens the storage demand function ( $\frac{\partial^2 D_s}{\partial p_1 \partial \Delta c} = -\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} \leq 0$ ). A more elastic storage demand strengthens the incentive to reduce the

<sup>12</sup>We refer to the proof of Corollary 5 in the Appendix for a formal derivation.

first period price. This implies that the limited commitment price unambiguously decreases with  $\Delta c$ . For instance, this is the case with linear demand (where the second effect disappears). When the demand is significantly convex, we know from the discussion after Proposition 3 that the first effect becomes negative, i.e.,  $\frac{\partial D_2^N}{\partial p_1} < 0$ . Since the second effect is still positive, the trade-off between the two effects yields the opposite results to those with concave demand, and the limited commitment price increases with  $\Delta c$  for  $\Delta c$  small enough but decreases with  $\Delta c$  otherwise.

## 7 Welfare analysis

We now investigate consumer surplus and total welfare associated with the firm's ability to commit to future prices. Total welfare is computed as the (unweighted) sum of consumer surplus and firm's profits. For illustrative purposes, we characterize in the Supplementary Appendix the equilibrium price sequence and the welfare effects for the case of linear demand.

The following proposition considers the situation in which consumer storage costs are relatively low, as in Proposition 3.

**Proposition 5** *Suppose  $s_c < s_c^l$ , where  $s_c^l$  is defined by (20) in the Appendix. Then, if  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$ ,*

*(i) consumer surplus is higher under limited commitment than under full commitment;*

*(ii) total welfare is higher under limited commitment than under full commitment if the second period net demand  $D_2^N(p_1^{*s})$  is small enough.*

*If  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \leq 0$ , consumer surplus and total welfare are lower under limited commitment than under full commitment.*

Proposition 5 characterizes the conditions under which a myopic pricing policy is welfare enhancing, when consumer storage costs are sufficiently low that consumer storage occurs irrespective of the firm's commitment powers, i.e.,  $s_c < s_c^l$ . Note that the quantity bought at the unit price  $p_1$  and stored by consumers in the first period is actually consumed in the second period at the additional unit cost  $s_c$ . This generates the same consumer surplus as if consumers had bought that quantity in the second period at the price  $p_1 + s_c$ . Since for  $s_c < s_c^l$  consumers are indifferent to storing under both commitment regimes, i.e.,  $p_2 = p_1 + s_c$ , consumer surplus is higher under a commitment regime if and only if prices are lower, irrespective of the level of consumer storage. It follows from Proposition 3 that limited commitment increases consumer surplus if and only if the second period net demand is positively sloped, i.e.,  $\frac{\partial D_2^N}{\partial p_1} > 0$ , which is the case when the demand is not significantly convex.

The comparison in terms of total welfare between the two regimes differs from the standard case of a price change. Intuitively, a lower price under limited commitment increases total welfare since it mitigates the deadweight loss from monopoly power. However, limited commitment harms per se the firm's profits, in addition to the mere price reduction. As Proposition 5 indicates, despite the firm's loss, a regime of limited commitment that increases consumer surplus is even total welfare superior, provided that the second period demand net of consumer storage is small enough. Recall from the discussion in Section 6 that for  $s_c < s_c^l$  the additional cost  $\Delta c$  of producing in the second period exceeds the additional price  $s_c$  that

consumers are available to pay ( $s_c^l \leq \Delta c$ ), which implies that production in the second period is socially inefficient. Since a myopic firm cannot refrain from serving the market in the second period, limited commitment improves total welfare if the second period sales are relatively low. As formally shown in the Supplementary Appendix, with a linear demand of the form  $D(p_\tau) = \alpha - \beta p_\tau$ ,  $\tau \in \{1, 2\}$ , limited commitment is total welfare enhancing for intermediate values of the cost increase, i.e.,  $\widetilde{\Delta c} < \Delta c < \Delta c^{\max}$ . The rationale is intuitive in the light of our analysis so far. A higher second period cost induces a firm with limited commitment powers to decrease prices in order to attract consumer storage and reduce the second period net demand, which translates into higher consumer surplus and higher total welfare. Note that the cost increase  $\Delta c$  must be below the threshold  $\Delta c^{\max}$  that guarantees a positive profit margin in the second period. If the magnitude of the cost increase is sufficiently pronounced, i.e.,  $\Delta c \geq \Delta c^{\max}$ , even a myopic firm does not produce in the second period, and the firm's problem of limited commitment becomes less interesting.<sup>13</sup> An additional critical factor is the slope  $\beta$  of the demand function. A higher  $\beta$  reduces  $\widetilde{\Delta c}$ , and therefore relaxes the condition about the cost level above which limited commitment is total welfare superior. A more elastic demand (due to a higher  $\beta$ ) induces the firm to charge lower prices and stimulates consumer storage, which reduces the net demand in the second period and therefore the firm's loss from the lack of commitment.

For the sake of completeness, the following remark formalizes the welfare results for higher values of consumer storage costs.

**Remark 1** *A. Suppose  $s_c^l \leq s_c < s_c^h$ , where  $s_c^l$  and  $s_c^h$  are defined by (20) and (21) in the Appendix. Then, if  $\Delta\mu^m > 0$ , consumer surplus and total welfare are lower under limited commitment than under full commitment.*

*B. Suppose  $s_c \geq s_c^h$ . Then, consumer surplus and total welfare are the same under the two regimes and coincide with the static monopoly level.*

The results in Remark 1 are a direct consequence of Proposition 4.<sup>14</sup> We know that, for intermediate storage costs, i.e.,  $s_c^l \leq s_c < s_c^h$ , prices are unambiguously higher under limited commitment as long as the demand is sufficiently convex ( $\Delta\mu^m > 0$ ), which leads to lower consumer surplus.<sup>15</sup> Given that the firm's profits are lower, limited commitment is definitely welfare detrimental. When the demand is not too convex ( $\Delta\mu^m \leq 0$ ), the first period price is higher under limited commitment, but no clear-cut result can be derived in the second period. A higher first period price and lower profits suggest that limited commitment tends to be still welfare detrimental, but a more rigorous welfare analysis can be only conducted in a more specific setting. For instance, as formally shown in the Supplementary Appendix, in a linear demand framework limited commitment definitively reduces consumer surplus and, a fortiori, overall welfare. When consumer storage costs are high enough, i.e.,  $s_c \geq s_c^h$ , the firm always

<sup>13</sup>As argued in Section 5, when the cost increase is so significant that the full commitment second period storing price is lower than the second period cost, i.e.,  $p_2^{cs} < c_2$ , the full commitment outcome can be achieved and the firm's problem of limited commitment trivially disappears.

<sup>14</sup>The proof of Remark 1 is omitted since it directly follows from Proposition 4.

<sup>15</sup>Since consumers are indifferent to storing under both commitment regimes (the storability constraint is binding), the level of consumer storage is irrelevant.

implements the static monopoly solution, and therefore the firm's commitment powers are welfare inconsequential.

## 8 Robustness and extensions

### 8.1 Cost uncertainty

A natural extension of our model is to allow for uncertainty about future production costs. To preserve the relevance of our analysis, the second period expected cost must be higher than the first period cost, i.e.,  $\mathbb{E}[c_2] > c_1$ . Cost expectations can be formed in several manners. For instance, systematic empirical evidence about the oil market indicates that crude oil prices evolve according to a mean reversion pattern (e.g., Anderson et al. forthcoming; Bessembinder et al. 1995; Deltas 2008). Following a negative (positive) cost shock, future costs are expected to be higher (lower) than current costs.<sup>16</sup>

Under full commitment the introduction of cost uncertainty does not crucially affect the firm's pricing policy, which is established in the first period according to future expected costs. Things are different under limited commitment, since the future price is set only when the relevant period commences, and therefore it depends on the realization of the cost shock. A stochastic cost process may yield significant welfare effects. To fix ideas, consider a continuum of risk-neutral, profit-maximizing, competitive arbitrageurs (or speculators) that purchase the good at the price  $p_1$  from the firm in the first period and, after incurring the storage cost  $s_c$ , resell it to final consumers in the second period. When the storage cost is sufficiently low that storage is profitable for the firm, competition among arbitrageurs implies that the second period expected price reflects the first period price inflated by the storage cost, i.e.,  $\mathbb{E}[p_2] = p_1 + s_c$ . Consumers are ex ante indifferent to buy in the second period from the firm or the competitive arbitrageurs. Aggregate expected consumer surplus is

$$\mathbb{E}[CS] = \int_{p_1} D(p_1) dp_1 + \mathbb{E} \int_{p_2} D(p_2) dp_2.$$

It follows from Waugh (1944)'s classical result that, since consumer surplus is convex in prices, consumers benefit from fluctuating prices. Hence, cost uncertainty creates a shift in the consumer preferences in favor of limited commitment.

