# Behavior of business investment in the USA under variable and proportional rates of replacement 

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By<br>George C Bitros ${ }^{1}$ and M. Ishaq Nadiri ${ }^{2}$


#### Abstract

Using data from the U. S. Bureau of Economic Analysis for the period 1947-2015, we test two investment models of neoclassical decent. Model A is based on the conceptualization that business firms have an active replacement investment policy, which renders the replacement rate $\delta$ a determinant of business investment behavior, whereas Model B is based on the traditional hypothesis that replacement investment is an engineering proportion of the capital stock, thus turning $\delta$ into a constant. The evidence that emerges from the estimations is heavily in favor of Model A on at least three grounds. Namely, first it establishes that the replacement rate is a decisive determinant of investment at all levels of aggregation; Second, it leads to estimates of investment equations with succinct short run and long run dynamics, thus facilitating policy applications; and thirdly, it gives rise to remarkably robust estimates of the elasticities of substitution of capital for labor, output and the replacement rate. When Model B is estimated for the period 1947-1960, it performs as expected, most likely because in short periods $\delta$ remains fairly constant due to long swings in replacement investment.


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## 1. Introduction

In Bitros, Nadiri (2017) we investigated the behavior of business investment in the U. S by substituting for the expression of the user cost in the neoclassical model of investment by a CobbDouglas approximation of its constituent variables. In particular, we adopted the equation that results from the neoclassical model of investment in the long-run:

$$
\begin{equation*}
I=\alpha \frac{Q^{\rho}}{c^{\sigma}} \delta, \alpha, \delta, \rho, \sigma>0 \tag{1}
\end{equation*}
$$

and substituted the expression for $c$ by:

$$
\begin{equation*}
c=\frac{q}{p}\left[\left(\frac{1-u v}{1-u}\right) \delta+\left(\frac{1-u w}{1-u}\right) r\right] \cong \frac{q}{p} \delta^{\alpha_{1}} r^{\alpha_{2}} u^{\alpha_{3}}, \quad \alpha_{1} \neq 0, \alpha_{2}, \alpha_{3} \geq 0 \tag{2}
\end{equation*}
$$

where the symbols are defined as follows: $Q$ stands for the quantity of output; $I$ is the quantity of gross investment; $c$ is the user cost of capital; $\delta, r, u$ represent the rates of depreciation, interest and taxes; $q, p$ are the prices of investment goods and output; $\sigma$ is the elasticity of substitution of capital for labor and coincides additively inversely with the elasticity of the user cost; $\rho$ is the elasticity of output and more technically the distribution parameter of the production function, which is assumed to be of the Constant-Elasticity-of-Substitution (CES) type; $v, w$ are the proportions of current replacement and interest cost allowable for tax purposes; capital gains are ignored; $\alpha$ is a shift parameter; and the $\alpha_{i}{ }^{\prime} s$, for $i=1,2,3$, are the elasticities of the user cost with respect to its constituent variables.

If the approximation to the user cost in equation (2) is abandoned, the logarithmic form of equation (1) in the long-run depends on the nature of the replacement rate $\delta$. If replacement investment is considered a decision variable on the part of business firms, equation (1) transforms into:

$$
\begin{gather*}
i=\beta_{0}+\beta_{1} o+\beta_{2} \delta+\beta_{3} c^{\prime} \\
i=\ln I, o=\ln Q, c^{\prime}=\ln c \\
\beta_{0}=\ln \alpha, \beta_{1}=\rho, \beta_{3}=-\sigma  \tag{3}\\
\beta_{0} \geq 0, \beta_{1}, \beta_{2}>0, \beta_{3}<0 .
\end{gather*}
$$

In this model, the variable $c^{\prime}$ is computed from the original expression of the user cost by treating $\delta$ as a variable rate of replacement investment. On the other hand, if the replacement invest-
ment $\delta$ is considered a constant proportion of the capital stock, (1) transforms into:

$$
\begin{gather*}
i=\gamma_{0}+\gamma_{1} o+\gamma_{2} c^{\prime \prime} \\
i=\ln I, o=\ln Q, c^{\prime \prime}=\ln c \\
\gamma_{0}=\ln \alpha+\ln \delta, \gamma_{1}=\rho, \gamma_{2}=-\sigma  \tag{4}\\
\gamma_{0} \geq 0 \quad \gamma_{1}>0, \gamma_{2}<0 .
\end{gather*}
$$

In this the user cost is computed by treating $\delta$ as constant. Hence, we have two distinct models, one in which the replacement rate is variable (henceforth called Model A) and another in which it is constant (henceforth called Model B).

In all results presented in this paper, the variables of gross investment and gross value added or output are defined and measured as in Bitros, Nadiri (2017). The only difference lies in the definition and measurement of the replacement rate, and hence in the computation of the user cost variable. To estimate these two distinct models, we compute $c^{\prime}$ and $c^{\prime \prime}$ by applying the following steps from page 218 of the paper by Jorgenson, Stephenson (1967):

1. The income tax rate is derived as (Profits - Profits after taxes)/ Profits, both series without inventory valuation and capital consumption adjustment.
2. The proportion of current replacement cost allowable for tax purposes, denoted $v_{t}$, is the ratio of depreciation to replacement in constant prices multiplied by the price of investment goods.
3. The proportion of total cost of capital allowable for tax purposes, denoted $w_{t}$, is the ratio of net monetary interest to the total cost of capital as measured below.
4. The cost of capital is defined as the ratio of corporate profits after taxes and net monetary interest to the value of all outstanding securities for the U.S. business sector.
5. The total cost of capital is equal to the product of the cost of capital, capital stock in constant prices, and the price of investment goods.
6. Value of securities. The value of equity is estimated as the ratio of corporate profits after taxes to the earnings-price ratio reported by Standard and Poor's. The value of debt is estimated as the ratio of net monetary interest to the bond yield reported by Moody's seasoned Baa corporate bond yield.

In the computation of $c^{\prime}$ the replacement rate $\delta$ is defined and measured by the inverse of the average age at historical prices ${ }^{1}$ of structures, equipment, intangibles and overall investment as reported by the U. S. Bureau of Economic Analysis (BEA). The way BEA computes the average ages is explained in their 2003 publication. ${ }^{2}$ From this we are informed that:
"...The average age is derived as the weighted average of the ages of all depreciated investment in the stock as of yearend. The weight for each age is based on the proportion of its value as part of the total net stock."(M-5)

In other words, by applying the well-known perpetual inventory technique they compute the average ages of more or less homogeneous classes of fixed assets, and then they derive the average age of structures, equipment, intangibles and overall investment by weighting the average age of each fixed asset by the proportion of its value in the total into which it is aggregated. As a result the average ages that result, and hence the rates of replacement for each of the four aggregates under consideration, are variable. On the contrary, in the computation of $c^{\prime \prime}$ the replacement rates for each of the four aggregates are computed by applying the perpetual inventory technique to the corresponding gross investment series in conjunction with some initial values of their net stocks in the year 1947. Consequently, in this case, the computed replacement rates remain constant throughout the sample period.

The average ages reported by BEA for structures, equipment, intangibles and overall investment in the U.S. business sector are uniquely suitable for our research. The main reason is that they allow us to study the impact of the changes in the replacement rate of an aggregate when the composition of the fixed assets of which it is composed changes. However, in view of the doubts that have been expressed in the relevant literature regarding the very demanding conditions for exact aggregation of two or more fixed assets that depreciate at different rates, ${ }^{3}$ it is advisable not to lose sight of their limitations. Here we use them as the best available approximations.

## 2. Tests of cointegration

The user cost series for structures, equipment, intangibles and overall investment that resulted from our computations are given in Columns 18-21 and 22-25 of Table A-1 in the Appendix.