Using (2) and (3), the firm's aggregate expected profits are given by

$$\begin{aligned} \mathbb{E}[\Pi] &= (p_1 - c_1) [D(p_1) + D_s(p_1)] + \mathbb{E}[(p_2 - c_2) (D(p_2) - D_s(p_1))] \\ &= (p_1 - c_1) [D(p_1) + D_s(p_1)] + (\mathbb{E}[p_2] - \mathbb{E}[c_2]) (\mathbb{E}[D(p_2)] - D_s(p_1)) \\ &\quad + cov[(p_2 - c_2), D(p_2)], \end{aligned}$$

where  $cov[\cdot]$  is the covariance operator. Note that, as long as the demand is not too convex, an increase in the second period cost  $c_2$  translates into a smaller increase in the second period price  $p_2$ . A lower value of the profit margin  $p_2 - c_2$  driven by a larger  $c_2$  is associated with a

<sup>16</sup>This stochastic process resembles the one adopted by Cabral and Fishman (2012) and Antoniou et al. (2017).

lower value of the demand  $D(p_2)$ , and therefore the covariance between  $p_2 - c_2$  and  $D(p_2)$  is positive. This suggests that the uncertainty about future costs also mitigates the firm's loss from the lack of commitment.

To obtain explicit results, suppose that the second period cost is  $c_2 = c_1 + \Delta c + \theta$ , where  $\theta$  is a stochastic term with zero mean, which implies that  $\mathbb{E}[c_2] = c_1 + \Delta c$ . With a linear demand function of the form  $D(p_\tau) = \alpha - \beta p_\tau$ ,  $\tau \in \{1, 2\}$ , it is straightforward to show that the aggregate expected consumer surplus and firm's profits are

$$\mathbb{E}[CS^{*s}] = CS^{*s} + \frac{\beta}{8} \text{var}[\theta] \quad \text{and} \quad \mathbb{E}[\Pi^{*s}] = \Pi^{*s} + \frac{\beta}{4} \text{var}[\theta],$$

where  $CS^{*s}$  and  $\Pi^{*s}$  denote the consumer surplus and firm's profits in a deterministic environment with  $c_2 = c_1 + \Delta c$  (see the Supplementary Appendix), and  $\text{var}[\cdot]$  is the variance operator. Both the consumers and the firm benefit from cost uncertainty, which is definitively (expected) total welfare enhancing.<sup>17</sup>

It is also of some interest to consider the case where competitive arbitrageurs do not operate in the market and final consumers may directly engage in storing activities. It follows from the previous discussion that, since consumer surplus is convex in prices, consumers prefer ex ante to buy a unit of the good in the second period at a random price  $p_2$  rather than at a deterministic price equal to  $\mathbb{E}[p_2]$ . Given that storing involves an aggregate deterministic cost  $p_1 + s_c$ , the condition under which consumers are indifferent to storing is such that  $p_1 + s_c < \mathbb{E}[p_2]$ . In this case, the firm must reduce the first period price to a larger extent to stimulate consumer storage, which tends to increase consumer surplus. The welfare superiority of limited commitment derived in Section 7 can be even more pronounced in the presence of cost uncertainty.

## 8.2 Number of periods

Our results can be generalized to a time horizon with more than two periods. There are various reasons that make a two-period model suitable for our purposes, aside from its analytical tractability. Predictions about the evolution of costs tend to be accurate only in the near future. Furthermore, storable goods typically depreciate over time and can be accumulated only for a limited amount of time. Nonetheless, the analysis of a larger time horizon warrants some attention. To fix ideas, consider a setting with  $T \geq 2$  periods, where production costs are expected to increase in each period, i.e.,  $\Delta c_\tau \equiv c_\tau - c_{\tau-1} > 0$ ,  $\tau \in \{2, \dots, T\}$ , and storage costs are sufficiently low that storing is profitable in each period irrespective of the firm's commitment powers, which requires  $s_c < \Delta c_\tau$ . It follows from our analysis that the storability constraint is binding in each period, i.e.,  $p_\tau = p_{\tau-1} + s_c$ ,  $\tau \in \{2, \dots, T\}$ . Formally, under full commitment the firm's maximization problem is given by

<sup>17</sup>Explicit calculations are available upon request. These results qualitatively persist under more general conditions. We refer to Antoniou and Tsakiris (2016) for a useful technique.

$$\max_{p_1} (p_1 - c_1) \sum_{\tau=1}^T D(p_\tau) \quad s.t. \quad p_\tau = p_{\tau-1} + s_c.$$

In line with the baseline model, a firm that can fully commit to a price sequence induces consumers to store in the first period the entire quantity consumed in the following periods, since the per period cost increase exceeds the consumer storage cost.

Under limited commitment the firm succumbs to the temptation to reduce prices below the full commitment level to serve any residual demand. Anticipating the firm's myopic behavior, consumers are less inclined to store. Formally, under limited commitment the firm's maximization problem is given by

$$\begin{aligned} \max_{p_1} (p_1 - c_1) \sum_{\tau=1}^T D(p_\tau) - \sum_{\tau=2}^T \{ [D(p_\tau) - D_s(p_{\tau-1}) + D_s(p_\tau)] \\ \times \left[ \sum_{t=2}^{\tau} \Delta c_t - (\tau - 1) s_c \right] \} \quad s.t. \quad p_\tau = p_{\tau-1} + s_c. \end{aligned}$$

The expression  $D(p_\tau) - D_s(p_{\tau-1}) + D_s(p_\tau)$  denotes the per period net future demand, namely, the demand for consumption  $D(p_\tau)$  in period  $\tau \in \{2, \dots, T\}$ , reduced by the consumer storage  $D_s(p_{\tau-1})$  in period  $\tau - 1$  and inflated by the consumer storage  $D_s(p_\tau)$  in period  $\tau$  (where  $D_s(p_\tau) = 0$  for  $\tau = T$  since no storage occurs in the final period). The amount of consumer storage in each period is determined in equilibrium by the condition of sequential optimality and the binding storability constraint. Each unit of the net future demand in period  $\tau \in \{2, \dots, T\}$  entails a loss equal to  $\sum_{t=2}^{\tau} \Delta c_t - (\tau - 1) s_c$ , which corresponds to the excess of the aggregate cost increases from the initial period over the aggregate storage costs. In line with the baseline model, when the net future demand is positively sloped, a myopic firm resorts to lower prices in order to stimulate consumer storage, which reduces future production that takes place at increasingly higher costs.<sup>18</sup> Notably, a longer sequence of periods with increasing costs aggravates the firm's loss from serving the future demand, which magnifies the firm's incentive for a downward price distortion. Hence, the result of higher consumer surplus under limited commitment derived in Section 7 can be stronger as the time horizon extends. This is consistent with the empirical evidence discussed in the introduction that a sequence of periods of increasing costs mitigates the firm's market power.

### 8.3 Firm's inventories

Throughout the paper we focus on the possibility of consumer storage. However, it is reasonable to think that in storable good markets firms can accumulate inventories as well. This option is naturally appealing when production costs are expected to increase over time. Consider the relevant case of relatively low consumer storage costs. We know from the previous analysis that a firm with full commitment powers is able to stick to a price sequence that induces

<sup>18</sup>For instance, in a three-period setting with a linear demand of the form  $D(p_\tau) = \alpha - \beta p_\tau$ ,  $\tau \in \{1, 2, 3\}$ , the net demand increases with the price in the second and third periods, i.e.,  $D'(p_2) - D'_s(p_1) + D'_s(p_2) = 3\beta > 0$  and  $D'(p_3) - D'_s(p_2) = \beta > 0$ .

full consumer storage in the first period, and therefore no production occurs in the following period(s). This implies that under full commitment there is no scope for firm's inventories.

Things differ under limited commitment, since the firm cannot refrain from serving the future demand, which alleviates the consumer storage incentives. Intuitively, a technological effect comes into play, since the firm finds it profitable to resort to inventories in order to save future production costs when the cost increase is sufficiently pronounced that it outweighs the inventory holding cost. Notably, inventories can also exhibit a strategic effect in markets characterized by consumer storage. As shown by Antoniou and Fiocco (2017), a firm with limited commitment powers can use inventories to mitigate the consumer storage incentives in a setting where demand increases over time. Since the cost of producing inventories is sunk when they are sold, inventories act as strategic device to reduce future costs, which translates in lower future prices and alleviates the consumer storage incentives. In our framework, this is profit detrimental, since the firm benefits from consumer storage. A trade-off emerges between the technological and strategic effects and, if the former effect dominates, the firm prefers to accumulate inventories. Notably, the combination of the technological and strategic effects is beneficial for consumers. The lower aggregate production costs stemming from inventories translate into lower prices and the cost sunkness of inventories exacerbates the firm's temptation to serve the future demand at lower prices, which induces the firm to decrease its prices in order to stimulate consumer storage. Therefore, the introduction of firm's inventories reinforces our result of welfare superiority of limited commitment.