[^0]Tables 1,2 and 3 below display the test results for cointegration in the series of the eight user cost variables and the equations for structures, equipment, intangibles and overall investment

Table 1: Phillips-Perron tests on the levels and first differences of the user cost variables

| Levels |  |  | First differences |  |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
| Series $^{1}$ | PP t-Statistic | P-Value | Series | PP t-Statistic | P-Value |
| cstru $_{2 t}$ | -3.001 | 0.0348 | Dcstru $_{2 t}$ | -5.587 | 0.0000 |
| ceq $_{2 t}$ | 0.869 | 0.9927 | cceq $_{2 t}$ | -5.038 | 0.0000 |
| int $_{2 t}$ | 0.923 | 0.9934 | Dint $_{2 t}$ | -5.745 | 0.0000 |
| cinv $_{2 t}$ | -0.637 | 0.8625 | D.inv $_{2 t}$ | -4.918 | 0.0000 |
| ccstru $_{2 t}$ | -2.397 | 0.1426 | Dcstru $_{2 t}$ | -6.138 | 0.0000 |
| cceq $_{2 t}$ | 0.916 | 0.9333 | Dcceq $_{2 t}$ | -5.627 | 0.0000 |
| cint $_{2 t}$ | 0.933 | 0.9935 | Dcint $_{2 t}$ | -5.993 | 0.0000 |
| cinv $_{2 t}$ | 0.386 | 0.9814 | Dcind $_{2 t}$ | -5.439 | 0.0000 |

Notes

1. Variables marked with one (two) $c$ imply user cost with a variable (constant) rate of replacement.

Table 2: Test results for cointegration under Model A

| Step A: OLS residuals from the four equations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Variables | Dependent Variables |  |  |  |
|  | strut | $e q_{t}$ | int $_{t}$ | $\operatorname{inv}_{t}$ |
| Constant | 0.956 (4.56) | -1.643 (-4.38) | -4.339 (-12.2) | -1.628 (-2.99) |
| $g v a_{t}$ | 0.372 (14.7) | 1.191 (28.9) | 1.812 (31.4) | 1.060 (15.5) |
| $\delta s t r u_{2 t}$ | 1.040 (7.02) | .... | $\ldots$ | $\ldots$ |
| $\delta e q_{2 t}$ | .. | $1.2076 .35)$ | .... | .. |
| $\operatorname{Sint}_{2 t}$ | $\ldots$ | $\ldots$ | 0.667(2.73) | $\ldots$ |
| $\operatorname{Sinv}_{2 t}$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.195 (0.72) |
| cstru $_{2 t}$ | 0.034 (0.35) | .... | $\ldots$ | $\ldots$ |
| ceq $_{2 t}$ | .... | -0.514 (7.82) | $\ldots$ |  |
| cint $_{2 t}$ | $\ldots$ | $\ldots$ | -0.273 (-2.64) | $\ldots$ |
| $\operatorname{cinv}_{2 t}$ | $\ldots$ | .... | .... | -0.059 (-0.51) |
| Adjusted R | 0.958 | 0.994 | 0.996 | 0.994 |
| Root MSE | 0.085 | 0.078 | 0.086 | 0.060 |
| Step B:Unit root tests on residuals |  |  |  |  |
| Phillips-Perron t-statistic | strut | $e q_{t}$ | int $_{t}$ | inv ${ }_{t}$ |
| $\mathrm{z}(\mathrm{t})$ | -3.684 | -3.608 | -2.653 | -3.444 |
| p-value | 0.0043 | 0.0056 | $0.0826^{1}$ | 0.0095 |

Notes:

1. The equation for intangibles is cointegrated only at the $10 \%$ level of confidence.

Table 3: Test results for cointegration under Model B

| Step A: OLS residuals from the four equations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Variables | Dependent Variables |  |  |  |
|  | strut | $e q_{t}$ | int $_{t}$ | inv ${ }_{t}$ |
| Constant | -0.095 (-0.35) | -3.74 (-126) | -5.310 (-205) | -2.006 (-58.6) |
| $g v a_{t}$ | 0.334 (8.39) | 1.317 (36.9) | 1.928 (55.9) | 1.116 (75.2) |
| ccstru $_{2 t}$ | 0.756 (6.79) | $\ldots$ | .... | .... |
| cceq $_{2 t}$ | .... | -0.283 (-5.68) | .... | .... |
| ccint $_{2 t}$ | $\ldots$ |  | -0.096 (1.27) | $\ldots$ |
| $\operatorname{ccinv}_{2 t}$ | $\ldots$ |  |  | 0.042 (1.14) |
| Adjusted R ${ }^{2}$ | 0.886 | 0.993 | 0.996 | 0.994 |
| Root MSE | 0.143 | 0.089 | 0.089 | 0.060 |
| Step B: Unit root tests on residuals |  |  |  |  |
| Phillips-Perron t -statistic | strut | $e q_{t}$ | int ${ }_{t}$ | inv ${ }_{t}$ |
| $\mathrm{z}(\mathrm{t})$ | -2.249 | -3.221 | -2.846 | -3.485 |
| p -value | $0.1888^{1}$ | $0.0188^{2}$ | $0.0520^{2}$ | $0.0084^{3}$ |

Notes:

1. Absence of cointegration.
2. These equations are cointegrated at the $5 \%$ confidence level.
3. Presence of cointegration.
with variable and proportional replacement rates. From them it turns out that the equations with variable replacement rates in Table 2 are cointegrated at high levels of confidence, with the possible exception of the equation for intangibles, which is cointegrated only at the $10 \%$ confidence level. On the contrary, cointegration in the second set of equations with proportional replacement rates is a bit more uncertain. For, as shown by the results in Table 3, cointegration holds only for the overall investment equation, whereas the equations for equipment and intangibles are cointegrated at the $5 \%$ level of confidence, and the equation for structures is not cointegrated at all. Hence, by drawing on these findings, we determined that with the possible exception of the equation for structures in the second set of equations, ${ }^{4}$ all other equations should be estimated by means of the Error-Correction Model (ECM).
[^1]
## 3. Results from the estimation of Model A

Upon embedding equation (3) into the error-correction specification, we obtain the following estimating form of the model with a variable replacement rate:

$$
\begin{align*}
& D i_{t}=d_{0}+ \sum_{s=1}^{n 1} \theta_{1 s} D i_{t-s}+\sum_{s=0}^{n 2} \theta_{2 s} D o_{t-s}+\sum_{s=0}^{n 3} \theta_{3 s} D \delta_{t-s}++\sum_{s=0}^{n 4} \theta_{4 s} D c_{t-s}^{\prime}+ \\
&+d_{1} i_{t-1}+d_{2} o_{t-1}+d_{3} \delta_{t-1}+d_{4} c_{t-1}^{\prime}+e_{t}  \tag{4}\\
& d_{0}=\theta_{0}-\lambda \beta_{0}, d_{1}=\lambda, \quad d_{2}=-\lambda \beta_{1}, d_{3}=-\lambda \beta_{2}, d_{4}=-\lambda \beta_{3} .
\end{align*}
$$

Before turning to the results of the estimations, one wholly new feature in this specification should be noted. This has to do with the appearance in the long run part of the model of the replacement rate $\delta_{t-1}$. In previous studies of investment the replacement rate did not appear as an independent determinant because in the analysis it was considered constant. It entered only through the user cost and it was held constant because $\delta$ was tied to the derivation of the capital stock through the perpetual inventory method. However, as long as the average ages of capital change, and here they change irrespective of whether they are calculated on current or historical prices, the rate of replacement does remain constant, and hence, in the long run relationship it cannot be subsumed in the $d_{0}$ parameter.

Table 4 exhibits the results of the estimations. From these it turns out that the estimated coefficients have the expected signs, with only few exceptions they are statistically significant with comfortable margins of confidence, and the explanatory power of the estimated equations is high. The values of the Breusch-Godfrey test for the equations of structures and intangibles signal the possibility that the estimated coefficients and t-statistics may reflect some influence from serial correlation. But as the values of the test statistics in these equations lie at the borderline of no serial correlation, the signal may be viewed only as a warning for caution.

Of particular interest to observe is that the replacement rate enters into the equations both directly and indirectly through the user cost, whereas the interest rate and the tax rate influences investment, if at all, only indirectly through the user cost. This is an important finding, because it confirms that the replacement rate of producer's goods is an important determinant of gross investment and that its omission in earlier studies may have biased the results in unknown magnitudes and directions.

Table 4: OLS estimates of equation (3)

| Variables | Dependent Variables ${ }^{\text {I, }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Dstru ${ }_{\text {t }}$ | Deq ${ }_{t}$ | Dint $_{t}$ | $\operatorname{Dinv}_{t}$ |
| Constant | 0.573 (3.44) | -.1.084 (-5.72) | .. | $\ldots$ |
| strut $_{t-1}$ | -0.483 (-6.31) | .... | $\ldots$ | $\ldots$ |
| $e q_{t-1}$ | .... | -0.319 (-4.86) | ... | $\ldots$ |
| int $t_{t-1}$ | $\ldots$ | .... | -0.073(-2.56) | $\ldots$ |
| inv $_{t-1}$ | .... | ... | .... | -0.215(-3.03) |
| $g v a_{t-1}$ | 0.276 (6.86) | 0.469 (5.67) | 0.079 (2.41) | 0.201 (3.14) |
| $\delta s t r u_{2 t-1}$ | 0.792 (5.11) | .... | .... | .... |
| $\delta e q_{2 t-1}$ | $\ldots$ | 0.113 (0.91) | $\ldots$ | .. |
| $\operatorname{Sint}_{2 t-1}$ | $\ldots$ | $\ldots$ | 0.237 (2.31) | $\ldots$ |
| $\operatorname{Sinv}_{2 t-1}$ | $\ldots$ | $\ldots$ | .... | 0.241 (3.36) |
| $\operatorname{cstr}_{\text {t-1 }}$ | -0.181 (-2.58) | $\ldots$ | $\ldots$ | $\ldots$ |
| $c e q_{t-1}$ | .... | -0.065 (-1.53) | $\ldots$ | $\ldots$ |
| $\operatorname{cint}_{t-1}$ | $\ldots$ | .... | -0.106 (-2.22) | $\ldots$ |
| $\operatorname{cinv}_{\mathrm{t}-1}$ | .... | $\ldots$ | $\ldots$ | -0.095 (-3.17) |
| Dstrut ${ }_{\text {t-1 }}$ | 0.138 (1.39) | $\ldots$ | $\ldots$ | .... |
| Deq $\mathrm{t}_{\mathrm{t}-1}$ | .... | 0.145 (2.62) | $\ldots$ | $\ldots$ |
| $\operatorname{Dint}_{\mathrm{t}-1}$ | $\ldots$ | .... | 0.037 (0.39) | .... |
| $\operatorname{Dinv}_{t-1}$ | .... | .... | $\ldots$ | 0.132 (1.28) |
| $D g v a_{t}$ | 0.887 (4.10) | 2.140 (12.07) | 0.663 (4.53) | 1.014 (5.81) |
| $D g v a_{t-1}$ | 0.653 (2.86) | .... | .... | 0.398 (2.00) |
| DSstru ${ }_{2 t}$ | 3.378 (6.39) | .... | $\ldots$ | .... |
| DSeq ${ }_{2 t}$ | .... | 1.138 (3.83) | .... | $\ldots$ |
| Dinit $_{2 t}$ | $\ldots$ | .... | 1.582 (5.75) | .... |
| $D \operatorname{Sinv} v_{2 t}$ | $\ldots$ | $\ldots$ | .... | 1.655 (4.70) |
| $D \operatorname{Sinv} v_{2 t-1}$ | $\ldots$ | $\ldots$ | $\ldots$ | -0.832 (-2.21) |
| Dint $_{1 t}$ | $\ldots$ | $\ldots$ | -0.306(2.55) | .... |
| $\mathrm{R}^{2}$ | 0.748 | 0.860 | 0.850 | 0.846 |
| $\mathrm{R}^{2}$-adjusted | 0.713 | 0.843 | 0.830 | 0.822 |
| D-W | 1.683 | 1.717 | 2.205 | 1.803 |
| Breusch-Godfrey | 0.065 | 0.158 | 0.043 | 0.184 |
| Root MSE | 0.041 | 0.033 | 0.033 | 0.029 |

Notes :

1. The numbers within the parentheses give the values of the $t$-statistic.

The next task is to compute the elasticities implied by the estimated equations in Table 4. These are exhibited in Table 5. To extract the standard errors of the elasticities, we estimated all four equations also non-linearly. In most cases the non-linear elasticities coincide with those obtained from the OLS estimates
and generally they are statistically significant and retain the right signs. This finding provides extra assurance that these long run elasticities are quite stable.

Table 5: Elasticities implied by the estimates in Table 4

| Variables | Investment ${ }^{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | strut ${ }_{t-1}$ | $e q_{t-1}$ | int $_{t-1}$ | $\operatorname{inv}_{t-1}$ |
| $\sigma^{2,3}$ | 0.376 (0.141) | 0.203 (0.106) | $\begin{aligned} & 1.453 \\ & 1.093(0.465) \end{aligned}$ | $\begin{aligned} & \hline 0.443 \\ & 0.421(0.059) \end{aligned}$ |
| $\rho$ | 0.570 (0.060) | 1.471 (0.076) | $\begin{aligned} & 1.088 \\ & 1.290(0.188) \end{aligned}$ | $\begin{aligned} & 0.931 \\ & 0.916(0.026) \end{aligned}$ |
| $\delta s t r u_{2 t-1}$ | 1.639 (0.210) | - ... | $\ldots$ | ... |
| $\delta e q_{2 t-1}$ |  | 0.354 (0.345) | $\ldots$ | $\ldots$ |
| int $_{2 t-1}$ | $\ldots$ | .... | $\begin{aligned} & \hline 3.248 \\ & 3.140(0.302) \\ & \hline \end{aligned}$ | $\ldots$ |
| $\operatorname{Sinv}_{2 t-1}$ | $\ldots$ | $\ldots$ | $\ldots$ | $\begin{aligned} & \hline 1.121 \\ & 1.080(0.041) \\ & \hline \end{aligned}$ |

Notes:

1. The numbers within the parenthesis are standard errors. These were computed by estimating the equations in Table B4 nonlinearly so as to factor out the parameter $\lambda$ in the equation (B3).
2. The estimates of the elasticities at the top of the rows were computed from the OLS estimates of the model shown in Table B4. It is observed that in the equations for structures, and equipment the linear and nonlinear estimates coincide, whereas in the equations for intangibles and overall investment they differ somewhat.
3. Recall that the parameter $\sigma$ is in the denominator of the investment equation and that therefore an increase in the user cost leads to a decline in gross investment.

Lastly, by way of passing to the results for Model B, a few comments are in order regarding the impact of the approximation we adopted in Bitros, Nadiri (2017). Juxtaposing Table 5 there with Table 5 here, we observe that: (a) in general the elasticities of substitution of capital for labor under the approximation are significantly higher than those obtained without it. For example, whereas the elasticity of substitution of capital for labor derived for overall investment from Table 5 there is 0.761 , the same elasticity from Table 5 here is 0.421 ; (b) the elasticities of gross value added with without the approximation are fairly closed to each other. For example, the ones for overall investment from Table 5 there and Table 5 here are 0.923 and 0.916 , respectively, and (c) the elasticities for the replacement rate vary within a narrow range, with those under the approximation tending to be lower than those without it.

## 4. Results from the estimation of Model B

Turning next to the model with constant replacement rates, upon embedding equation (4) into the er-ror-correction specification yields:

$$
\begin{align*}
D i_{t}=d_{0} & +\sum_{s=1}^{n 1} \zeta_{1 s} D i_{t-s}+\sum_{s=0}^{n 2} \zeta_{2 s} D o_{t-s}+\sum_{s=0}^{n 3} \zeta_{4 s} D c_{t-s}^{\prime \prime}+ \\
& +d_{1} i_{t-1}+d_{2} o_{t-1}+d_{3} c_{t-1}^{\prime \prime}+e_{t},  \tag{5}\\
d_{0}=\zeta_{0} & -\lambda \gamma_{0}, d_{1}=\lambda, \quad d_{2}=-\lambda \gamma_{1}, d_{3}=-\lambda \gamma_{2} .
\end{align*}
$$

The objective in this section is to estimate equation (5) twice. That is, once using all observations in the sample, so as to compare the results with those obtained above from Model A, and another using a subsample of the available observations, so as to compare the results with those reported in a benchmark study.

### 4.1 Estimation of Model B using all sample observations

Table 6 displays a representative sample of the results we obtained by using all sample observations from 1947 to 2015. Focusing again on the long run segment of the estimates in the upper part of this table, we observe that the coefficient of gross value added is statistically significant and that it has the expected sign across all equations. This finding is in line with the estimates both from Model A, as well as with the evidence from the voluminous literature in this area. Invariably gross value added has been found to influence positively gross investment, irrespective of the specification of the user cost variable and the disaggregation of gross investment. However, with the exception of the equation for equipment, in which the user cost variable performs as expected from the neoclassical theory of investment behavior, the user cost coefficients in the remaining equations in Table 6 are either inconsistent or lack adequate robustness. In particular, in the equation for structures this coefficient is statistically significant but has the wrong sign; in the equation for overall investment it does have the expected sign but its statistical significance is quite low; and in the equation for intangibles this coefficient is missing altogether.