#### **8.4 Competition**

It is also of some interest to explore the impact of competition among firms on our results. In a two-period Cournot duopoly setting where consumers can store for future consumption, Anton and Das Varma (2005) show that firms behave more aggressively to attract consumer storage. In a two-period differentiated good Bertrand framework, Guo and Villas-Boas (2007) find that the opportunity of consumer storage exacerbates price competition. When production costs are expected to increase over time, the firms' incentives to compete for consumer storage are magnified, which creates further pressure to lower prices. Therefore, the forces described in our paper complement those identified by Anton and Das Varma (2005) and Guo and Villas-Boas (2007). Their mutual reinforcement definitively enhances consumer surplus. Our analysis suggests that, when consumer storage costs are relatively low, the benefits for consumers tend to be larger under limited commitment.

#### **8.5 Discount factor**

In the baseline model, the firm's current and future profits share the same weight. In line with Dudine et al. (2006), this assumption is imposed for the sake of simplicity and our qualitative results carry over with a more general discount factor  $\delta$ , provided that the firm cares about the future to a sufficient extent. This is because the firm's commitment problem is relevant as long as the discounted second period costs exceed the first period costs, i.e.,  $\delta c_2 > c_1$ . Otherwise, the preferences of the firm and the consumers are trivially aligned against consumer storage

and the firm's commitment powers are inconsequential.

The consumer discount factor can be captured by the consumer storage cost  $s_c$ . It follows from our analysis that the firm's problem of limited commitment is more relevant when  $s_c$  is relatively low, namely, consumers are sufficiently forward-looking.

## 9 Concluding remarks

The interaction between the intertemporal strategies of firms and consumers is relevant in many dynamic settings, such as markets for storable goods. In this paper, we characterize a firm's pricing policy and the welfare effects associated with the firm's ability to commit to future prices in a dynamic storable good market where consumers are willing to store and production costs change over time. We show that, when costs are expected to increase, the firm's lack of commitment generally leads to lower prices as long as consumer storage costs are sufficiently small. The rationale for this result arises in a non-trivial manner from the firm's manipulation of the price strategy in order to affect the future demand net of consumer storage. A firm with limited commitment powers cannot refrain from serving the future net demand, even though this is *ex ante* suboptimal because of higher costs. A reduction in the first period price stimulates consumer storage, which generally reduces the net demand in the second period and therefore alleviates the firm's loss from the lack of commitment. Since the storability constraint is binding, the second period price is also lower under limited commitment. As a result, consumer surplus definitively increases. Despite the firm's loss, under certain circumstances total welfare is also higher. For instance, this is the case for an intermediate increase in production costs in a linear demand framework. For intermediate values of consumer storage costs, the inefficient behavior of a firm with limited commitment powers when dealing with consumer storage can result in higher prices, which harms consumers and, *a fortiori*, the whole economy. When consumer storage costs are large enough, the static monopoly solution is implemented irrespective to the firm's commitment powers.

Our results can substantiate the stance of regulators and antitrust authorities on the firm's adoption of instruments that mitigate the commitment problem in storable good markets. A well-known contractual policy that the firm can implement to restore (or approach) the full commitment solution is a money-back guarantee — sometimes called “most-favored nation” clause — that commits the firm to reimburse its customers if the future price falls below the pre-announced level. In markets for durable goods where full commitment leads to higher prices, these price protection policies harm consumers, and therefore they should be prohibited. As shown by Dudine et al. (2006), in markets for storable goods where demand increases over time, the firm's lack of commitment is unambiguously welfare detrimental, which induces a positive evaluation of such contracts that improve the firm's commitment ability. Our analysis suggests that antitrust authorities should conduct a more sophisticated assessment in relatively mature markets for storable goods, where demand tends to remain stable while production costs vary over time. Specifically, contractual clauses that enhance the firm's commitment powers should be banned when consumer storage costs are relatively small, since they reduce consumer surplus and possibly total welfare as well. Conversely, when consumer storage

costs are larger, antitrust authorities should approve these policies, since they tend to benefit consumers and, a fortiori, the whole economy.

In the light of our result that a myopic pricing policy enhances consumer surplus and even total welfare for intermediate cost increases (when consumer storage costs are sufficiently small), we also provide support for the target of moderate inflation commonly adopted by central banks. As discussed in the introduction, our analysis sheds new light on the negative empirical relation between the rate of inflation and the firm's average markup. The predictions of our model lend themselves to an empirically testable validation and can stimulate further empirical investigation in markets for storable goods.

## Appendix

**Proof of Proposition 1.** The firm faces the following three pricing options: (I)  $p_1 + s_c > p_2$ ; (II)  $p_1 + s_c = p_2$ ; (III)  $p_1 + s_c < p_2$ .

**Option (I)**  $p_1 + s_c > p_2 \Rightarrow D_s(p_1) = 0$ . The firm's maximization problem is

$$\max_{p_1, p_2} (p_1 - c_1) D(p_1) + (p_2 - c_2) D(p_2).$$

The first order condition for  $p_\tau$ ,  $\tau \in \{1, 2\}$ , is given by

$$D(p_\tau) + (p_\tau - c_\tau) D'(p_\tau) = 0, \quad (7)$$

which yields the equilibrium static monopoly prices

$$p_1^m = c_1 + \mu_1^m \text{ and } p_2^m = c_2 + \mu_2^m, \quad (8)$$

where  $\mu_\tau^m$ ,  $\tau \in \{1, 2\}$ , is defined in Section 3. Consumer storage is  $D_s^m = 0$ . This solution is implementable if and only if the static monopoly feasibility constraint (6) holds.

**Option (II)**  $p_1 + s_c = p_2 \Rightarrow D_s(p_1) \in [0, D(p_1 + s_c)]$ . The firm's maximization problem is

$$\max_{p_1} (p_1 - c_1) [D(p_1) + D_s(p_1)] + (p_1 + s_c - c_2) [D(p_2) - D_s(p_1)]. \quad (9)$$

The following two cases emerge:

**(IIa)**  $s_c \leq \Delta c$ . Since the firm's profits in the maximand of (9) increase with  $D_s(p_1)$ , the firm prefers to induce full consumer storage, i.e.,  $D_s(p_1) = D(p_1 + s_c)$ . The firm's maximization problem reduces to

$$\max_{p_1} (p_1 - c_1) [D(p_1) + D(p_1 + s_c)]. \quad (10)$$

The first order condition for  $p_1$  is

$$D(p_1) + D(p_1 + s_c) + (p_1 - c_1) [D'(p_1) + D'(p_1 + s_c)] = 0. \quad (11)$$

Using (5) yields the equilibrium full commitment storing prices

$$p_1^{cs} = c_1 - \frac{D(p_1^{cs} + s_c) + \phi_1(p_1^{cs})}{D'(p_1^{cs} + s_c)} \text{ and } p_2^{cs} = p_1^{cs} + s_c. \quad (12)$$

**(IIb)**  $s_c > \Delta c$ . Since the firm's profits in the maximand of (9) decrease with  $D_s(p_1)$ , the firm prefers to deter consumer storage, i.e.,  $D_s(p_1) = 0$ . The firm's maximization problem becomes

$$\max_{p_1} (p_1 - c_1) D(p_1) + (p_1 + s_c - c_2) D(p_1 + s_c).$$

The first order condition for  $p_1$  is

$$D(p_1) + D(p_1 + s_c) + (p_1 - c_1) D'(p_1) + (p_1 + s_c - c_2) D'(p_1 + s_c) = 0. \quad (13)$$

Using (5) yields the equilibrium full commitment no-storing prices

$$p_1^{cn} = c_1 - \frac{D(p_1^{cn}) + \phi_2(p_1^{cn} + s_c)}{D'(p_1^{cn})} \text{ and } p_2^{cn} = p_1^{cn} + s_c. \quad (14)$$

**Option (III)**  $p_1 + s_c < p_2 \Rightarrow D_s(p_1) = D(p_1 + s_c)$ . This yields the same profits as in (IIa), and therefore it is irrelevant.

We obtain the following results.

**(i)** Suppose  $s_c < \min\{\Delta c + \Delta\mu^m, \Delta c\}$ . The only feasible option is (IIa), and the equilibrium prices are described by (12).