Table 6: Estimates of equation (4) for the period 1947-2015

| Variables | Dependent Variables ${ }^{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Dstrut ${ }^{2}$ | Deq ${ }_{t}$ | Dint $^{\text {t }}$ | Dinv $_{t}$ |
| Constant | .... | -1.242 (-5.85) | -0.727 (-2.56) | -0.826 (-5.60) |
| stru $_{t-1}$ | -0.178 (-3.79) | $\ldots$ | $\ldots$ | .... |
| $e q_{t-1}$ | .... | -0.303 (-5.40) | $\ldots$ | $\ldots$ |
| int $t_{\text {t-1 }}$ | $\ldots$ | .... | -0.144 (-2.72) | $\ldots$ |
| inv $_{t-1}$ | ..... | .... | .... | -0.357 (-4.96) |
| $g v a_{t-1}$ | 0.053 (3.00) | 0.456 (5.83) | 0.272 (2.59) | 0.433 (4.96) |
| $\operatorname{ccstru}_{\mathrm{t}-1}$ | 0.173 (4.34) | .... | .... | .... |
| cceq $_{\text {t-1 }}$ | $\ldots$ | -0.098 (-3.07) | $\ldots$ | $\ldots$ |
| $\operatorname{cing}_{\text {t-1 }}$ | $\ldots$ | .... | $\ldots$ | -0.027 (-1.28) |
| Dstru ${ }_{\text {t-1 }}$ | 0.254 (2.47) | ... | $\ldots$ | .... |
| Deq ${ }_{\text {t-1 }}$ | ... | 0.231 (4.48) | $\ldots$ | $\ldots$ |
| $\operatorname{Dint}_{\mathrm{t}-1}$ | $\ldots$ | .... | 0.242 (2.39) | .... |
| $\operatorname{Dinv}_{\mathrm{t}-1}$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.310 (3.13) |
| $D g v a_{t}$ | 1.414 (5.76) | 2.427 (15.24) | 0.798 (4.66) | 1.438 (10.2) |
| $D g v a_{t-1}$ | 1.033 (3.97) | .... | .... | .... |
| Dccstru $_{t-1}$ | 0.291 (2.78) | .... | $\ldots$ | .... |
| Dccint $_{\text {lt-1 }}$ | .... | .... | -0.255 (-2.06) | .... |
| Dummy $_{t}$ | .... | -0.065 (-2.95) | .... | -0.059 (-3.60) |
| $\mathrm{R}^{2}$ | 0.616 | 0.847 | 0.384 | 0.775 |
| $\mathrm{R}^{2}$-adjusted | 0.572 | 0.832 | 0.334 | 0.748 |
| D-W | 1.809 | 2.021 | 2.127 | 1.810 |
| Breusch-Godfrey | 0.383 | 0.866 | 0.094 | 0.278 |
| Root MSE | 0.051 | 0.034 | 0.038 | 0.029 |

Notes :

1. The numbers within the parentheses give the values of the $t$-statistic.
2. Since the equation for structures lacks cointegration, the OLS error-correction results shown in this column should be considered only as indicative because they may suffer from the socalled "spurious regression" problem. This equation was run also by the Autoregressive Distributed Lag method and the results were the following:

$$
\begin{array}{ccccc}
\text { stru }_{t}=-0.192 \text { stru }_{t-1}+0.299 g v a_{t}+0.962 \text { ccstru }_{t}+0.281 \text { Dstru }_{t}+1.384 D g v a_{t}+1.073 D g v a_{t-1} \\
(-4.23) & (6.99) & (17.0) \quad(2.82) \quad(4.16) \\
& R^{2}=0.609, \quad \bar{R}^{2}=0.570 \quad \text { R MSE }=0.051
\end{array}
$$

The contrast of the results from the two models is equally sharp if glimpsed through the differences in the respective elasticities. To corroborate this assessment, Table 7 presents the long run elasticities which are implied by the estimates in Table 6. Comparing them to those in Table 5 from Model A, it turns out that their crucial difference lies in the elasticities of substitution, the additive inverse of which coincide with the elasticities of gross investment in the particular fixed assets with respect to the user cost. Once again we observe that, with the exception of the elasticities for equipment, which come close to those from Table 5,
the elasticities of substitution from Table 7 are of questionable validity since their statistical significance is low; in the equation for structure the elasticity of substitution has the wrong sign, and in the equation for intangible it is zero. Moreover, aside from this difference, notice that the explanatory power of the equations for structures and intangibles in Table 6 is much lower than that of the corresponding equations in Table 4.

Table 7: Elasticities implied by the estimates in Table 6

| Variables | Investment $^{1,2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | strut $_{t-1}$ | $e q_{t-1}$ | int $_{t-1}$ | inv $_{t-1}$ |
|  | $-0.972(-0.063)$ | $0.325(0.083)$ | $\ldots .$. | $0.075(0.063)$ |
| $\rho$ | $0.298(0.046)$ | $1.507(0.057)$ | $1.885(0.062)$ | $1.213(0.034)$ |

Notes:

1. The numbers within the parenthesis are standard errors. These were computed by estimating the equations in Table 6 nonlinearly so as to factor out the parameter $\lambda$ in the equation (B4).
2. Across all equations linear and nonlinear estimates of elasticities coincide.

In the light of the above comparison, the question that comes to mind is this: How can we explain the profound inferiority of Model B? Recall that its only difference from Model A lies in the treatment of the replacement rate as an engineering constant. Therefore, the only reasonable explanation is that $\delta$ is not a constant, implying further that business firms do have and follow active replacement policies. But this explanation contradicts the evidence from most previous investment studies which find that the user cost variable, as computed traditionally, does performs well. To shed light on this issue, and perhaps resolve it in a convincing manner, we conjectured that the said contradiction would be expected to emerge if $\delta$ is variable over long periods, due to long replacement investment cycles, but relatively constant over short ones. The objective in the next sub-section is to test this hypothesis.

### 4.2 Estimates of Model B using a subsample of the observations

Tables 8 and 9 present the results that we obtained by fitting Model B to the segment of the sample observations for the years 1947-1960. We shall explain the reasons for choosing this particular period shortly. But for now, it takes precedent to offer a few comments regarding the properties of the estimated equations and the long run elasticities computed from them.

Turning first to Table 8, observe that in their great majority the estimated coefficients are statistically significant at comfortable levels of confidence, their signs are consistent with those expected from theory, and the explanatory power of the equations is very high. Moreover, notice that the D-W and BreuschGodfrey tests for serial correlation signal a cautionary warning for the present of serial correlation in the
equation for equipment. But in general the model performs very well and this confirms that in sample with a limited number of time series observations the replacement rate may be approximated as a constant.

Table 8: Estimates of equation (4) for the period 1947-1960

| Variables | Dependent Variables ${ }^{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Dstrut ${ }^{2}$ | Deq ${ }_{t}$ | Dint $_{t}$ | Dinv $_{t}$ |
| Constant | -4.227 (-5.53) | -5.454 (-5.89) | -6.752 (-3.47) | -3.370 (-5.30) |
| strut $_{t-1}$ | -1.227 (-5.14) | $\ldots$ |  | .... |
| $e q_{t-1}$ | .... | -1.481 (-5.50) | $\ldots$ | $\ldots$ |
| int $t_{t-1}$ | .... | .... | -1.135 (-3.63) | $\ldots$ |
| inv $v_{t-1}$ | .... | .... | ... | -1.246 (-5.03) |
| $g v a_{t-1}$ | 1.823 (5.63) | 1.496 (7.12) | 3.293 (3.44) | 1.803 (5.50) |
| $\operatorname{ccstru}_{\mathrm{t}-1}$ | -0.454 (-4.43) | $\ldots$ | .... | $\ldots$ |
| cceq $_{\text {t-1 }}$ | .... | -0.726 (-5.10) | $\ldots$ | $\ldots$ |
| $\operatorname{ccint}_{\text {t-1 }}$ | $\ldots$ | .... | -0735 (-1.65) |  |
| $\operatorname{cinv} \mathrm{t}_{\mathrm{t}-1}$ | $\ldots$ | $\ldots$ | ... | -0.536 (-4.09) |
| Dstrut ${ }_{\text {t-1 }}$ | 0.399 (2.32) | $\ldots$ |  | .... |
| Deq $_{\text {t-1 }}$ | .... | 0.533 (5.37) |  | $\ldots$ |
| $\operatorname{Dint}_{\mathrm{t}-1}$ | .... | .... | -0.481 (-1.26) | $\ldots$ |
| $\operatorname{Dinv}_{t-1}$ | .... | $\ldots$ | .... | 0.469 (2.58) |
| Dgva ${ }_{\text {d }}$ | 0.618 (2.84) | 1.555(4.15) | 1.006 (1.66) | 0.951 (4.21) |
| $D g v a_{t-1}$ | -0.385 (-1.29) | .... | -1.153 (-1.94) | -0.472 (-1.60) |
| $D g v a_{t-2}$ | .... | 1.081 (5.06) |  | .... |
| $\mathrm{R}^{2}$ | 0.939 | 0.985 | 0.865 | 0.952 |
| $\mathrm{R}^{2}$-adjusted | 0.867 | 0.962 | 0.703 | 0.895 |
| D-W | 1.983 | 2.662 | 2.462 | 2.112 |
| Breusch-Godfrey | 0.528 | 0.047 | 0.189 | 0.579 |
| Root MSE | 0.019 | 0.015 | 0.043 | 0.018 |

Notes:

1. The numbers within the parentheses give the values of the $t$-statistic.
2. When the cointegration tests were run for the truncated sample period, all equations were found to be cointegrated at adequate significance levels.

Table 9: Elasticities implied by the estimates in Table 8

| Variables | Investment $^{1,2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | stru $_{t-1}$ | $e q_{t-1}$ | int $_{t-1}$ | inv $_{t-1}$ |
| $\sigma$ | $0.371(0.072)$ | $0.490(0.072)$ | $0.648(0.288)$ | $0.430(0.081)$ |
| $\rho$ | $1.493(0.073)$ | $1.010(0.075)$ | $2.902(0.143)$ | $1.447(0.071)$ |

## Notes:

1. The numbers within the parenthesis are standard errors. These were computed by estimating the equations in Table B6 nonlinearly so as to factor out the parameter $\lambda$ in the equation (B4).
2. Across all equations linear and nonlinear estimates of elasticities coincide.

Table 9 reports the elasticities of investment for this period. The figures in the upper row show the values of the elasticity of substitution at the overall and the disaggregate levels of the fixed assets under consideration. From them it turns out that this elasticity varies narrowly around 0.5 depending on the type of investment. Also, notice from the extreme left column that the same elasticity for overall investment is 0.430 . The lower row of the table shows the elasticities of investment with respect to gross value added. With the exemption of the elasticity for equipment, which is closed to 1 , in all other equations this elasticity is significantly higher than 1 , with that for overall investment being 1.447. Hence, if we must draw a single conclusion from these findings, this is that the elasticity of substitution over the 1947-1960 period was much less than 1 , whereas the elasticity of investment with respect to the value added was considerably higher than 1 .

The reason for placing emphasis in these two elasticities is twofold. The first emanates from the long and heated debate about their size that took place in the 1960s between Jorgenson and his associates $(1967, \underline{1969})$, on the one hand, and Eisner and Nadiri $(1968,1970)$ on the other. Just for a quick reminder, the controversy started with Jorgenson's (1963) classic paper. In this he formalized the neoclassical theory of investment and tested it empirically using quarterly data from the U. S. Manufacturing sector over the period 1948-1960. He claimed then and in many publications over the following years that he found the above two elasticities to be respectively equal to 1 . By contrast, Eisner and Nadiri argued that what Jorgenson and associates had found was what they had assumed in the first place and that in fact the data they had used showed the elasticity of substitution to be closer to zero. We re-estimated these elasticities for the period 1947-1960 not to rekindle the debate, which in any way will continue for as long as these elasticities elude the research efforts in this area. We run this experiment in the hope that our results may contribute to the narrowing of the uncertainty that surrounds the true size of these crucial elasticities for policy applications. From our tests it has emerged that, even in short period data samples that the replacement rate may be approximated by a constant, because then the traditional conceptualization of the use cost performs well, the elasticities are closer to the ones reported above.

## 5. Overall assessment

We tested two investment models of neoclassical decent using data from the U.S private economy for the period 1947-2015. Model A was based on the conceptualization that business firms have an active replacement investment policy, which renders the replacement rate $\delta$ a determinant of business investment behavior, whereas Model B was based on the traditional hypothesis
that replacement investment is an engineering proportion of the capital stock, thus turning $\delta$ into a constant. The evidence that emerged from the estimations is in heavily in favor of Model A on at least four grounds. Namely, first it establishes that the replacement rate is a decisive determinant of investment at all levels of aggregation; Second, it leads to estimates of investment equations with succinct short run and long run dynamics, thus facilitating policy applications; Third, it gives rise to remarkably robust estimates of the elasticities of substitution, gross value added and replacement; and lastly, the estimates obtained are very closed to those under the approximation of the expression of the user cost as per equation (2) in the introduction.

Moreover, upon contrasting Model B to the one that Jorgenson, Stephenson (1967) estimated using quarterly data for the period 1948-1960 and from the same source, we found that it performed as expected, most likely because in short periods $\delta$ remains fairly constant due to long swings in replacement investment. But it gave rise to estimates of elasticities of substitution and output that differ significantly from theirs.

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## Appendix A

Table A-1: Series entering in the computation of the user cost

| Years | Variables ${ }^{1,2}$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1947 | 0.002 | 0.006 | 0.001 | 0.010 | 0.075 | 0.080 | 0.007 | 0.145 | 10.7 | 0.001 | 0.138 | 0.034 |
| 1948 | 0.003 | 0.008 | 0.001 | 0.012 | 0.082 | 0.081 | 0.008 | 0.159 | 10.2 | 0.001 | 0.193 | 0.029 |
| 1949 | 0.003 | 0.009 | 0.001 | 0.013 | 0.087 | 0.076 | 0.008 | 0.166 | 10.5 | 0.001 | 0.199 | 0.032 |
| 1950 | 0.003 | 0.010 | 0.002 | 0.014 | 0.094 | 0.072 | 0.008 | 0.176 | 11.3 | 0.001 | 0.205 | 0.037 |
| 1951 | 0.003 | 0.011 | 0.002 | 0.016 | 0.101 | 0.069 | 0.008 | 0.186 | 12.1 | 0.001 | 0.226 | 0.038 |
| 1952 | 0.003 | 0.012 | 0.002 | 0.017 | 0.108 | 0.064 | 0.009 | 0.194 | 12.9 | 0.002 | 0.262 | 0.043 |
| 1953 | 0.004 | 0.013 | 0.002 | 0.019 | 0.115 | 0.062 | 0.011 | 0.204 | 11.8 | 0.002 | 0.235 | 0.048 |
| 1954 | 0.004 | 0.014 | 0.002 | 0.021 | 0.121 | 0.058 | 0.012 | 0.210 | 15.8 | 0.002 | 0.344 | 0.063 |
| 1955 | 0.004 | 0.015 | 0.003 | 0.022 | 0.129 | 0.058 | 0.013 | 0.217 | 18.9 | 0.002 | 0.532 | 0.068 |
| 1956 | 0.005 | 0.016 | 0.003 | 0.024 | 0.138 | 0.057 | 0.014 | 0.229 | 17.2 | 0.003 | 0.476 | 0.064 |
| 1957 | 0.005 | 0.018 | 0.004 | 0.026 | 0.148 | 0.058 | 0.016 | 0.240 | 13.7 | 0.003 | 0.379 | 0.064 |
| 1958 | 0.005 | 0.019 | 0.004 | 0.028 | 0.153 | 0.055 | 0.017 | 0.242 | 17.4 | 0.004 | 0.432 | 0.078 |
| 1959 | 0.006 | 0.020 | 0.004 | 0.030 | 0.158 | 0.054 | 0.018 | 0.247 | 18.6 | 0.003 | 0.592 | 0.063 |
| 1960 | 0.006 | 0.020 | 0.005 | 0.031 | 0.162 | 0.054 | 0.019 | 0.251 | 17.6 | 0.003 | 0.560 | 0.066 |
| 1961 | 0.006 | 0.021 | 0.005 | 0.033 | 0.167 | 0.054 | 0.021 | 0.255 | 22.0 | 0.004 | 0.730 | 0.085 |
| 1962 | 0.007 | 0.022 | 0.006 | 0.035 | 0.172 | 0.055 | 0.022 | 0.263 | 18.6 | 0.005 | 0.744 | 0.100 |
| 1963 | 0.007 | 0.024 | 0.007 | 0.037 | 0.177 | 0.057 | 0.024 | 0.269 | 21.0 | 0.005 | 0.930 | 0.109 |
| 1964 | 0.008 | 0.025 | 0.007 | 0.040 | 0.184 | 0.060 | 0.026 | 0.280 | 22.8 | 0.006 | 1.128 | 0.133 |
| 1965 | 0.008 | 0.028 | 0.008 | 0.043 | 0.195 | 0.068 | 0.028 | 0.300 | 23.7 | 0.008 | 1.379 | 0.156 |
| 1966 | 0.009 | 0.030 | 0.009 | 0.047 | 0.206 | 0.075 | 0.031 | 0.322 | 19.7 | 0.010 | 1.230 | 0.168 |
| 1967 | 0.009 | 0.033 | 0.010 | 0.052 | 0.215 | 0.082 | 0.035 | 0.339 | 21.8 | 0.012 | 1.327 | 0.186 |
| 1968 | 0.010 | 0.036 | 0.011 | 0.056 | 0.224 | 0.087 | 0.037 | 0.355 | 22.3 | 0.013 | 1.384 | 0.180 |
| 1969 | 0.010 | 0.039 | 0.012 | 0.061 | 0.233 | 0.094 | 0.041 | 0.378 | 17.3 | 0.018 | 1.014 | 0.229 |
| 1970 | 0.011 | 0.042 | 0.013 | 0.066 | 0.244 | 0.097 | 0.042 | 0.390 | 15.9 | 0.023 | 0.817 | 0.257 |
| 1971 | 0.012 | 0.045 | 0.014 | 0.071 | 0.251 | 0.101 | 0.044 | 0.404 | 16.6 | 0.025 | 1.038 | 0.293 |
| 1972 | 0.013 | 0.049 | 0.015 | 0.077 | 0.259 | 0.107 | 0.046 | 0.416 | 18.7 | 0.027 | 1.397 | 0.330 |
| 1973 | 0.014 | 0.054 | 0.017 | 0.085 | 0.271 | 0.118 | 0.048 | 0.442 | 13.5 | 0.033 | 1.125 | 0.398 |
| 1974 | 0.015 | 0.060 | 0.018 | 0.094 | 0.282 | 0.128 | 0.050 | 0.466 | 8.3 | 0.045 | 0.605 | 0.471 |
| 1975 | 0.017 | 0.066 | 0.020 | 0.102 | 0.286 | 0.131 | 0.053 | 0.476 | 10.3 | 0.052 | 0.895 | 0.487 |
| 1976 | 0.018 | 0.072 | 0.022 | 0.112 | 0.291 | 0.137 | 0.056 | 0.487 | 11.6 | 0.050 | 1.266 | 0.509 |
| 1977 | 0.020 | 0.081 | 0.025 | 0.125 | 0.296 | 0.147 | 0.059 | 0.509 | 9.7 | 0.058 | 1.271 | 0.647 |
| 1978 | 0.022 | 0.092 | 0.027 | 0.141 | 0.311 | 0.164 | 0.064 | 0.539 | 9.0 | 0.065 | 1.385 | 0.684 |
| 1979 | 0.024 | 0.106 | 0.031 | 0.160 | 0.332 | 0.182 | 0.069 | 0.583 | 8.8 | 0.077 | 1.391 | 0.721 |
| 1980 | 0.027 | 0.119 | 0.035 | 0.182 | 0.356 | 0.188 | 0.075 | 0.620 | 9.4 | 0.107 | 1.281 | 0.781 |
| 1981 | 0.032 | 0.133 | 0.040 | 0.205 | 0.386 | 0.194 | 0.082 | 0.669 | 7.8 | 0.140 | 1.278 | 0.874 |
| 1982 | 0.037 | 0.149 | 0.046 | 0.231 | 0.407 | 0.193 | 0.089 | 0.684 | 8.5 | 0.161 | 1.384 | 0.998 |
| 1983 | 0.041 | 0.162 | 0.052 | 0.255 | 0.408 | 0.194 | 0.094 | 0.692 | 9.8 | 0.162 | 1.955 | 1.198 |
| 1984 | 0.045 | 0.177 | 0.060 | 0.281 | 0.416 | 0.207 | 0.104 | 0.716 | 9.6 | 0.193 | 2.307 | 1.360 |
| 1985 | 0.049 | 0.195 | 0.068 | 0.312 | 0.431 | 0.216 | 0.113 | 0.755 | 11.7 | 0.204 | 2.990 | 1.603 |
| 1986 | 0.053 | 0.211 | 0.077 | 0.340 | 0.429 | 0.224 | 0.122 | 0.762 | 14.1 | 0.213 | 3.025 | 2.045 |
| 1987 | 0.056 | 0.227 | 0.085 | 0.368 | 0.426 | 0.230 | 0.127 | 0.757 | 13.4 | 0.215 | 3.153 | 2.030 |
| 1988 | 0.059 | 0.242 | 0.094 | 0.395 | 0.420 | 0.242 | 0.136 | 0.767 | 14.7 | 0.233 | 4.018 | 2.152 |
| 1989 | 0.062 | 0.257 | 0.105 | 0.424 | 0.419 | 0.250 | 0.148 | 0.791 | 17.7 | 0.274 | 4.734 | 2.688 |
| 1990 | 0.066 | 0.271 | 0.117 | 0.453 | 0.420 | 0.256 | 0.157 | 0.804 | 15.9 | 0.265 | 4.306 | 2.555 |
| 1991 | 0.069 | 0.281 | 0.129 | 0.479 | 0.407 | 0.257 | 0.171 | 0.798 | 18.4 | 0.219 | 5.766 | 2.239 |
| 1992 | 0.071 | 0.291 | 0.141 | 0.503 | 0.394 | 0.262 | 0.184 | 0.793 | 20.5 | 0.189 | 6.679 | 2.105 |