**(iia)** Suppose  $\Delta c + \Delta\mu^m \leq s_c \leq \Delta c$ . This interval is non-empty if and only if  $\Delta\mu^m \leq 0$ . The feasible options are (I) and (IIa), whose associated profits are  $\Pi^m$  and  $\Pi^{cs}$ . It follows from the feasibility constraint (6) that at the lower bound  $s_c = \Delta c + \Delta\mu^m$  it holds  $p_2^m = p_1^m + s_c$ . Substituting  $p_1^m$  and  $p_2^m = p_1^m + s_c$  into the maximand of (9) yields  $\Pi^{cs}(p_1^m) = \Pi^m(p_1^m)$ , where the equality follows since  $D_s(p_1^m) = 0$ . Then, the profit outcome in (I) can be replicated by (IIa). Since  $\Pi^{cs}(\cdot)$  is maximized at  $p_1^{cs}$ , which differs from  $p_1^m$ , we find that  $\Pi^{cs} > \Pi^m$  at the lower bound  $s_c = \Delta c + \Delta\mu^m$ . Now, consider the upper bound  $s_c = \Delta c$ . Then,  $\Pi^{cs}$  does not depend on consumer storage. As  $\Pi^m$  is characterized by no storage, without any loss of generality, we focus on the no-storage case. It holds  $\Pi^m > \Pi^{cs}$ , since  $\Pi^m$  is the solution to an unconstrained maximization problem. Taking the derivative of  $\Pi^{cs}$  with respect to  $s_c$  and using (11) yields  $\frac{\partial \Pi^{cs}}{\partial s_c} = (p_1^{cs} - c_1) D'(p_1^{cs} + s_c) < 0$ . As  $\Pi^m$  is independent of  $s_c$ , there exists a unique threshold  $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$  such that for  $s_c < \bar{s}_c^c$  it holds  $\Pi^{cs} > \Pi^m$  and the equilibrium prices are described by (12), while for  $s_c \geq \bar{s}_c^c$  it holds  $\Pi^m \geq \Pi^{cs}$  and the equilibrium prices are described by (8), where  $\Pi^{cs} = \Pi^m$  if and only if  $s_c = \bar{s}_c^c$ .

**(iib)** Suppose, alternatively,  $\Delta c \leq s_c < \Delta c + \Delta\mu^m$ . This interval is non-empty if and only if  $\Delta\mu^m > 0$ . The only feasible option is (IIb),<sup>19</sup> and the equilibrium prices are described by (14).

**(iii)** Suppose  $s_c \geq \max\{\Delta c + \Delta\mu^m, \Delta c\}$ . The feasible options are (I) and (IIb), whose associated profits are  $\Pi^m$  and  $\Pi^{cn}$ . Since consumer storage is absent under both options and  $\Pi^m$  is the solution to an unconstrained maximization problem, it holds  $\Pi^m > \Pi^{cn}$  and the

<sup>19</sup>Clearly, for  $s_c = \Delta c$  cases (IIa) and (IIb) coincide.

equilibrium prices are described by (8). ■

**Proof of Corollary 1.** Since  $\Delta\mu^m \leq 0$ , the equilibrium consumer storage and prices in points (i) and (ii) of the corollary are a direct consequence of the outcomes in points (i), (iia) and (iii) of Proposition 1. Taking the derivative of the left-hand side of the first order condition for  $p_1^{cs}$  in (11) with respect to  $s_c$  yields  $D'(p_1^{cs} + s_c) + (p_1^{cs} - c_1) D''(p_1^{cs} + s_c) < 0$ , where the inequality holds if and only if  $D''(p_1^{cs} + s_c) < \tilde{D}'' \equiv -\frac{D'(p_1^{cs} + s_c)}{p_1^{cs} - c_1}$ , with  $\tilde{D}'' > 0$ . It follows from the implicit function theorem that  $\frac{\partial p_1^{cs}}{\partial s_c} < 0$  for  $D''(\cdot) < \tilde{D}''$ . Moreover, the derivative of the left-hand side of the first order condition for  $p_2^{cs}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (11) — with respect to  $s_c$  is  $-2D'(p_2^{cs} - s_c) - (p_2^{cs} - s_c - c_1) D''(p_2^{cs} - s_c) - D'(p_2^{cs}) > 0$ , where the inequality follows from Assumption 1 and  $D'(\cdot) < 0$ . By the implicit function theorem, we obtain  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ . Now, we turn to the price comparisons. Substituting the first order condition for  $p_1^m$  in (7) into the left-hand side of the first order condition for  $p_1^{cs}$  in (11) yields  $D(p_1^m + s_c) + (p_1^m - c_1) D'(p_1^m + s_c)$ . This expression vanishes if and only if  $s_c = 0$ , which implies  $p_1^m = p_1^{cs}$ . Given that  $\frac{\partial p_1^{cs}}{\partial s_c} < 0$  for  $D''(\cdot) < \tilde{D}''$  while  $p_1^m$  is independent of  $s_c$ , if  $s_c > 0$  we obtain  $p_1^m > p_1^{cs}$  for  $D''(\cdot) < \tilde{D}''$ . ■

**Proof of Corollary 2.** Since  $\Delta\mu^m > 0$ , the equilibrium consumer storage and prices in points (i), (ii) and (iii) of the corollary are a direct consequence of the outcomes in points (i), (iib) and (iii) of Proposition 1. It follows from the proof of Corollary 1 that  $\frac{\partial p_1^{cs}}{\partial s_c} > 0$  for  $D''(\cdot) > \tilde{D}''$  and that  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ . Taking the derivative of the left-hand side of the first order condition for  $p_1^{cn}$  in (13) with respect to  $s_c$  yields  $2D'(p_1^{cn} + s_c) + (p_1^{cn} + s_c - c_2) D''(p_2^{cn} + s_c) < 0$ , where the inequality follows from Assumption 1. Using the implicit function theorem, we obtain  $\frac{\partial p_1^{cn}}{\partial s_c} < 0$ . Moreover, the derivative of the left-hand side of the first order condition for  $p_2^{cn}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (13) — with respect to  $s_c$  is  $-2D'(p_2^{cn} - s_c) - (p_2^{cn} - s_c - c_1) D''(p_2^{cn} - s_c) > 0$ , where the inequality follows from Assumption 1. We find from the implicit function theorem that  $\frac{\partial p_2^{cn}}{\partial s_c} > 0$ . Now, we turn to the price comparisons. Recall from the proof of Corollary 1 that  $p_1^{cs} = p_1^m$  if and only if  $s_c = 0$ . If  $s_c > 0$  we have  $p_1^{cs} > p_1^m$  for  $D''(\cdot) > \tilde{D}''$ , since  $\frac{\partial p_1^{cs}}{\partial s_c} > 0$  while  $p_1^m$  is independent of  $s_c$ . Moreover, we obtain  $p_1^{cn} > p_1^m$ , because  $\frac{\partial p_1^{cn}}{\partial s_c} < 0$  and the price strategy is continuous. Finally, it follows from  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ ,  $\frac{\partial p_2^{cn}}{\partial s_c} > 0$ ,  $\frac{\partial p_2^m}{\partial s_c} = 0$  and the continuity of the price strategy that  $p_2^m > p_2^{cn} > p_2^{cs}$ . ■

**Proof of Proposition 2.** Proceeding backwards, in the second period the firm maximizes the profits in (3). This yields the sequentially optimal second period price

$$p_2 = c_2 - \frac{D(p_2) - D_s(p_1)}{D'(p_2)}. \quad (15)$$

Moving to the first period, the firm faces the following three pricing options: (I)  $p_1 + s_c > p_2$ ; (II)  $p_1 + s_c = p_2$ ; (III)  $p_1 + s_c < p_2$ .

**Option (I)**  $p_1 + s_c > p_2 \Rightarrow D_s(p_1) = 0$ . As under full commitment, the equilibrium prices are set at the static monopoly level in (8). This solution is feasible if and only if the feasibility constraint (6) is fulfilled.

**Option (II)**  $p_1 + s_c = p_2 \Rightarrow D_s(p_1) \in [0, D(p_1 + s_c)]$ . The firm's first period maximization program is given by (9), subject to the sequential optimality constraint (15). Using (15) and  $p_1 + s_c = p_2$  yields  $D_s(p_1) = \max\{0, D(p_1 + s_c) + (p_1 + s_c - c_1 - \Delta c) D'(p_1 + s_c)\}$ . The following

two cases emerge.

**(IIa)** Let  $D_s(p_1) > 0$ . The firm's first period maximization problem becomes

$$\max_{p_1} (p_1 - c_1) [D(p_1) + D(p_1 + s_c)] + (\Delta c - s_c) (p_1 + s_c - c_1 - \Delta c) D'(p_1 + s_c). \quad (16)$$

The first order condition for  $p_1$  is

$$D(p_1) + D(p_1 + s_c) + (p_1 - c_1) [D'(p_1) + D'(p_1 + s_c)] + (\Delta c - s_c) [D'(p_1 + s_c) + (p_1 + s_c - c_1 - \Delta c) D''(p_1 + s_c)] = 0. \quad (17)$$

Using (5) yields the equilibrium limited commitment storing prices

$$p_1^{*s} = c_1 - \frac{D(p_1^{*s}) + \phi_2(p_1^{*s} + s_c) + (\Delta c - s_c) \phi_2'(p_1^{*s} + s_c)}{D'(p_1^{*s})} \text{ and } p_2^{*s} = p_1^{*s} + s_c. \quad (18)$$