| 1993 | 0.074 | 0.305 | 0.152 | 0.531 | 0.382 | 0.276 | 0.191 | 0.804 | 21.2 | 0.180 | 7.427 | 2.275 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1994 | 0.076 | 0.326 | 0.163 | 0.565 | 0.373 | 0.296 | 0.199 | 0.821 | 19.9 | 0.182 | 8.537 | 2.107 |
| 1995 | 0.079 | 0.351 | 0.175 | 0.605 | 0.371 | 0.325 | 0.213 | 0.857 | 25.0 | 0.183 | 12.145 | 2.228 |
| 1996 | 0.083 | 0.380 | 0.190 | 0.652 | 0.374 | 0.352 | 0.231 | 0.900 | 27.7 | 0.185 | 15.374 | 2.297 |
| 1997 | 0.087 | 0.410 | 0.210 | 0.706 | 0.380 | 0.384 | 0.254 | 0.953 | 33.0 | 0.216 | 20.489 | 2.749 |
| 1998 | 0.091 | 0.443 | 0.234 | 0.768 | 0.387 | 0.429 | 0.286 | 1.016 | 38.8 | 0.270 | 21.576 | 3.741 |
| 1999 | 0.096 | 0.478 | 0.263 | 0.836 | 0.396 | 0.474 | 0.316 | 1.086 | 44.2 | 0.271 | 25.260 | 3.447 |
| 2000 | 0.101 | 0.513 | 0.296 | 0.910 | 0.406 | 0.523 | 0.355 | 1.179 | 37.3 | 0.333 | 19.235 | 3.983 |
| 2001 | 0.107 | 0.539 | 0.325 | 0.971 | 0.416 | 0.547 | 0.369 | 1.227 | 30.5 | 0.321 | 16.799 | 4.033 |
| 2002 | 0.113 | 0.553 | 0.345 | 1.010 | 0.411 | 0.560 | 0.377 | 1.237 | 23.1 | 0.235 | 16.514 | 3.012 |
| 2003 | 0.117 | 0.563 | 0.360 | 1.040 | 0.405 | 0.565 | 0.387 | 1.247 | 26.6 | 0.207 | 21.648 | 3.064 |
| 2004 | 0.122 | 0.576 | 0.376 | 1.074 | 0.399 | 0.586 | 0.408 | 1.279 | 27.1 | 0.157 | 26.524 | 2.451 |
| 2005 | 0.129 | 0.596 | 0.394 | 1.119 | 0.398 | 0.624 | 0.423 | 1.318 | 26.4 | 0.234 | 28.169 | 3.853 |
| 2006 | 0.137 | 0.624 | 0.415 | 1.175 | 0.403 | 0.667 | 0.448 | 1.382 | 27.3 | 0.306 | 32.002 | 4.728 |
| 2007 | 0.148 | 0.655 | 0.437 | 1.240 | 0.418 | 0.708 | 0.473 | 1.467 | 26.0 | 0.385 | 28.128 | 5.943 |
| 2008 | 0.161 | 0.677 | 0.461 | 1.299 | 0.432 | 0.720 | 0.495 | 1.525 | 15.4 | 0.408 | 15.011 | 5.477 |
| 2009 | 0.172 | 0.675 | 0.481 | 1.328 | 0.429 | 0.678 | 0.500 | 1.490 | 20.3 | 0.301 | 22.913 | 4.123 |
| 2010 | 0.179 | 0.670 | 0.493 | 1.342 | 0.414 | 0.680 | 0.506 | 1.481 | 22.4 | 0.252 | 30.820 | 4.174 |
| 2011 | 0.187 | 0.682 | 0.507 | 1.375 | 0.402 | 0.697 | 0.525 | 1.503 | 20.5 | 0.268 | 29.498 | 4.737 |
| 2012 | 0.195 | 0.706 | 0.528 | 1.429 | 0.397 | 0.746 | 0.545 | 1.540 | 21.2 | 0.334 | 32.935 | 6.761 |
| 2013 | 0.206 | 0.737 | 0.550 | 1.493 | 0.397 | 0.784 | 0.554 | 1.581 | 24.9 | 0.337 | 38.911 | 6.598 |
| 2014 | 0.217 | 0.771 | 0.575 | 1.563 | 0.400 | 0.836 | 0.577 | 1.645 | 26.8 | 0.365 | 43.384 | 7.530 |
| 2015 | 0.227 | 0.808 | 0.603 | 1.638 | 0.401 | 0.875 | 0.602 | 1.708 | 26.0 | 0.376 | 39.846 | 7.528 |

## Notes

1. Depreciation of nonresidential structures at historical-cost prices, BEA, Table 2.6, Line 36.
2. Depreciation of nonresidential equipment at historical-cost prices, BEA, Table 2.6, Line 3.
3. Depreciation of nonresidential intangibles at historical-cost prices, BEA, Table 2.6, Line 77.
4. Column $4=$ Columns $(1+2+3)$.
5. Replacement of nonresidential structures at constant historical-cost prices, computed as the product of the replacement rate of structures times the capital stock in nonresidential structures.
6. Replacement of nonresidential equipment at constant historical-cost prices, computed as the product of the replacement rate of equipment times the capital stock in nonresidential equipment.
7. Replacement of nonresidential intangibles at constant historical-cost prices, computed as the product of the replacement rate of intangibles times the capital stock in nonresidential intangibles.
8. Replacement of nonresidential business investment at constant historical-cost prices, computed as the product of the replacement rate of business investment times the capital stock in nonresidential business investment.
9. Price-earnings ratio from Standard and Poor's (S\&P) 500 Index Data including Dividend, Earnings and P/E Ratio, http://data.okfn.org/data/core/s-and-p-500.
10. Net monetary interest BEA, Table 7.11, Line 99.
11. Value of equity computed as the ratio of corporate profits after taxes without inventory valuation and capital consumption adjustment to the earnings-price ratio.
12. Value of debt is estimated as the ratio of net monetary interest to the Moody's seasoned Baa bond yield.

Table A-1: Continued from above

| Years | Variables ${ }^{1,2}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 1947 | 0.002 | 0.026 | 0.034 | 0.013 | 0.082 | 0.052 | 0.622 | 0.534 | 0.196 | 0.073 | 0.628 | 0.601 | 0.265 |
| 1948 | 0.002 | 0.034 | 0.040 | 0.015 | 0.090 | 0.057 | 0.685 | 0.528 | 0.213 | 0.078 | 0.634 | 0.592 | 0.270 |
| 1949 | 0.002 | 0.043 | 0.043 | 0.017 | 0.087 | 0.058 | 0.735 | 0.521 | 0.224 | 0.077 | 0.659 | 0.597 | 0.277 |
| 1950 | 0.002 | 0.053 | 0.045 | 0.018 | 0.080 | 0.065 | 0.838 | 0.585 | 0.258 | 0.085 | 0.738 | 0.670 | 0.305 |
| 1951 | 0.003 | 0.067 | 0.050 | 0.021 | 0.075 | 0.083 | 1.011 | 0.670 | 0.323 | 0.106 | 0.876 | 0.781 | 0.360 |
| 1952 | 0.003 | 0.079 | 0.052 | 0.022 | 0.071 | 0.082 | 0.961 | 0.646 | 0.318 | 0.102 | 0.833 | 0.738 | 0.346 |
| 1953 | 0.003 | 0.089 | 0.053 | 0.023 | 0.077 | 0.087 | 0.960 | 0.682 | 0.332 | 0.104 | 0.834 | 0.750 | 0.350 |
| 1954 | 0.003 | 0.105 | 0.055 | 0.025 | 0.059 | 0.079 | 0.883 | 0.642 | 0.309 | 0.094 | 0.779 | 0.694 | 0.327 |
| 1955 | 0.003 | 0.115 | 0.059 | 0.026 | 0.051 | 0.081 | 0.875 | 0.648 | 0.313 | 0.094 | 0.774 | 0.699 | 0.327 |
| 1956 | 0.003 | 0.132 | 0.062 | 0.029 | 0.056 | 0.089 | 0.905 | 0.662 | 0.336 | 0.100 | 0.801 | 0.700 | 0.340 |
| 1957 | 0.003 | 0.151 | 0.066 | 0.031 | 0.069 | 0.099 | 0.952 | 0.693 | 0.368 | 0.108 | 0.847 | 0.718 | 0.356 |
| 1958 | 0.003 | 0.175 | 0.070 | 0.034 | 0.056 | 0.096 | 0.933 | 0.698 | 0.365 | 0.105 | 0.844 | 0.723 | 0.353 |
| 1959 | 0.003 | 0.187 | 0.075 | 0.036 | 0.053 | 0.100 | 0.943 | 0.714 | 0.379 | 0.107 | 0.868 | 0.740 | 0.361 |
| 1960 | 0.004 | 0.194 | 0.079 | 0.037 | 0.056 | 0.100 | 0.928 | 0.717 | 0.379 | 0.107 | 0.866 | 0.741 | 0.360 |
| 1961 | 0.004 | 0.203 | 0.081 | 0.038 | 0.046 | 0.098 | 0.894 | 0.713 | 0.371 | 0.105 | 0.847 | 0.737 | 0.356 |
| 1962 | 0.004 | 0.209 | 0.083 | 0.040 | 0.053 | 0.095 | 0.843 | 0.687 | 0.358 | 0.100 | 0.810 | 0.712 | 0.343 |
| 1963 | 0.004 | 0.211 | 0.085 | 0.041 | 0.048 | 0.094 | 0.827 | 0.680 | 0.352 | 0.099 | 0.792 | 0.704 | 0.339 |
| 1964 | 0.004 | 0.212 | 0.088 | 0.042 | 0.044 | 0.092 | 0.796 | 0.665 | 0.343 | 0.096 | 0.763 | 0.688 | 0.330 |
| 1965 | 0.004 | 0.207 | 0.088 | 0.044 | 0.043 | 0.091 | 0.784 | 0.638 | 0.335 | 0.095 | 0.729 | 0.660 | 0.322 |
| 1966 | 0.004 | 0.204 | 0.088 | 0.045 | 0.051 | 0.099 | 0.809 | 0.664 | 0.355 | 0.102 | 0.742 | 0.673 | 0.330 |
| 1967 | 0.005 | 0.210 | 0.090 | 0.048 | 0.048 | 0.103 | 0.832 | 0.680 | 0.365 | 0.105 | 0.754 | 0.676 | 0.337 |
| 1968 | 0.005 | 0.220 | 0.095 | 0.051 | 0.048 | 0.114 | 0.877 | 0.721 | 0.397 | 0.117 | 0.795 | 0.715 | 0.357 |
| 1969 | 0.005 | 0.227 | 0.101 | 0.055 | 0.061 | 0.125 | 0.912 | 0.761 | 0.425 | 0.126 | 0.818 | 0.741 | 0.371 |
| 1970 | 0.006 | 0.247 | 0.115 | 0.060 | 0.070 | 0.136 | 0.916 | 0.761 | 0.444 | 0.136 | 0.838 | 0.756 | 0.385 |
| 1971 | 0.007 | 0.263 | 0.126 | 0.065 | 0.066 | 0.133 | 0.861 | 0.721 | 0.424 | 0.132 | 0.787 | 0.718 | 0.371 |
| 1972 | 0.007 | 0.275 | 0.133 | 0.071 | 0.059 | 0.130 | 0.804 | 0.686 | 0.402 | 0.129 | 0.733 | 0.682 | 0.357 |
| 1973 | 0.008 | 0.279 | 0.146 | 0.077 | 0.076 | 0.132 | 0.770 | 0.665 | 0.394 | 0.130 | 0.694 | 0.662 | 0.346 |
| 1974 | 0.010 | 0.306 | 0.167 | 0.088 | 0.109 | 0.146 | 0.785 | 0.673 | 0.417 | 0.142 | 0.700 | 0.670 | 0.356 |
| 1975 | 0.012 | 0.377 | 0.189 | 0.107 | 0.101 | 0.159 | 0.845 | 0.696 | 0.455 | 0.154 | 0.758 | 0.680 | 0.384 |
| 1976 | 0.013 | 0.420 | 0.203 | 0.121 | 0.089 | 0.152 | 0.828 | 0.684 | 0.441 | 0.147 | 0.729 | 0.654 | 0.371 |
| 1977 | 0.015 | 0.466 | 0.223 | 0.137 | 0.099 | 0.149 | 0.804 | 0.651 | 0.431 | 0.143 | 0.695 | 0.622 | 0.359 |
| 1978 | 0.018 | 0.502 | 0.242 | 0.156 | 0.106 | 0.157 | 0.814 | 0.651 | 0.439 | 0.149 | 0.684 | 0.610 | 0.359 |
| 1979 | 0.021 | 0.553 | 0.267 | 0.177 | 0.112 | 0.172 | 0.834 | 0.658 | 0.461 | 0.160 | 0.685 | 0.605 | 0.369 |
| 1980 | 0.024 | 0.657 | 0.303 | 0.205 | 0.118 | 0.200 | 0.894 | 0.713 | 0.521 | 0.185 | 0.748 | 0.646 | 0.410 |
| 1981 | 0.029 | 0.761 | 0.342 | 0.235 | 0.141 | 0.233 | 0.920 | 0.746 | 0.574 | 0.211 | 0.782 | 0.667 | 0.440 |
| 1982 | 0.034 | 0.892 | 0.382 | 0.274 | 0.136 | 0.235 | 0.859 | 0.739 | 0.558 | 0.210 | 0.743 | 0.648 | 0.434 |
| 1983 | 0.037 | 0.969 | 0.425 | 0.298 | 0.115 | 0.204 | 0.749 | 0.702 | 0.499 | 0.180 | 0.651 | 0.611 | 0.387 |
| 1984 | 0.040 | 0.984 | 0.449 | 0.319 | 0.118 | 0.209 | 0.737 | 0.734 | 0.503 | 0.185 | 0.642 | 0.624 | 0.389 |
| 1985 | 0.044 | 1.040 | 0.483 | 0.339 | 0.100 | 0.199 | 0.660 | 0.701 | 0.474 | 0.173 | 0.579 | 0.592 | 0.365 |
| 1986 | 0.048 | 1.106 | 0.509 | 0.373 | 0.084 | 0.194 | 0.588 | 0.692 | 0.456 | 0.167 | 0.518 | 0.576 | 0.350 |
| 1987 | 0.052 | 1.166 | 0.552 | 0.410 | 0.087 | 0.190 | 0.566 | 0.658 | 0.436 | 0.165 | 0.509 | 0.565 | 0.343 |
| 1988 | 0.058 | 1.198 | 0.586 | 0.445 | 0.082 | 0.182 | 0.547 | 0.633 | 0.413 | 0.160 | 0.490 | 0.544 | 0.335 |
| 1989 | 0.063 | 1.254 | 0.607 | 0.474 | 0.073 | 0.173 | 0.491 | 0.598 | 0.387 | 0.152 | 0.445 | 0.510 | 0.322 |
| 1990 | 0.069 | 1.320 | 0.643 | 0.509 | 0.078 | 0.174 | 0.482 | 0.568 | 0.381 | 0.154 | 0.446 | 0.500 | 0.315 |
| 1991 | 0.076 | 1.385 | 0.670 | 0.553 | 0.066 | 0.159 | 0.458 | 0.541 | 0.351 | 0.144 | 0.430 | 0.475 | 0.297 |
| 1992 | 0.082 | 1.407 | 0.672 | 0.583 | 0.059 | 0.149 | 0.428 | 0.515 | 0.325 | 0.136 | 0.406 | 0.451 | 0.280 |
| 1993 | 0.089 | 1.382 | 0.702 | 0.608 | 0.055 | 0.142 | 0.395 | 0.483 | 0.303 | 0.132 | 0.372 | 0.432 | 0.263 |
| 1994 | 0.098 | 1.375 | 0.726 | 0.638 | 0.057 | 0.145 | 0.406 | 0.469 | 0.304 | 0.138 | 0.382 | 0.432 | 0.265 |
| 1995 | 0.107 | 1.334 | 0.748 | 0.661 | 0.046 | 0.141 | 0.392 | 0.460 | 0.289 | 0.135 | 0.359 | 0.422 | 0.255 |
| 1996 | 0.113 | 1.301 | 0.750 | 0.674 | 0.042 | 0.139 | 0.377 | 0.449 | 0.277 | 0.133 | 0.344 | 0.411 | 0.247 |
| 1997 | 0.121 | 1.251 | 0.758 | 0.685 | 0.036 | 0.135 | 0.355 | 0.433 | 0.260 | 0.130 | 0.322 | 0.395 | 0.238 |