Consumer storage is  $D_s^{*s} = D(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_1 - \Delta c) D'(p_1^{*s} + s_c) = \phi_2(p_1^{*s} + s_c)$ . To derive the condition under which this solution is feasible, i.e.,  $D_s^{*s} > 0$ , we compute the derivative of  $D_s^{*s}$  with respect to  $s_c$ . This yields

$$\frac{\partial D_s^{*s}}{\partial s_c} = \frac{\partial p_2^{*s}}{\partial s_c} [2D'(p_2^{*s}) + (p_2^{*s} - c_1 - \Delta c) D''(p_2^{*s})] < 0,$$

where the inequality holds since  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$  (see Corollaries 3 and 4) and the expression in square brackets is negative by Assumption 1. Now, we prove that  $D_s^{*s} > 0$  at  $s_c = 0$ . Note that this is the case if and only if  $D(p_1^{*s}) + (p_1^{*s} - c_1 - \Delta c) D'(p_1^{*s}) > 0$ , which means from the first order condition for  $p_2^m$  in (7) that  $p_1^{*s} = p_2^{*s} < p_2^m$ . Substituting (7) into the left-hand side of the first order condition for  $p_1^{*s}$  in (17) evaluated at  $s_c = 0$  yields after some manipulation  $\Delta c [2D'(p_2^m) + (p_2^m - c_2) D''(p_2^m)] + \Delta c D'(p_2^m) < 0$ , where the inequality follows since the expression in square brackets is negative (by Assumption 1) and  $D'(\cdot) < 0$ . This implies that  $p_1^{*s} = p_2^{*s} < p_2^m$  and therefore  $D_s^{*s} > 0$  at  $s_c = 0$ . By continuity, we have  $D_s^{*s} > 0$  for  $s_c$  small enough. Since  $D_s^{*s} < 0$  for  $s_c$  arbitrarily large, we can conclude that there exists a unique threshold  $\tilde{s}_c^* > 0$  such that  $D_s^{*s} > 0$  if and only if  $s_c < \tilde{s}_c^*$ .

**(IIb)** For  $s_c \geq \tilde{s}_c^*$ , consumers do not store, i.e.,  $D_s^{*n} = 0$ . It follows from (15) that the equilibrium limited commitment no-storing prices are

$$p_1^{*n} = p_2^{*n} - s_c \text{ and } p_2^{*n} = p_2^m = c_2 + \mu_2^m. \quad (19)$$

**Option (III)**  $p_1 + s_c < p_2 \Rightarrow D_s(p_1) = D(p_1 + s_c)$ . As discussed in Section 5, this option is not implementable, since the firm has an incentive to reduce  $p_2$  and serve the market in the second period.

We obtain the following results.

**(i)** Suppose  $s_c < \min\{\Delta c + \Delta\mu^m, \tilde{s}_c^*\}$ . The only feasible option is (IIa), and the equilibrium prices are described by (18).

**(ii)** Suppose  $\Delta c + \Delta\mu^m \leq s_c \leq \tilde{s}_c^*$ . The feasible options are (I) and (IIa).<sup>20</sup> We know from

<sup>20</sup>Clearly, for  $s_c = \tilde{s}_c^*$  cases (IIa) and (IIb) coincide.

the feasibility constraint (6) that at the lower bound  $s_c = \Delta c + \Delta\mu^m$  it holds  $p_2^m = p_1^m + s_c$ . Substituting  $p_1^m$  and  $p_2^m = p_1^m + s_c$  into the maximand of (9) yields  $\Pi^{*s}(p_1^m) = \Pi^m(p_1^m)$ , where the equality follows since  $D_s(p_1^m) = 0$ . Hence, the profit outcome in (I) can be replicated by (IIa). Since  $\Pi^{*s}(\cdot)$  is maximized at  $p_1^{*s}$ , which differs from  $p_1^m$ , we find that  $\Pi^{*s} > \Pi^m$  at the lower bound  $s_c = \Delta c + \Delta\mu^m$ . At the upper bound  $s_c = \tilde{s}_c^*$ , there is no storing in (IIa), and the same holds in (I). This implies that  $\Pi^m > \Pi^{*s}$ , since  $\Pi^m$  is the solution to an unconstrained maximization problem. Taking the derivative of  $\Pi^{*s}$  with respect to  $s_c$  and using (17) yields after some manipulation  $\frac{\partial \Pi^{*s}}{\partial s_c} = (\Delta c - s_c) [2D'(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_2) D''(p_1^{*s} + s_c)]$ , where the expression in square brackets is negative by Assumption 1. In principle, the following three cases emerge: (1) if  $\tilde{s}_c^* < \Delta c$ , then  $\frac{\partial \Pi^{*s}}{\partial s_c} < 0$ ; (2) if  $\Delta c + \Delta\mu^m \leq \Delta c \leq \tilde{s}_c^*$ , then  $\frac{\partial \Pi^{*s}}{\partial s_c} < 0$  for  $s_c < \Delta c$  and  $\frac{\partial \Pi^{*s}}{\partial s_c} > 0$  for  $s_c > \Delta c$  (with a minimum at  $s_c = \Delta c$ ); (3) if  $\Delta c < \Delta c + \Delta\mu^m$ , i.e.,  $\Delta\mu^m > 0$ , then  $\frac{\partial \Pi^{*s}}{\partial s_c} > 0$ . Note that case (3) is impossible, since it contradicts the previous result that  $\Pi^{*s} > \Pi^m$  at the lower bound  $s_c = \Delta c + \Delta\mu^m$  and  $\Pi^m > \Pi^{*s}$  at the upper bound  $s_c = \tilde{s}_c^*$ , where  $\Pi^m$  is independent of  $s_c$ . First, consider case (1). Since  $\frac{\partial \Pi^{*s}}{\partial s_c} < 0$ , there exists a unique point of equalization between  $\Pi^{*s}$  and  $\Pi^m$ . Now, consider case (2). Given that the impossibility of case (3) implies  $\Delta\mu^m \leq 0$ , we know from point (ii) of Corollary 1 that for  $s_c \geq \bar{s}_c^c$  the static monopoly solution is implemented under full commitment. Since it is sequentially optimal, the static monopoly solution must be also implemented under limited commitment. Given that  $\Pi^{*s} > \Pi^m$  at the lower bound  $s_c = \Delta c + \Delta\mu^m$  and  $\Pi^m > \Pi^{*s}$  for  $s_c \geq \bar{s}_c^c$ , where  $\bar{s}_c^c < \Delta c$ , the point of equalization between  $\Pi^{*s}$  and  $\Pi^m$  must lie in the region where  $s_c < \Delta c$ , i.e., in the declining part of  $\Pi^{*s}$ , which implies that the point of equalization is again unique. Summarizing, in either case (1) or (2), we find that  $\frac{\partial \Pi^{*s}}{\partial s_c} < 0$  in the relevant range for  $s_c$ . Hence, there exists a unique threshold  $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \tilde{s}_c^*)$  such that for  $s_c < \bar{s}_c^*$  it holds  $\Pi^{*s} > \Pi^m$  and the equilibrium prices are described by (18), while for  $s_c \geq \bar{s}_c^*$  it holds  $\Pi^m \geq \Pi^{*s}$  and the equilibrium prices are described by (8), where  $\Pi^m = \Pi^{*s}$  if and only if  $s_c = \bar{s}_c^*$ .

(iib) Suppose, alternatively,  $\tilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$ . The only feasible option is (IIb), and the equilibrium prices are described by (19).

(iii) Suppose  $s_c \geq \max\{\Delta c + \Delta\mu^m, \tilde{s}_c^*\}$ . The feasible options are (I) and (IIb). Since consumer storage is absent under both options and  $\Pi^m$  is the solution to an unconstrained maximization problem, it holds  $\Pi^m > \Pi^{*s}$  and the equilibrium prices are described by (8). ■

**Proof of Corollary 3.** The equilibrium consumer storage and prices in points (i) and (ii) of the corollary are a direct consequence of the outcomes in points (i), (iia) and (iii) of Proposition 2. The derivative of the left-hand side of the first order condition for  $p_1^{*s}$  in (17) with respect to  $s_c$  vanishes for  $D''(\cdot) = 0$ , which implies  $\frac{\partial p_1^{*s}}{\partial s_c} = 0$  from the implicit function theorem. Moreover, the derivative of the left-hand side of the first order condition for  $p_2^{*s}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (17) — with respect to  $s_c$  is  $-[2D'(p_2^{*s} - s_c) + (p_2^{*s} - s_c - c_1) D''(p_2^{*s} - s_c)] - [2D'(p_2^{*s}) + (p_2^{*s} - c_2) D''(p_2^{*s})] > 0$ , where the inequality holds because each expression in square brackets is negative by Assumption 1. We find from the implicit function theorem that  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ . We now turn to the price comparisons. Substituting the first order condition for  $p_1^m$  in (7) into the left-hand side of the first order condition for  $p_1^{*s}$  in (17) with  $D'' = 0$  yields  $D(p_1^m + s_c) + (p_1^m - c_1) D' + (\Delta c - s_c) D'$ . Since for  $s_c = 0$  this expression reduces to  $\Delta c D' < 0$  and both  $p_1^{*s}$  (with  $D'' = 0$ ) and  $p_1^m$  do not depend on  $s_c$ , we obtain by Assumption 1 that

$p_1^m > p_1^{*s}$  for  $D'' = 0$ . It follows from  $D_s^{*s} = D(p_2^{*s}) + (p_2^{*s} - c_2) D'(p_2^{*s}) > 0$  that  $p_2^m > p_2^{*s}$ . ■

**Proof of Corollary 4.** The equilibrium consumer storage and prices in points (i), (ii) and (iii) of the corollary are a direct consequence of the outcomes in points (i), (iib) and (iii) of Proposition 2. It follows from the proof of Corollary 3 that  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ . We now turn to the price comparisons. Since  $\frac{\partial p_1^{*n}}{\partial s_c} < 0$  and the price strategy is continuous, we obtain  $p_1^{*n} > p_1^m$ . It follows from  $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ ,  $\frac{\partial p_2^m}{\partial s_c} = 0$  and the continuity of the price strategy that  $p_2^m > p_2^{*s}$ . ■

**Proof of Proposition 3.** The proof proceeds through the following four cases: (I)  $\Delta\mu^m \leq 0$  and  $\Delta c + \Delta\mu^m \leq \bar{s}_c^*$ ; (II)  $\Delta\mu^m \leq 0$  and  $\Delta c + \Delta\mu^m > \bar{s}_c^*$ ; (III)  $\Delta\mu^m > 0$  and  $\Delta c + \Delta\mu^m > \bar{s}_c^*$ ; (IV)  $\Delta\mu^m > 0$  and  $\Delta c + \Delta\mu^m \leq \bar{s}_c^*$ . Note that the thresholds  $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$  and  $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \bar{s}_c^*)$  defined in Propositions 1 and 2 are such that  $\bar{s}_c^* < \bar{s}_c^c$ . The rationale is the following. The full commitment profits are strictly higher than the limited commitment profits (as long as they differ) and they both decrease with  $s_c$  in the presence of consumer storage (see the proofs of Propositions 1 and 2). Therefore, the point of equalization between the static monopoly profits and the full commitment storing profits, which identifies  $\bar{s}_c^c$ , must be strictly higher than the corresponding point under limited commitment, which identifies  $\bar{s}_c^*$ .

**Case (I)**  $\Delta\mu^m \leq 0$  and  $\Delta c + \Delta\mu^m \leq \bar{s}_c^*$ . It follows from Corollaries 1 and 3 that the following three subcases emerge.

**(Ia)** If  $s_c < \bar{s}_c^*$ , the full commitment prices are  $p_1^{cs}$  and  $p_2^{cs}$  and the limited commitment prices are  $p_1^{*s}$  and  $p_2^{*s}$ . Substituting the first order condition for  $p_1^{*s}$  in (17) into the left-hand side of the first order condition for  $p_1^{cs}$  in (11) yields

$$-(\Delta c - s_c) [D'(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_2) D''(p_1^{*s} + s_c)] > 0,$$

where the inequality holds if and only if the expression in square brackets is negative, i.e.,  $D''(p_1^{*s} + s_c) < -\frac{D'(p_1^{*s} + s_c)}{p_1^{*s} + s_c - c_2}$  (recall  $s_c < \bar{s}_c^* < \bar{s}_c^c < \Delta c$ ). This condition corresponds to  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$ , where  $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1)$ . Since  $p_2^{cs} = p_1^{cs} + s_c$  and  $p_2^{*s} = p_1^{*s} + s_c$ , it follows from Assumption 1 that  $p_\tau^{*s} < p_\tau^{cs}$ ,  $\tau \in \{1, 2\}$ , if and only if  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$ .

**(Ib)** If  $\bar{s}_c^* \leq s_c < \bar{s}_c^c$ , the full commitment prices are  $p_1^{cs}$  and  $p_2^{cs}$  and the limited commitment prices are  $p_1^m$  and  $p_2^m$ . Substituting the first order condition for  $p_1^m$  in (7) into the left-hand side of the first order condition for  $p_1^{cs}$  in (11) yields

$$D(p_1^m + s_c) + (p_1^m - c_1) D'(p_1^m + s_c) < 0,$$

where the inequality holds since  $p_1^m + s_c > p_2^m$  (that follows from  $s_c \geq \bar{s}_c^* > \Delta c + \Delta\mu^m$  and (6)) implies  $D(p_1^m + s_c) + (p_1^m - c_1) D'(p_1^m + s_c) < (\Delta c - s_c) D'(p_1^m + s_c) < 0$  (recall  $s_c < \bar{s}_c^c < \Delta c$ ). By Assumption 1, we obtain  $p_1^m > p_1^{cs}$ . Substituting the first order condition for  $p_2^m$  in (7) into the left-hand side of the first order condition for  $p_2^{cs}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (11) — yields

$$D(p_2^m - s_c) + (p_2^m - s_c - c_1) D'(p_2^m - s_c) + (\Delta c - s_c) D'(p_2^m),$$

whose sign is ambiguous. Computing this expression at  $s_c = \Delta c + \Delta\mu^m < \bar{s}_c^*$  and at  $s_c = \Delta c > \bar{s}_c^c$  we find  $-\Delta\mu^m D'(p_2^m) < 0$  (for  $\Delta\mu^m < 0$ ) and  $D(p_2^m - \Delta c) + (p_2^m - \Delta c - c_1) D'(p_2^m - \Delta c) >$

0 (as  $p_1^m + \Delta c > p_2^m$ ), respectively. As  $\frac{\partial p_2^{cs}}{\partial s_c} > 0$  (see Corollaries 1 and 2) while  $\frac{\partial p_2^m}{\partial s_c} = 0$ , there exists a unique threshold for  $s_c$  that lies between  $\Delta c + \Delta\mu^m$  and  $\Delta c$  such that it holds  $p_2^m > p_2^{cs}$  if and only if  $s_c$  is below this threshold. However, it cannot be established a priori whether the threshold lies within the relevant interval for  $s_c$ .

**(Ic)** If  $s_c \geq \bar{s}_c^c$ , the full commitment prices coincide with the limited commitment prices and correspond to the static monopoly prices  $p_1^m$  and  $p_2^m$ .

**Case (II)**  $\Delta\mu^m \leq 0$  and  $\Delta c + \Delta\mu^m > \tilde{s}_c^*$ . It follows from Corollaries 1 and 4 that the following four subcases emerge.

**(IIa)** If  $s_c < \tilde{s}_c^*$ , case (Ia) applies.

**(IIb)** If  $\tilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$ , the full commitment prices are  $p_1^{cs}$  and  $p_2^{cs}$  and the limited commitment prices are  $p_1^{*n}$  and  $p_2^{*n}$ . Substituting the first order condition for  $p_2^{*n}$  in (7) into the left-hand side of the first order condition for  $p_2^{cs}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (11) — yields

$$D(p_2^m - s_c) + (p_2^m - s_c - c_1) D'(p_2^m - s_c) + (\Delta c - s_c) D'(p_2^m) < 0,$$

where the inequality follows from  $p_1^m + s_c < p_2^m$  (recall  $s_c < \Delta c + \Delta\mu^m$  and (6)). Since  $p_1^{cs} = p_2^{cs} - s_c$  and  $p_1^{*n} = p_2^{*n} - s_c$ , it follows from Assumption 1 that  $p_\tau^{*n} > p_\tau^{cs}$ ,  $\tau \in \{1, 2\}$ .

**(IIc)** If  $\Delta c + \Delta\mu^m \leq s_c < \bar{s}_c^c$ , case (Ib) applies.

**(IId)** If  $s_c \geq \bar{s}_c^c$ , case (Ic) applies.

**Case (III)**  $\Delta\mu^m > 0$  and  $\Delta c + \Delta\mu^m > \tilde{s}_c^*$ . It follows from Corollaries 2 and 4 that the following five subcases emerge.

**(IIIa)** If  $s_c < \min\{\Delta c, \tilde{s}_c^*\}$ , case (Ia) applies.

**(IIIb)** If  $\Delta c \leq s_c < \tilde{s}_c^*$ , the full commitment prices are  $p_1^{cn}$  and  $p_2^{cn}$  and the limited commitment prices are  $p_1^{*s}$  and  $p_2^{*s}$ . Substituting the first order condition for  $p_1^{*s}$  in (17) into the left-hand side of the first order condition for  $p_1^{cn}$  in (13) yields

$$-(\Delta c - s_c) [2D'(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_2) D''(p_1^{*s} + s_c)] \leq 0,$$

where the expression in square brackets is negative by Assumption 1 and the equality holds if and only if  $s_c = \Delta c$ . Since  $p_2^{cn} = p_1^{cn} + s_c$  and  $p_2^{*s} = p_1^{*s} + s_c$ , it follows from Assumption 1 that  $p_\tau^{*s} \geq p_\tau^{cn}$ ,  $\tau \in \{1, 2\}$ , where the equality holds if and only if  $s_c = \Delta c$ .