| 1998 | 0.131 | 1.155 | 0.748 | 0.688 | 0.033 | 0.132 | 0.328 | 0.428 | 0.240 | 0.128 | 0.288 | 0.376 | 0.229 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1999 | 0.139 | 1.085 | 0.774 | 0.694 | 0.029 | 0.137 | 0.327 | 0.432 | 0.239 | 0.133 | 0.288 | 0.381 | 0.231 |
| 2000 | 0.149 | 1.034 | 0.800 | 0.700 | 0.037 | 0.146 | 0.330 | 0.456 | 0.248 | 0.143 | 0.291 | 0.390 | 0.234 |
| 2001 | 0.162 | 1.008 | 0.850 | 0.716 | 0.042 | 0.147 | 0.320 | 0.430 | 0.240 | 0.144 | 0.290 | 0.382 | 0.220 |
| 2002 | 0.179 | 0.991 | 0.872 | 0.735 | 0.049 | 0.150 | 0.316 | 0.413 | 0.235 | 0.148 | 0.292 | 0.379 | 0.212 |
| 2003 | 0.196 | 0.982 | 0.885 | 0.750 | 0.041 | 0.143 | 0.286 | 0.385 | 0.216 | 0.143 | 0.273 | 0.361 | 0.199 |
| 2004 | 0.223 | 0.968 | 0.875 | 0.765 | 0.039 | 0.146 | 0.275 | 0.375 | 0.210 | 0.147 | 0.265 | 0.350 | 0.193 |
| 2005 | 0.265 | 0.943 | 0.894 | 0.796 | 0.041 | 0.153 | 0.262 | 0.351 | 0.200 | 0.155 | 0.252 | 0.336 | 0.188 |
| 2006 | 0.314 | 0.919 | 0.902 | 0.822 | 0.040 | 0.168 | 0.258 | 0.349 | 0.202 | 0.170 | 0.248 | 0.334 | 0.192 |
| 2007 | 0.347 | 0.913 | 0.917 | 0.834 | 0.043 | 0.173 | 0.251 | 0.344 | 0.201 | 0.174 | 0.241 | 0.328 | 0.191 |
| 2008 | 0.382 | 0.928 | 0.938 | 0.854 | 0.068 | 0.192 | 0.259 | 0.357 | 0.215 | 0.191 | 0.253 | 0.342 | 0.199 |
| 2009 | 0.401 | 0.996 | 0.961 | 0.892 | 0.053 | 0.179 | 0.246 | 0.341 | 0.204 | 0.179 | 0.250 | 0.333 | 0.195 |
| 2010 | 0.428 | 0.966 | 0.979 | 0.899 | 0.047 | 0.161 | 0.225 | 0.318 | 0.184 | 0.163 | 0.231 | 0.318 | 0.179 |
| 2011 | 0.473 | 0.967 | 0.985 | 0.920 | 0.050 | 0.156 | 0.216 | 0.312 | 0.178 | 0.161 | 0.225 | 0.312 | 0.174 |
| 2012 | 0.521 | 0.945 | 0.998 | 0.948 | 0.047 | 0.147 | 0.208 | 0.299 | 0.166 | 0.154 | 0.214 | 0.298 | 0.166 |
| 2013 | 0.560 | 0.941 | 1.031 | 0.973 | 0.042 | 0.148 | 0.207 | 0.290 | 0.164 | 0.156 | 0.213 | 0.296 | 0.166 |
| 2014 | 0.608 | 0.926 | 1.044 | 0.992 | 0.039 | 0.147 | 0.205 | 0.285 | 0.161 | 0.156 | 0.208 | 0.290 | 0.162 |
| 2015 | 0.636 | 0.936 | 1.057 | 1.008 | 0.040 | 0.149 | 0.208 | 0.286 | 0.163 | 0.159 | 0.211 | 0.292 | 0.163 |

Notes:
13. Proportion of replacement cost allowable for tax purposes regarding structures.
14. Proportion of replacement cost allowable for tax purposes regarding equipment.
15. Proportion of replacement cost allowable for tax purposes regarding intangibles.
16. Proportion of replacement cost allowable for tax purposes regarding business investment.
17. Proportion of total cost of capital allowable for tax purposes.
18. User cost of structures with variable rate of replacement.
19. User cost of equipment with variable rate of replacement.
20. User cost of intangibles with variable rate of replacement.
21. User cost of investment with variable rate of replacement.
22. User cost of structures with constant rate of replacement.
23. User cost of equipment with constant rate of replacement.
24. User cost of intangibles with constant rate of replacement.
25. User cost of investment with constant rate of replacement.


[^0]:    ${ }^{1}$ We have chosen to limit our attention to average ages at historical prices because in all experiments they performed better that the average ages at current prices by reference to standard statistical criteria.
    ${ }_{3}^{2}$ See the U.S. Department of Commerce, Bureau of Economic Analysis (2003).
    ${ }^{3}$ See in this regard Haavelmo (1960), Zarembka (1975), Brown, Chang (1976), and Bitros (2009) more recently.

[^1]:    ${ }^{4}$ For this equation the method of estimation which is suggested as appropriate is the Autoregressive Distributed Lag Model (ADLM). However, as it will become evident later on, the results from the two models of estimation are fairly similar.