**(IIIc)** If, alternatively,  $\tilde{s}_c^* \leq s_c < \Delta c$ , case (IIb) applies.

**(IIId)** If  $\max\{\Delta c, \tilde{s}_c^*\} \leq s_c < \Delta c + \Delta\mu^m$ , the full commitment prices are  $p_1^{cn}$  and  $p_2^{cn}$  and the limited commitment prices are  $p_1^{*n}$  and  $p_2^{*n}$ . Substituting the first order condition for  $p_2^{*n}$  in (7) into the left-hand side of the first order condition for  $p_2^{cn}$  — obtained by replacing  $p_1$  with  $p_2 - s_c$  in (13) — yields

$$D(p_2^m - s_c) + (p_2^m - s_c - c_1) D'(p_2^m - s_c) < 0,$$

where the inequality follows from  $p_1^m + s_c < p_2^m$  (recall  $s_c < \Delta c + \Delta\mu^m$  and (6)). Since  $p_1^{cn} = p_2^{cn} - s_c$  and  $p_1^{*n} = p_2^{*n} - s_c$ , it follows from Assumption 1 that  $p_\tau^{*n} > p_\tau^{cn}$ ,  $\tau \in \{1, 2\}$ .

**(IIIe)** If  $s_c \geq \Delta c + \Delta\mu^m$ , case (Ic) applies.

**Case (IV)**  $\Delta\mu^m > 0$  and  $\Delta c + \Delta\mu^m \leq \bar{s}_c^*$ . We show by contradiction that this case is impossible. It follows from Corollaries 2 and 3 that for  $s_c \in (\Delta c + \Delta\mu^m, \bar{s}_c^*)$  the full commitment prices are  $p_1^m$  and  $p_2^m$  and the limited commitment prices are  $p_1^{*s}$  and  $p_2^{*s}$ . Note that, since the static monopoly solution is sequentially optimal, this solution should be also implemented under limited commitment. Hence, case (IV) is impossible.<sup>21</sup>

After defining

$$s_c^l \equiv \begin{cases} \bar{s}_c^* & \text{if case (I) applies} \\ \bar{s}_c^* & \text{if case (II) applies} \\ \min\{\Delta c, \bar{s}_c^*\} & \text{if case (III) applies} \end{cases}, \quad (20)$$

we find from cases (Ia), (IIa) and (IIIa) that for  $s_c < s_c^l$  in each period the price under limited commitment is lower than the price under full commitment, i.e.,  $p_\tau^{*s} < p_\tau^{cs}$ ,  $\tau \in \{1, 2\}$ , if and only if  $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$ . ■

**Proof of Proposition 4.** Using the results in the proof of Proposition 3, we define  $s_c^h$  as

$$s_c^h \equiv \begin{cases} \bar{s}_c^c & \text{if case (I) or (II) applies} \\ \Delta c + \Delta\mu^m & \text{if case (III) applies} \end{cases}. \quad (21)$$

A. Suppose  $s_c^l \leq s_c < s_c^h$ . It follows from cases (Ib), (IIb), (IIc), (IIIb), (IIIc) and (IIId) in the proof of Proposition 3 that in the first period the price under limited commitment is higher than the price under full commitment. If  $\Delta\mu^m > 0$ , it follows from cases (IIIb), (IIIc) and (IIId) in the proof of Proposition 3 that the price under limited commitment is also higher in the second period.

B. Suppose  $s_c \geq s_c^h$ . It follows from cases (Ic), (IIc) and (IIIc) in the proof of Proposition 3 that the prices under full and limited commitment coincide with the static monopoly prices in each period. ■

**Proof of Corollary 5.** To see the results in point (i) of the corollary, note from (11) that  $p_1^{cs}$  does not depend on  $\Delta c$  (for a given  $c_1$ ). Since  $p_2^{cs} = p_1^{cs} + s_c$ , it holds  $\frac{\partial p_\tau^{cs}}{\partial \Delta c} = 0$ ,  $\tau \in \{1, 2\}$ . To see the results in point (ii), note from the implicit function theorem that the sign of  $\frac{\partial p_1^{*s}}{\partial \Delta c}$  is equal to the sign of the derivative of the left-hand side of the first order condition for  $p_1^{*s}$  in (17) with respect to  $\Delta c$ , which is given by

$$D'(p_1^{*s} + s_c) + (p_1^{*s} + 2s_c - c_1 - 2\Delta c) D''(p_1^{*s} + s_c).$$

To establish the sign of this expression, consider the second period net demand  $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1) = -(p_1 + s_c - c_1 - \Delta c) D'(p_1 + s_c)$ . Then,

$$\frac{\partial}{\partial \Delta c} \left[ (\Delta c - s_c) \frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \right] = -D'(p_1^{*s} + s_c) - (p_1^{*s} + 2s_c - c_1 - 2\Delta c) D''(p_1^{*s} + s_c).$$

Since  $p_2^{*s} = p_1^{*s} + s_c$ , it holds  $\frac{\partial p_\tau^{*s}}{\partial \Delta c} < 0$ ,  $\tau \in \{1, 2\}$ , if and only if  $\frac{\partial}{\partial \Delta c} \left[ (\Delta c - s_c) \frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \right] > 0$ . ■

<sup>21</sup>Alternatively, it follows from point (ia) of the proof of Proposition 2 that  $\Delta\mu^m > 0$  is impossible when  $\Delta c + \Delta\mu^m \leq \bar{s}_c^*$ .

**Proof of Proposition 5.** Consumer surplus under full and limited commitment is respectively given by

$$CS^{CS} = \int_{p_1^{CS}} D(p) dp + \int_{p_1^{CS}+s_c} D(p) dp \quad \text{and} \quad CS^{*S} = \int_{p_1^{*S}} D(p) dp + \int_{p_1^{*S}+s_c} D(p) dp.$$

Taking the difference between  $CS^{*S}$  and  $CS^{CS}$  yields

$$\Delta CS \equiv CS^{*S} - CS^{CS} = \int_{p_1^{*S}}^{p_1^{CS}} D(p) dp + \int_{p_1^{*S}+s_c}^{p_1^{CS}+s_c} D(p) dp. \quad (22)$$

We know from Proposition 3 that  $p_\tau^{*S} < p_\tau^{CS}$ ,  $\tau \in \{1, 2\}$ , if and only if  $\frac{\partial D_2^N(p_1^{*S})}{\partial p_1} > 0$ . This implies  $\Delta CS > 0$  if and only if  $\frac{\partial D_2^N(p_1^{*S})}{\partial p_1} > 0$ .

We now turn to total welfare, which is computed as the (unweighted) sum of consumer surplus and firm's profits, i.e.,  $W \equiv CS + \Pi$ . The firm's full commitment profits  $\Pi^{CS}$  are given by the maximand of (10). The firm's limited commitment profits  $\Pi^{*S}$  are given by the maximand of (16) and can be rewritten as

$$\begin{aligned} \Pi^{*S} &= (p_1^{*S} - c_1) [D(p_1^{CS}) + D(p_1^{CS} + s_c)] + (p_1^{*S} - c_1) [D(p_1^{*S}) - D(p_1^{CS})] \\ &\quad + D(p_1^{*S} + s_c) - D(p_1^{CS} + s_c) + (\Delta c - s_c) (p_1^{*S} + s_c - c_1 - \Delta c) D'(p_1^{*S} + s_c). \end{aligned}$$

Taking the difference between  $\Pi^{*S}$  and  $\Pi^{CS}$ , we obtain

$$\begin{aligned} \Delta \Pi \equiv \Pi^{*S} - \Pi^{CS} &= -(p_1^{CS} - p_1^{*S}) [D(p_1^{CS}) + D(p_1^{CS} + s_c)] + (p_1^{*S} - c_1) [D(p_1^{*S}) - D(p_1^{CS})] \\ &\quad + D(p_1^{*S} + s_c) - D(p_1^{CS} + s_c) + (\Delta c - s_c) (p_1^{*S} + s_c - c_1 - \Delta c) D'(p_1^{*S} + s_c). \end{aligned} \quad (23)$$

Summing (22) and (23) yields

$$\begin{aligned} \Delta W \equiv \Delta CS + \Delta \Pi &= \int_{p_1^{*S}}^{p_1^{CS}} D(p) dp + \int_{p_1^{*S}+s_c}^{p_1^{CS}+s_c} D(p) dp - (p_1^{CS} - p_1^{*S}) [D(p_1^{CS}) + D(p_1^{CS} + s_c)] \\ &\quad + (p_1^{*S} - c_1) [D(p_1^{*S}) - D(p_1^{CS}) + D(p_1^{*S} + s_c) - D(p_1^{CS} + s_c)] \\ &\quad + (\Delta c - s_c) (p_1^{*S} + s_c - c_1 - \Delta c) D'(p_1^{*S} + s_c). \end{aligned} \quad (24)$$

Suppose  $\frac{\partial D_2^N(p_1^{*S})}{\partial p_1} > 0$ , which implies from Proposition 3 that  $p_\tau^{*S} < p_\tau^{CS}$ ,  $\tau \in \{1, 2\}$ . The aggregate expression in the first line of (24) is positive. To see this, note that this expression can be rewritten as

$$\int_{p_1^{*S}}^{p_1^{CS}} D(p) dp - \int_{p_1^{*S}}^{p_1^{CS}} D(p_1^{CS}) dp + \int_{p_1^{*S}+s_c}^{p_1^{CS}+s_c} D(p) dp - \int_{p_1^{*S}+s_c}^{p_1^{CS}+s_c} D(p_1^{CS} + s_c) dp > 0,$$

where the inequality follows from  $D'(\cdot) < 0$ . The expression in the second line of (24) is positive as well. The expression in the third line of (24) corresponds to  $-(\Delta c - s_c) D_2^N(p_1^{*S})$ , where  $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1)$  is the second period net demand. If  $D_2^N(\cdot)$  is small enough, we obtain  $\Delta W > 0$ . Now, suppose  $\frac{\partial D_2^N(p_1^{*S})}{\partial p_1} \leq 0$ . This implies  $\Delta CS \leq 0$ , where the equality holds if and only if  $\frac{\partial D_2^N(p_1^{*S})}{\partial p_1} = 0$ . Since  $\Delta \Pi < 0$  (the full commitment profits are

higher than the limited commitment profits), we obtain  $\Delta W \equiv \Delta CS + \Delta \Pi < 0$ . ■

## Supplementary Appendix

This Supplementary Appendix formalizes the results with a linear demand of the form  $D(p_\tau) = \alpha - \beta p_\tau$ ,  $\tau \in \{1, 2\}$ . The threshold values are defined in the proofs.

**Remark 2** Suppose  $s_c < \bar{s}_c^*$ . Then,

(i) under full commitment, consumer storage is  $D_s^{cs} = \frac{2\alpha - 2\beta c_1 - 3\beta s_c}{4}$ , and prices are  $p_1^{cs} = \frac{2\alpha + 2\beta c_1 - \beta s_c}{4\beta}$  and  $p_2^{cs} = \frac{2\alpha + 2\beta c_1 + 3\beta s_c}{4\beta}$ ;

(ii) under limited commitment, consumer storage is  $D_s^{*s} = \frac{3}{2}\beta\Delta c - 2\beta s_c$ , and prices are  $p_1^{*s} = \frac{2\alpha + 2\beta c_1 - \beta\Delta c}{4\beta}$  and  $p_2^{*s} = \frac{2\alpha + 2\beta c_1 - \beta\Delta c + 4\beta s_c}{4\beta}$ .

Consumer surplus is higher under limited commitment than under full commitment. Total welfare is higher under limited commitment than under full commitment if and only if  $\widetilde{\Delta c} < \Delta c < \Delta c^{\max}$ .

**Proof of Remark 2.** Since  $\Delta\mu^m = -\frac{\Delta c}{2} < 0$  and  $\Delta c + \Delta\mu^m = \frac{\Delta c}{2} < \bar{s}_c^* = \frac{3}{4}\Delta c$ , it follows from the proof of Proposition 3 that case (I) applies and  $s_c^l$  in (20) corresponds to  $\bar{s}_c^*$ . The results in points (i) and (ii) of the remark are a direct application of Corollaries 1 and 3 (recall from the proof of Proposition 3 that  $\bar{s}_c^* < \bar{s}_c^c$ ). Consumer surplus and firm's profits under full commitment are respectively

$$CS^{cs} = \frac{4(\alpha - \beta c_1)^2 - 4\beta(\alpha - \beta c_1)s_c + 5\beta^2 s_c^2}{16\beta} \quad (25)$$

and

$$\Pi^{cs} = \frac{[2\alpha - \beta(2c_1 + s_c)]^2}{8\beta}. \quad (26)$$

Consumer surplus and firm's profits under limited commitment are respectively

$$CS^{*s} = \frac{[2\alpha - \beta(2c_1 - \Delta c)]^2 - 4\beta[2\alpha - \beta(2c_1 - \Delta c)]s_c + 8\beta^2 s_c^2}{16\beta} \quad (27)$$

and

$$\Pi^{*s} = \frac{4\alpha^2 - 4\alpha\beta(2c_1 + \Delta c) + \beta^2(4c_1^2 + 4c_1\Delta c + 9\Delta c^2 - 16s_c\Delta c + 8s_c^2)}{8\beta}. \quad (28)$$

Consumer surplus and firm's profits under static monopoly are respectively

$$CS^m = \frac{2(\alpha - \beta c_1)^2 - 2\beta(\alpha - \beta c_1)\Delta c + \beta^2\Delta c^2}{8\beta} \quad (29)$$

and

$$\Pi^m = \frac{2(\alpha - \beta c_1)^2 - 2\beta(\alpha - \beta c_1)\Delta c + \beta^2\Delta c^2}{4\beta}. \quad (30)$$

To compute the threshold  $\bar{s}_c^*$ , we use (28) and (30), which yields  $\Pi^{*s} > \Pi^m$  if and only if  $s_c < \bar{s}_c^*$ , where  $\bar{s}_c^* = \left(1 - \frac{1}{2\sqrt{2}}\right) \Delta c$  (see Corollary 3). Now, we turn to the welfare analysis. Taking the difference between (27) and (25) yields  $\Delta CS \equiv CS^{*s} - CS^{cs} = \frac{4\alpha - \beta(4c_1 - \Delta c + 3s_c)}{16} (\Delta c - s_c) > 0$ , where the inequality follows from the assumptions on the parameters of the model. Taking the difference between (28) and (26) yields  $\Delta \Pi \equiv \Pi^{*s} - \Pi^{cs} = -\frac{4\alpha - \beta(4c_1 + 9\Delta c - 7s_c)}{8} (\Delta c - s_c) < 0$  (the limited commitment profits are lower than the full commitment profits). Then, we obtain  $\Delta W \equiv \Delta CS + \Delta \Pi = -\frac{4\alpha - \beta(4c_1 + 19\Delta c - 17s_c)}{16} (\Delta c - s_c)$ . It holds  $\Delta W > 0$  if and only if  $\widetilde{\Delta c} < \Delta c < \Delta c^{\max}$ , where  $\widetilde{\Delta c} \equiv \frac{4\alpha - 4\beta c_1 + 17\beta s_c}{19\beta}$  and  $\Delta c^{\max} \equiv \frac{2\alpha - 2\beta c_1 + 4\beta s_c}{5\beta}$ . Note that  $\Delta c^{\max}$  is the highest value for  $\Delta c$  such that the second period profit margin is positive. ■

**Remark 3** A. Suppose  $\bar{s}_c^* \leq s_c < \bar{s}_c^c$ . Then,

(i) under full commitment, consumer storage is  $D_s^{cs} = \frac{2\alpha - 2\beta c_1 - 3\beta s_c}{4}$ , and prices are  $p_1^{cs} = \frac{2\alpha + 2\beta c_1 - \beta s_c}{4\beta}$  and  $p_2^{cs} = \frac{2\alpha + 2\beta c_1 + 3\beta s_c}{4\beta}$ ;

(ii) under limited commitment, consumer storage is  $D_s^m = 0$ , and prices are  $p_1^m = \frac{\alpha + \beta c_1}{2\beta}$  and  $p_2^m = \frac{\alpha + \beta c_1 + \beta \Delta c}{2\beta}$ .

Consumer surplus and total welfare are lower under limited commitment than under full commitment.

B. Suppose  $s_c \geq \bar{s}_c^c$ . Then, the static monopoly solution in point (ii) applies under the two regimes.

**Proof of Remark 3.** A. It follows from the proof of Proposition 4 that  $s_c^h$  in (21) corresponds to  $\bar{s}_c^c$ . Using (26) and (30), we obtain  $\Pi^{cs} > \Pi^m$  if and only if  $s_c < \bar{s}_c^c$ , where  $\bar{s}_c^c = \frac{2(\alpha - \beta c_1)}{\beta} - \frac{\sqrt{2}}{\beta} \sqrt{2(\alpha - \beta c_1)^2 - \beta(2\alpha - 2\beta c_1 - \beta \Delta c) \Delta c}$  (see Corollary 1). The results in points (i) and (ii) of the remark are a direct application of Corollaries 1 and 3. Taking the difference between (29) and (25) yields  $\Delta CS \equiv CS^m - CS^{cs} = -\frac{4\alpha(\Delta c - s_c) - \beta[2\Delta c(2c_1 + \Delta c) - 4c_1 s_c - 5s_c^2]}{16} < 0$ , where the inequality follows from the assumptions on the parameters of the model. Since  $\Delta \Pi \equiv \Pi^m - \Pi^{cs} < 0$ , it holds  $\Delta W \equiv \Delta CS + \Delta \Pi < 0$ .

B. The proof follows from Corollaries 1 and 3. ■

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